Semi-Generalized Closed Mappings and Generalized-Semi Closed Mappings and Its Relationships with Semi-Normal and Semi-Regular Spaces

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Abstract

In this paper, the semi-generalized closed maps (sg-closed maps) and generalized semi-closed maps (gs-closed maps) are studied and some results are presented and proved including the study of some of their basic properties which are related to such type of mappings. Also, a study of s-normal spaces and s-regular spaces is given using its relationships with sg-closed and gs-closed mappings.

1- Introduction

P. Bahtharyya and P. K. Lahiri [2] introduced the concept of semi-generalized closed sets (sg-closed sets) and investigated some of their properties and semi-$T_{1/2}$ spaces.

S. P. Arya and T. Nour [1] defined the generalized semi-open sets (gs-open sets) and studied some of their properties and characterizations of s-normal spaces by using semi-open sets.

Recently, P. Sundaram introduced the concept of semi-generalized continuous map and generalized semi-continuous.

In this paper, the study of sg and gs-closed maps and some of their basic properties are given, then its relationship with s-normal and s-regular spaces are also presented as the main results.

2- Preliminaries

In this section, the basic definitions and concepts related to this paper are given for completeness:

Definition (2.1), [3]:

A subset of a topological space (X, τ) is said to be semi-open set, if there exists an open set U of X such that $U \subset S \subset cl(U)$, where cl refers to the closure.

It is remarkable that the complement of semi-open set is said to be semi-closed set. The semi-closure of subset A of (X, τ) denoted by $scl_X(A)$ or briefly scl(A), is defined to be the intersection of all semi-closed sets containing A.

Definition (2.2), [2]:

A subset A of (X, τ) is said to be semi-generalized closed (written in short as sg-closed) in (X, τ) if $scl(A) \subseteq O$, whenever $A \subseteq O$ and O is semi-open in (X, τ).

Also, a subset B is said to be semi-generalized open (written as sg-open) in (X, τ) if its complement X–B is sg-closed in (X, τ).

$A \subseteq X$ is called sg-closed in X if and only if for all U semi-open set in X, $A \subseteq U \longrightarrow scl(A) \subseteq U$.

Following are some of the basic well known results and remarks concerning gs-closed sets:

1. The complement of sg-closed is called sg-open.
2. A set A is sg-closed if and only if scl(A) – A contains no non-empty semi-closed.
3. Let A be sg-closed, then A is semi-closed if and only if scl(A) – A is semi-closed.
4. If A is sg-closed and $A \subset B \subset scl(A)$, then B is sg-closed.
5. Every s-closed set is sg-closed, but the converse is not true.
6. g-closed and sg-closed are in general independent and every semi-closed set is sg-closed.

Now, we consider the other type of topological sets concerning this paper.

Definition (2.3), [1]:

A subset A of (X, τ) is said to be generalized semi-open (written as gs-open) in (X, τ) if $F \subset sint(A)$, whenever $F \subset A$ and F is closed in (X, τ). A subset B is generalized semi-closed (written as gs-closed) if its complement X–B is gs-open in (X, τ).

The following results appeared in [1] and [2] which are given here for completeness:

Theorem (2.4):

A subset A of (X, τ) is gs-closed in X if and only if $scl(A) \subset U$, whenever $A \subset U$ and U is open in (X, τ).

Proposition (2.5):

If A is an open and gs-closed set of (X, τ), then A is semi-closed set.

Proposition (2.6):

Let $F \subseteq A \subseteq X$, where A is an open set in X and also gs-closed in X. If F is gs-closed set in A, then F is gs-closed set in X.

Proposition (2.7):

Let $F \subseteq A \subseteq X$, where A is an open in X and if F is gs-closed set in X, then F is gs-closed set in A.

3- Semi-Generalized And Generalized Semi-Closed Maps

In this section, we will give the definition of semi-generalized closed maps and generalized semi-closed maps and some related results.

Definition (3.1), [7]:

A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be semi-closed if for any closed set F o X, f(F) is semi-closed in Y.

Definition (3.2), [6]:

A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is said to be g-closed if for any closed set of X, f(F) is g-closed in Y.

Definition (3.3), [8]:

A map $f : X \longrightarrow Y$ is called a generalized semi-closed map (written as gs-closed map) if for each closed set for X, f(F) is gs-closed set of Y.

Next, we give some of the obtained results concerning the mappings of this section:

Theorem (3.4):

A map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is sg-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$, there is a sg-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof:

(Necessity) Let S be a subset of Y and U be an open set of X, such that $f^{-1}(S) \subset U$.

Then $Y - f(X - U)$, say V, is a sg-open set containing S, such that $f^{-1}(V) \subseteq U$.

(Sufficiency) Let F be a closed set of X, then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F$ is open
By hypothesis, there is a sg-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X \subset X - F$
Therefore, we have $F \subset X - f^{-1}(V)$ and hence:

$$Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$$

Which implies $f(F) = Y - V$, since $Y - V$ is sg-closed
f(F) is sg-closed and thus f is a sg-closed map.

