

Reliable Algorithm of Homotopy Analysis Method for Solving System of Integral Equations

Osama H. Mohammed and Omar I. Khaleel

Department of Mathematics and Computer Applications, College of Science, Al-Nahrain University.

Abstract

This paper presents an approximate solution for systems of Volterra integral equations of the second kind using a reliable algorithm of homotopy analysis method. The solution is calculated in the form of convergent series with easily compatible components. The approach is tested for some examples and the results demonstrate reliability and efficiency of the proposed method.

1. Introduction

Differential equations, integral equations as well as combinations of them, integro-differential equations, are obtained in modeling real-life engineering phenomena that are inherently nonlinear with variable coefficients. Most of these types of equations do not have an analytical solution. Therefore, these problems should be solved by using numerical or semi-analytical techniques. In numerical methods, computer codes and more powerful processors are required to achieve accurate results. Acceptable results are obtained via semi-analytical methods which are more convenient than numerical methods. The main advantage of semi-analytical methods, compared with other methods, is based on the fact that they can be conveniently applied to solve various complicated problems. Several analytical methods including the linear superposition technique [1], the exp-function method [2], the Laplace decomposition method [3], the matrix exponential method [4], the homotopy perturbation method [5], variational iteration methods [6] and the Adomian decomposition method [7] have been developed for solving linear or nonlinear non-homogeneous partial differential equations. One of these semi-analytical solution methods is the Homotopy analysis method (HAM).

HAM was first proposed by Liao in 1997 by employing the basic ideas of homotopy analysis in topology to produce an analytical method for solving various nonlinear problems [8],[9],[10],[11],[12],[13]. This method has been successfully applied to solve different classes of nonlinear problems.

In this paper, a reliable algorithm of HAM which was given in [14] was used for solving

system of Volterra integral equations of the second kind, such as:

$$y(t) = f(t) + \int_0^t K(s,t,y(s))ds$$

where

$$y(t) = (y_1, y_2, \dots, y_n)^T,$$

$$f(t) = (f_1(t), f_2(t), \dots, f_n(t))^T,$$

$$K(s,t,y(s)) = (K_1(s,t,y(s)), K_2(s,t,y(s)), \dots, K_n(s,t,y(s)))^T$$

The kernel of the integral K and f are known as functions, y is an unknown function should be determined.

2. Basic Idea of Homotopy Analysis Method

We consider the following integral equation

$$N[y(t)] = 0, \dots\dots\dots(1)$$

where N is a nonlinear integral operator, t denotes the independent variable, y is an unknown function, respectively. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the traditional homotopy method, Liao in [11] constructed the so-called zero-order deformation equation

$$(1-q)L[\Phi(t;q) - u_0(t)] = qhH(t)N[\Phi(t;q)], \dots\dots\dots(2)$$

where $q \in [0,1]$ is the embedding parameter, $h \neq 0$ is a non-zero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, $y_0(t)$ is an initial guess of $y(t)$ and $\Phi(t;p)$ is an unknown function and L an auxiliary linear operator with the property

$$L[f(t)] = 0 \text{ when } f(t) = 0 \dots\dots\dots(3)$$

It is important, that one has a great freedom to choose auxiliary things in HAM. Obviously, when $q = 0$ and $q = 1$, it holds

$$\Phi(t;0) = y_0(t), \quad \Phi(t;1) = y(t), \dots\dots\dots(4)$$

respectively. Thus, as q increases from 0 to 1, the solution $\Phi(t;q)$ vary from the initial guess $y_0(t)$ to the solution $y(t)$. Expanding $\Phi(t;q)$ in Taylor series with respect to q , we have

$$\Phi(t;q) = y_0(t) + \sum_{m=1}^{+\infty} y_m(t)q^m, \dots\dots\dots(5)$$

where

$$y_m(t) = \left[\frac{1}{m!} \frac{\partial^m \Phi(t;q)}{\partial q^m} \right]_{q=0} \dots\dots\dots(6)$$

If the auxiliary linear operator, the initial guess, the auxiliary parameter h , and the auxiliary function are so properly chosen, the series (5) converges at $q = 1$, then we have

$$y(t) = y_0(t) + \sum_{m=1}^{+\infty} y_m(t), \dots\dots\dots(7)$$

which must be one of solutions of original nonlinear equation, as proved by [8]. As $h = -1$ and $H(t) = 1$, equation (2) becomes

$$(1-q)L[\Phi(t;q) - y_0(t)] + qN[\Phi(t;q)] = 0, \dots\dots\dots(8)$$

which is used mostly in the homotopy perturbation method [15], whereas the solution obtained directly, without using Taylor series [5]. According to the definition (5), the governing equation can be deduced from the zero-order deformation equation (2).

Define the vector field

$$\vec{y}_n(t) = \{y_0(t), y_1(t), \dots, y_n(t)\}$$

Differentiating equation (2) m -times with respect to the embedding parameter q and then setting $q = 0$ and finally dividing them by $m!$, we have the so-called m^{th} -order deformation equation

$$L[y_m(t) - X_m y_{m-1}(t)] = hH(t)R_m(\vec{y}_{m-1}), \dots\dots\dots(9)$$

where

$$R_m(\vec{y}_{m-1}) = \left[\frac{1}{(m-1)!} \frac{\partial^{m-1} N[\Phi(t;q)]}{\partial q^{m-1}} \right]_{q=0} \dots\dots(10)$$

and

$$X_m = \begin{cases} 0, & m \leq 1, \\ 1, & m > 1, \end{cases} \dots\dots\dots(11)$$

It should be emphasized that $y_m(t)$ for $m \geq 1$ is governed by the linear equation (8) under the linear boundary conditions that come from original problem, which can be easily solved by symbolic computation software such as MathCad.

3. The Reliable Algorithm of HAM

The homotopy analysis method provides an analytical solution which is applied to various nonlinear problems. This section relates reliable approach of the HAM have been given in [13].

To illustrate the basic idea of this approach, we consider the following nonlinear integral equation

$$y(t) = f(t) + N(y(t)), \quad t > 0 \dots\dots\dots(12)$$

where $N(y(t))$ is a nonlinear operator which include integer order integration and $f(t)$ is a known analytic function.

In view of the homotopy technique, we can construct the following homotopy

$$(1-q)L[\Phi(t,q) - \Phi_0(t)] = qhH[\Phi(t,q) - N(\Phi(t,q)) - f(t)] \dots\dots\dots(13)$$

where $q \in [0,1]$ in the embedding parameter $h \neq 0$ is a nonzero auxiliary parameter, $H(t) \neq 0$ is an auxiliary function, $\Phi_0(t)$ is an initial guess of $y(t)$ and L is an auxiliary linear operator that may be defined as $L[\Phi(t,q)] = \Phi(t,q)$.

When $q = 0$ equation (13) becomes

$$L[\Phi(t,0) - \Phi_0(t)] = 0 \dots\dots\dots(14)$$

which implies that $\Phi(t,0) = \Phi_0(t)$, and it is obvious when $q = 1$, equation (13) becomes the original nonlinear equation (12).

Thus as q vary from 0 to 1, the solution $\Phi(t,0)$ vary from the initial guess $\Phi_0(t)$ to the solution $\Phi(t)$.

The basic assumption of this approach is that the solution of equation (12) can be

expressed as a power series in q as:

$$\Phi = \Phi_0 + q\Phi_1 + q^2\Phi_2 + \dots \dots\dots(15)$$

substituting the series (15) into the homotopy (13) and then equating the coefficients of the like powers of q , we obtain the high-order deformation equations,

$$\left. \begin{aligned} L[\Phi_1] &= hH(\Phi_0(t) - N_0(\Phi_0) - f(t)), \\ L[\Phi_2] &= L[\Phi_1] + hH(\Phi_1 - N_1(\Phi_0, \Phi_1)), \\ L[\Phi_3] &= L[\Phi_2] + hH(\Phi_2 - N_2(\Phi_0, \Phi_1, \Phi_2)), \\ &\vdots \end{aligned} \right\} \dots\dots\dots(16)$$

and so on.