Theorem (3.5):

A mapping $f : (X, \tau) \longrightarrow (Y, \sigma)$ is gs-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$, there exists a gs-open set V of Y containing S and $f^{-1}(V) \subset U$.

Proof:

(Necessity) Let S be a subset of Y and U be an open set of X, such that $f^{-1}(S) \subset U$
Then $Y - f(X - U)$, say V, is a gs-open set containing S such that $f^{-1}(V) \subset U$

(Sufficiency) Let F be a closed set of X, we claim that f(F) is gs-closed in Y, that is, $f^{-1}(Y - f(F)) \subset X - F$

By taking $S = Y - f(F)$ and $U = X - F$ in hypothesis there exists a gs-open set V of Y containing $Y - f(F)$ and $f^{-1}(V) \subset X - F$
Then we have $F \subset X - f^{-1}(V)$ and $Y - V = f(F)$
Since $Y - V$ is gs-closed, f(F) is gs-closed and thus f is a gs-closed map.

The main results of this paper are given in the next section:

4- S-Normal And S-Regular Spaces

In this section, we study s-normal and s-regular spaces and we give sufficient conditions on $f : (X, \tau) \longrightarrow (Y, \sigma)$ so that f preserved s-normality and s-regularity.

First, recall the following definitions:

Definition (4.1), [5]:

Let (X, τ) be a topological space, then X is s-regular if and only if given a closed set $F \subseteq X$ and $x \notin F$, then there exists two semi-open sets W_1 and W_2 such that $x \in W_1$, $F \subseteq W_2$ and $W_1 \cap W_2 = \varnothing$.

Definition (4.2), [5]:

Let (X, τ) be a topological space, we say that (X, τ) is s-normal if and only if given two disjoint closed sets, F_1 and F_2 in X, then there exists two disjoint semi-open sets, W_1 and W_2 such that $F_1 \subseteq W_1$, $F_2 \subseteq W_2$.

Theorem (4.3):

If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a continuous and onto gs-closed map from a normal space (X, τ) to a space (Y, σ), then (Y, σ) is s-normal.

Proof:

Let A and B be disjoint closed sets of Y
Since X is normal, then there exist disjoint open sets U and V in X such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$ (by theorem (3.5))
Then there exist gs-open sets G and H in Y such that $f^{-1}(G) \subseteq U$, $f^{-1}(H) \subseteq V$ and $f^{-1}(G) \cap f^{-1}(H) = \varnothing$
Hence $G \cap H = \varnothing$, since G is gs-open and A is closed
$G \supseteq A$ implies that $s\ int(G) \supseteq A$
Similarly, $s\ int\ (H) \supseteq B$
Hence $s\ int\ (G) \cap s\ int\ (H) = G \cap H = \varnothing$
Therefore, Y is s-normal.

The next corollary is given in [7], which may be considered as a result of the above theorem:

Corollary (4.5):

(i) Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a continuous semi-closed onto mapping, if (X, τ) is normal then (Y, σ) is s-normal.
(ii) Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a continuous, sg-closed onto mapping, if (X, τ) is normal then (Y, σ) is s-normal.

Proof:

(i) Since f is semi-closed and then f is gs-closed
Then by theorem (4.3), we get that Y is s-normal.
(ii) Since f is sg-closed, then f is gs-closed
Then by theorem (4.3), we get that Y is s-normal.

Theorem (4.6):

If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is a continuous semi-open and gs-closed onto mapping from a regular space (X, τ) to a space (Y, σ), then (Y, σ) is s-regular.

Proof:

Let $y \in Y$, let U be an open set containing y in Y
F is onto, then there exists $x \in X$ such that $f(x) = y$
Now, $f^{-1}(U)$ is an open set in X containing x
But X is regular, then there exist an open set V such that:

$x \in V \subseteq cl(U) \subseteq f^{-1}(U)$
$y \in f(V) \subseteq f(cl(V)) \subset U$

But f(cl(V)) is gs-closed
Then we have $scl(f(cl(V)) \subset U$
Therefore, $Y \in f(V) \subset scl(f(V)) \subset U$ and f(V) is semi-open in Y (Because f is semi-open)
Hence Y is s-regular.

Corollary (4.7):

If $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a continuous, semi-open and sg-closed onto mapping, if (X, τ) is a regular space then (Y, σ) is s-regular.

Proof:

Since f is sg-closed, then f is gs-closed (by theorem (4.6))
Hence we get that Y is s-regular.

5- References

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الخلاصة

في هذا البحث، قمنا بدراسة التطبيقات شبه العامة المغلقة (sg-closed maps) والتطبيقات العامة شبه المغلقة (gs-closed maps)وا إعطاء بعض النتائج حولهما مع البرهان حيث تضمنت دراسة بعض من خواصهما الرئيسة المرافقة لهذا النوع من التطبيقات. كما وتمت دراسة الفضاءات شبه الطبيعية (s-normal) والفضاءات شبه المنتظمة (s-regular) بالاعتماد على علاقتها مع التطبيقات شبه العامة المغلقة و التطبيقات العامة شبه المغلقة.