where

$$N(\Phi_0 + q\Phi_1 + q^2\Phi_2 + \dots) = N_0(\Phi_0) + qN_1(\Phi_0, \Phi_1) + q^2N_2(\Phi_0, \Phi_1, \Phi_2) + \dots$$

Using the definition of L and by seeking $h = -1, H(t) = 1$, equations (16) can be simplified as

$$\left. \begin{aligned} \Phi_1(t) &= -[\Phi_0(t) - N_0(\Phi_0) - f(t)], \\ \Phi_2 &= N_1(\Phi_0, \Phi_1), \\ \Phi_3 &= N_2(\Phi_0, \Phi_1, \Phi_2), \\ &\vdots \end{aligned} \right\} \dots\dots\dots(17)$$

and so on.

The approximate solution of equation (12), therefore, can be readily obtained as

$$y(t) = \lim_{q \rightarrow -1} \Phi(t, q) = \Phi_0(t) + \Phi_1(t) + \Phi_2(t) + \dots$$

the success of the technique is based on the proper selection of the initial guess $\Phi_0(t)$.

Hence

$$\begin{aligned} qL_i[\Phi_{i1}] + q^2L_i[\Phi_{i2} - \Phi_{i1}] + \dots &= qhH[\Phi_{i0} - N_{i0}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0})] + \\ q^2hH[\Phi_{i1} - N_{i1}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1})] + \\ q^3hH[\Phi_{i2} - N_{i2}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}, \Phi_{12}, \Phi_{22}, \dots, \Phi_{n2})] + \dots \end{aligned}$$

where

$$N_i(\Phi(t, q)) = N_{i0}(\Phi_{10}, \Phi_{20}, \Phi_{30}, \dots, \Phi_{n0}) + qN_{i1}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}) + q^2N_{i2}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}, \Phi_{12}, \Phi_{22}, \dots, \Phi_{n2}) + \dots$$

Equating the coefficients of the like powers of q , we obtain the higher-order deformation equations

$$\left. \begin{aligned} L_i[\Phi_{i1}] &= hH(\Phi_{i0}(t) - f(t) - N_{i0}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0})) \\ L_i[\Phi_{i2} - \Phi_{i1}] &= hH(\Phi_{i1} - N_{i1}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1})) \\ L_i[\Phi_{i3} - \Phi_{i2}] &= hH(\Phi_{i2} - N_{i2}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}, \Phi_{12}, \Phi_{22}, \dots, \Phi_{n2})) \\ &\vdots \end{aligned} \right\} \dots\dots\dots(22)$$

According to equation (12), the initial guess of the solution $\Phi_0(t)$ may be chooses as

$$\Phi_0(t) = f(t) \dots\dots\dots(18)$$

4. The Reliable Algorithm of HAM for solving System of Integral Equations:

In this section, the reliable algorithm of the HAM that given in section three in order to find the approximate solution of the system of integral equations given by

$$y(t) = f(t) + \int_0^t K(s, t, y(s)) ds \dots\dots\dots(19)$$

where

$$y(t) = (y_1, y_2, \dots, y_n)^T, f(t) = (f_1(t), f_2(t), \dots, f_n(t))^T$$

and

$$K(s, t, y(s)) = (K_1(s, t, y(s)), K_2(s, t, y(s)), \dots, K_n(s, t, y(s)))^T$$

Construct the following homotopy

$$\begin{aligned} (1-q)L_i[\Phi_i(t, q) - \Phi_{i0}(t)] &= \\ qhH[\Phi_i(t, q) - N_i(\Phi(t, q)) - f(t)], & i = 1, 2, \dots, n \end{aligned} \dots\dots\dots(20)$$

where $\Phi(t, q) = (\Phi_1, \Phi_2, \dots, \Phi_n)^T$ such that:

$$\Phi_i = \Phi_{i0} + q\Phi_{i1} + q^2\Phi_{i2} + \dots, \quad \forall i = 1, 2, \dots, n \dots\dots\dots(21)$$

Substituting (21) into (20) we get

$$\begin{aligned} (1-q)L_i[(\Phi_{i0} + q\Phi_{i1} + \dots) - \Phi_{i0}(t)] &= \\ qhH[(\Phi_{i0} + q\Phi_{i1} + \dots) - N_i(\Phi(t, q)) - f(t)] \end{aligned}$$

and so on, for $i = 1, 2, \dots, n$.

By choosing

$$L_i[\Phi_{ij}] = \Phi_{ij}, \quad i = 1, 2, \dots, n; j = 1, 2, \dots, n$$

and according to equation (19) the suitable choice for $\Phi_{i0}(t)$, may be given as

$$\Phi_{i0}(t) = f_i(t), \quad i = 1, 2, \dots, n \dots\dots\dots(23)$$

Thus with $h = -1$ and $H = 1$ equations (22) can be written as:

$$N_{i0}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}) = \int_0^t K_i(t, s, A_0(s)) ds$$

$$N_{i1}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}) = \int_0^t K_i(t, s, A_1(s)) ds$$

⋮

and so on.

where A_n are the Adomain Polynomials given by

$$A_n = \frac{1}{n!} \left[\frac{d^n}{d\lambda^n} F \left(\sum_{i=0}^n \lambda^i y_i \right) \right]_{\lambda=0}$$

The approximate solution of equation (19), therefore, can be obtain as

$$y_i(t) = \lim_{q \rightarrow 1} \Phi_i(t, q) = \Phi_{i0}(t) + \Phi_{i1}(t) + \Phi_{i2}(t) + \dots, \quad i = 1, 2, \dots, n$$

5. Illustrative Examples

In order to illustrate the approach given in section four, two examples are presented in this section.

Example (1):

Consider the following linear system of integral equation

$$\begin{aligned} y_1(t) &= t + \frac{t^3}{2} + \frac{t^4}{12} + \frac{t^5}{5} + \int_0^t (s^2 - t)[y_1(s) + y_2(s)] ds \\ y_2(t) &= t^2 + \frac{t^3}{3} + \frac{t^4}{4} + \int_0^t s[y_1(s) + y_2(s)] ds \end{aligned} \dots\dots\dots(25)$$

with the exact solution which was given in [16] as

$$y_1(t) = t \quad \text{and} \quad y_2(t) = t^2$$

According to equations (23) and (24) thus, we have:

$$\left. \begin{aligned} \Phi_{i1} &= N_{i0}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}) \\ \Phi_{i2} &= N_{i1}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}) \\ \Phi_{i3} &= N_{i2}(\Phi_{10}, \Phi_{20}, \dots, \Phi_{n0}, \Phi_{11}, \Phi_{21}, \dots, \Phi_{n1}, \\ &\quad \Phi_{12}, \Phi_{22}, \dots, \Phi_{n2}) \\ &\quad \vdots \end{aligned} \right\} \dots\dots\dots(24)$$

and so on.

where

$$\Phi_{10} = t + \frac{t^3}{2} + \frac{t^4}{12} + \frac{t^5}{5}$$

$$\Phi_{20} = t^2 + \frac{t^3}{3} + \frac{t^4}{4}$$

and in general

$$\Phi_{1n} = \int_0^t (s^2 - t)[\Phi_{1(n-1)}(s) + \Phi_{2(n-1)}(s)] ds$$

$$\Phi_{2n} = \int_0^t s[\Phi_{1(n-1)}(s) + \Phi_{2(n-1)}(s)] ds$$

⋮

and so on, $n=1, 2, 3, \dots$

The approximate solution of equations (25) can be readily given as

$$y_1(t) = \Phi_{10}(t) + \Phi_{11}(t) + \dots$$

and

$$y_2(t) = \Phi_{20}(t) + \Phi_{21}(t) + \dots$$

Following, Table (1) represent a comparison of the exact solution of example (1) with the approximate solution up to four terms.

Table (1)
Comparison between the exact solution and the approximate solution of example (1).

t_i	$y_1(t)$			$y_2(t)$		
	Approximate Value	Exact value	Absolute error	Approximate Value	Exact value	Absolute error
0	0	0	0	0	0	0
0.1	0.1	0.1	0	0.01	0.01	0
0.2	0.2	0.2	0	0.03999999999999999	0.04	8.327×10^{-17}
0.3	0.30000000000000004	0.3	3.886×10^{-15}	0.08999999999999954	0.09	4.594×10^{-15}
0.4	0.3999999999999526	0.4	4.737×10^{-13}	0.1600000000007150	0.16	7.151×10^{-13}
0.5	0.499999999981542	0.5	1.846×10^{-11}	0.2500000000309280	0.25	3.093×10^{-11}
0.6	0.599999999844846	0.6	1.552×10^{-10}	0.3600000002860870	0.36	2.861×10^{-10}
0.7	0.700000000367263	0.7	3.673×10^{-10}	0.4899999985686660	0.49	1.431×10^{-9}
0.8	0.800000014437179	0.8	1.444×10^{-8}	0.6399999469901210	0.64	5.301×10^{-8}
0.9	0.900000114439351	0.9	1.144×10^{-17}	0.8099994084737840	0.81	5.915×10^{-7}
1	1.000000457565610	1	4.576×10^{-7}	0.9999956792821380	1	4.321×10^{-6}

Example (2):

Consider the following nonlinear system of integral equation

$$y_1(t) = \sin t - t + \int_0^t [y_1^2 + y_2^2] ds$$

$$y_2(t) = \cos t - \frac{1}{2} \sin^2 t + \int_0^t [y_1(s)y_2(s)] ds$$

.....(26)

with the exact solution which was given in [17] as

$$y_1(t) = \sin(t) \text{ and } y_2(t) = \cos(t)$$

According to equations (23) and (24) we have

$$\Phi_{10} = \sin t - t$$

$$\Phi_{20} = \cos t - \frac{1}{2} \sin^2 t$$

$$\Phi_{11} = N_{10}(\Phi_{10}, \Phi_{20}) = \int_0^t [(\Phi_{10}(s))^2 + (\Phi_{20}(s))^2] ds$$

$$\Phi_{21} = N_{20}(\Phi_{10}, \Phi_{20}) = \int_0^t [\Phi_{10}(s) \Phi_{20}(s)] ds$$

$$\Phi_{12} = N_{11}(\Phi_{10}, \Phi_{11}, \Phi_{20}, \Phi_{21}) = \int_0^t [2\Phi_{10}(s)\Phi_{11}(s) + 2\Phi_{20}(s)\Phi_{21}(s)] ds$$

$$\Phi_{22} = N_{21}(\Phi_{10}, \Phi_{11}, \Phi_{20}, \Phi_{21}) = \int_0^t [\Phi_{10}(s)\Phi_{21}(s) + \Phi_{11}(s)\Phi_{20}(s)] ds$$

$$\vdots$$

and so on.

terms and the exact solution of example (2).

Following Table (2) represent a comparison between the approximate solution up to 6

Table (2)
Comparison between the exact solution and the approximate solution of example (2).

t_i	$y_1(t)$			$y_2(t)$		
	Approximate Value	Exact value	Absolute error	Approximate Value	Exact value	Absolute error
0	0	0	0	1	1	0.00E+00
0.1	0.099833389	0.0998334	2.76×10^{-8}	0.99500399	0.9950042	$1.75E \times 10^{-7}$
0.2	0.198665915	0.1986693	3.42×10^{-6}	0.98005568	0.9800666	1.09×10^{-5}
0.3	0.295465085	0.2955202	5.51×10^{-5}	0.95521776	0.9553365	1.19×10^{-4}
0.4	0.389037000	0.3894183	3.81×10^{-4}	0.92043432	0.921061	6.27×10^{-4}
0.5	0.477783222	0.4794255	1.64×10^{-3}	0.87537776	0.8775826	2.20×10^{-3}
0.6	0.559442143	0.5646425	5.20×10^{-3}	0.81937618	0.8253356	5.96×10^{-3}
0.7	0.630982875	0.6442177	1.32×10^{-2}	0.75149556	0.7648422	1.33×10^{-2}
0.8	0.688797621	0.7173561	2.86×10^{-2}	0.67079997	0.6967067	2.59×10^{-2}
0.9	0.729228556	0.7833269	5.41×10^{-2}	0.57674625	0.62161	4.49×10^{-2}
1	0.749322120	0.8414710	9.21×10^{-2}	0.46961235	0.5403023	7.07×10^{-2}

Conclusions

1.The numerical results showed that this reliable algorithm of HAM has good accuracy and reduces the calculations compared with the HAM.

2.This technique can be considered as an easy and efficient for solving various kinds of non-linear problems in science and engineering without any assumptions and restrictions.

References

- [1] **MA W. X., FAN E. G.**, Linear superposition principle applying to Hirota bilinear equations, *Comput. Math. Appl.* 61, pp. 950-959, 2011.
- [2] **MA W. X., HUANG T. W., ZHANG Y.**, A multiple exp-function method for nonlinear differential equations and its application, *Phys. Scr.*, 82, 065003, 2010.
- [3] **Khan Y., Faraz N.**, Application of modified Laplace decomposition method for solving boundary layer equation, *Journal of King Saud University (Science)*, 23, pp. 115-119, 2011.
- [4] **MA W. X., GU X., GAO L.**, A note on exact solutions to linear differential equations by the matrix exponential, *Adv. Appl. Math. Mech.*, 1, pp. 573-580, 2009.
- [5] **Jafari H., Zabihi M., Saidy M.**, Application of homotopy perturbation method for solving gas dynamic equation, *Appl. Math. Sci.*, 2, pp. 2393-2396, 2008.
- [6] **Chena X. M., Wang L. J.**, The variational iteration method for solving a neutral functional-differential equation with proportional delays, *Comput. Math. Appl.* 59, pp.2696-2702, 2010.
- [7] **Li J. L.**, Adomian decomposition method and homotopy perturbation method in solving nonlinear equations, *J. Comput. Appl. Math.*, 228, pp. 168-173, 2009.
- [8] **Liao S. J.**, *Beyond Perturbation: Introduction to Homotopy Analysis Method*, Chapman Hall/CRC Press, London/Boca Raton, 2003.
- [9] **Liao S. J.**, Notes on the homotopy analysis method: Some definitions and theorems, *Commun Nonlinear sci*, 14, pp. 983-997, 2009.
- [10] **Liao S. J.**, on the homotopy analysis method for nonlinear problems, *J. Comput. Appl. Math.*, 147, pp. 499-513, 2004.
- [11] **Liao S. J.**, The proposed homotopy analysis technique for the solution of nonlinear problems, PHD thesis, Shanghai Jiao Tong University, 1992.
- [12] **Liao S. J.**, A kind of approximate solution technique which does not depend upon small parameters: a special example, *Int. J. of Non-Linear Mech.*, 30, pp.371-380, 1995.
- [13] **Liao S. J.**, Homotopy Analysis Method: A New Analytical Technique for Nonlinear Problems, *Commun Nonlinear Science Num Sim*, 2, pp.95-100, 1997.
- [14] **Odibat Z., Momani S., Xu H.**, A reliable algorithm of Homotopy Analysis Method for solving nonlinear fractional differential equations. *Applied mathematics modeling*. 34, pp.593-600, 2010.
- [15] **Matinfar M., Saeidy M.**, The homotopy perturbation method for solving higher dimensional initial boundary value problems of variable coefficients, *World. J. Model. Sim.*, 4, pp. 72-80, 2009.
- [16] **Noor M. A., Mohyud-din S. T.**, A reliable approach for higher-order integro-differential equations, *Appl. Appl. Math.*, 3(2), pp. 188-199, 2008.
- [17] **Saeed, R.K., Ahmed, C.S.**, Numerical solution of the system of linear Volterra integral equations of the second kind using Monte-Carlo method. *J. Appl. Sci. Res.* 4 (10), pp. 1174-1181, 2008

الخلاصة

في هذا البحث تم تقديم الحل التقريبي لمنظومة معادلات تكاملية من نوع فولتيرا ومن النوع الثاني باستخدام خوارزمية معول عليها لطريقة تحليل الهوموتوبي.

جرى حساب الحل على شكل متسلسلة متقاربة بحيث يمكن حساب كل حد من حدودها بسهولة.

هذا الأسلوب تم اختباره بواسطة بعض الامثلة والنتائج التي تم الحصول عليها توضح كفاءة ودقة الأسلوب المتبع.