Some Results on (σ,τ)-Left Jordan Ideals in Prime Rings

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Abstract

In this paper we have proved the following results. Let R be a prime ring, U be (σ,τ)-left Joradan ideal of R where $\sigma,\tau: R\rightarrow R$ be two automorphisms of R and d be a nonzero derivation of R . (1) If $(R,a)_{\sigma,\tau}=0$, then $a\in Z(R)$. (2) If $aU=0$(or $Ua=0$) and $a\in R$, then $a=0$ or $U\subset Z(R)$. (3) If characteristic of R not equal 2 and $U\subset C_{\sigma,\tau}$, then $\sigma(u)+\tau(u)\in Z(R)$ for all $u\in U$.(4) If $d(U)=0$, $d\tau=\tau d$ and $d\sigma=\sigma d$, then $\sigma(u)+\tau(u)\in Z(R)$ for all $u\in U$.

Introduction

Over the last thirty years, many authors studied Lie ideals & Lie ideals with derivation and proved many results when the ring is prime or semiprime see [8],[9].In the end of the twentieth century and the beginning of this century, Neset Aydin , H. Kandamar, K. Kaya, Ogolbasi O., Studied (σ,τ)-Lie ideals with derivation and generalized many results from Lie ideals to (σ,τ) Lie ideals, see [1], [2], [3], [4], [5].

Neset Aydin ,H. Kandamar and K. Kaya in [5] proved that if R is a prime ring and U is a (σ,τ)-right Jordan ideal of R, then (i) If $U\subset Z(R)$, then R is commutative. (ii) If $aU=0$ (or $Ua=0$) and $a\in R$, then $a=0$. (iii) If $U\subset C_{\sigma,\tau}$, then R is commutative .

In this paper we want to study the above results in (σ,τ)-left Jordan ideal of R and generalized some results from Lie ideals to (σ,τ)-left jordan ideals . So, we must recall the basic terms that we need them in this research. Let U be an additive subgroup of R. $\sigma,\tau: R\rightarrow R$ be two mapping of R. Then we can defined U is a (σ,τ)-right Jordan Ideal of R if $(U,R)_{\sigma,\tau}\subset U$. Also, U is (σ,τ)-Left Jordan Ideal of R if $(R,U)_{\sigma,\tau}\subset U$. So, a (σ,τ)-Jordan ideal of R, if U is a (σ,τ)-right and Left Jordan Ideal of R [5] .

Let $d: R \rightarrow R$ be an additive mapping. If $d(xy)=d(x)y+xd(y)$ for all $x,y\in R$, then d is called a derivation of R [6]. Recall that a ring R is a prime if $aRb=0$, $a,b\in R$, implies that either $a=0$ or $b=0$ [6]. Also, a (σ,τ)-centralizer of R which is denoted $C_{\sigma,\tau}$ is the set $\{c\in R; c\sigma(x)= \tau(x)c$, for all $x\in R\}$, see[7].

In this paper we consider R to be a prime ring, U is a (σ,τ)-Left Jordan ideal of R and σ,τ are two automorphisms of R. Also, we considered d be a derivation of R to prove some of our results. Also, the following identities are used in this paper.

For all $x, y ,z \in R$, see[8].

(1) $[xy,z]= x [y,z] + [x,z] y$
(2) $[x,yz]= [x,y] z + y [x,z]$
(3) $[x,y]_{\sigma,\tau}=x \sigma(y)-\tau(y)x$
(4) $[xy,z]_{\sigma,\tau} = x [y, \sigma(z)]+ [x,z]_{\sigma,\tau} y$
$= x[y,z]_{\sigma,\tau} +[x, \tau(z)]y$ .Also the Jordan product is define as follows.
(5) $(x,y)_{\sigma,\tau}=x \sigma(y)+ \tau(y)x$
(6) $(xy,z)_{\sigma,\tau} =x(y,z)_{\sigma,\tau} - [x,\tau(z)]y$
(6) $(xy,z)_{\sigma,\tau}=(x,z)_{\sigma,\tau} y+x[y,\sigma(z)]$

Results

Theorem(2.1):

If $(R,a)_{\sigma,\tau}=0$, then $a\in Z(R)$.

Proof:

By hypothesis $(R,a)_{\sigma,\tau}=0$, then for all $x,y\in R$, we have

$0 =(xy,a)_{\sigma,\tau}=x(y,a)_{\sigma,\tau} -[x,\tau(a)]y =-[x,\tau(a)]y$

by using hypothesis.So,we have $[x,\tau(a)]y =0$ for all $x, y\in R$. Then $x\tau(a)y-\tau(a)xy=0$.Since τ is automorphism then τ^{-1} exists and $\tau^{-1}(x\tau(a)y-\tau(a)xy)=0$. Also, , τ^{-1} is automorphism ,
then $\tau^{-1}(x)a\tau^{-1}(y)-a\tau^{-1}(x)\tau^{-1}(y)=0$.So,
we get $[\tau^{-1}(x),a]\tau^{-1}(y) =0$, for all $x, y\in R$. Thus $[\tau^{-1}(R),a] \tau^{-1}(R) =0$. So, we get $a\in Z(R)$.

Theorem(2.2):

If $aU=0$ (or $Ua=0$) and $a\in R$, then
$a=0$ or $U\subset Z(R)$.

Proof:

By hypothesis $aU=0$,then for all $x,y\in R$, $u\in U$ we have

$0 = a(xy,u)_{\sigma,\tau} =a(x,u)_{\sigma,\tau} y+ax[y, \sigma(u)]$

$0 = ax[y, \sigma(u)]$. Then $ax[y, \sigma(u)]=0$ for all $x,y\in R$, $u\in U$ .Since R is prime ring , we get $a=0$ or $U\subset Z(R)$.

For another side if we have $Ua=0$, then for all $x\in R$, $u,v\in U$

$0 = (xy,u)_{\sigma,\tau}\, a = x\, (y,u)_{\sigma,\tau}\, a-[x, \tau(u)]ya$

$=-[x, \tau(u)]ya$ .Then $[x, \tau(u)]ya =0$ for all x , $y\in R$, $u\in U$. Since τ is automorphism ,we get $[\tau^{-1}(x),u]\tau^{-1}(y)a=0$ for all x, $y\in R$. Also, we have$[\tau^{-1}(R),U]\tau^{-1}(R)a=0$ .This implies $[R,U]Ra=0$. Since R is prime ring , we get $a=0$ or $U\subset Z(R)$.

Theorem(2.3):

If R has a characteristic not equal 2 and $U\subset C_{\sigma,\tau}$,then $\sigma(u)+\tau(u)\in Z(R)$ for all $u\in U$.

Proof:

If σ,τ are any two automorphisms , then by the hypothesis, we have

$(v,u)_{\sigma,\tau}\in C_{\sigma,\tau}$ for all u, $v\in U$. Then for all $r\in R$,we get

$0=[v\sigma(u)+\tau(u)v,r]_{\sigma,\tau}$

$= [v\sigma(u) ,r]_{\sigma,\tau}+[\tau(u)v,r]_{\sigma,\tau}$

$=v[\sigma(u),\sigma(r)]+[v,r]_{\sigma,\tau}\sigma(u) +\tau(u)[v,r]_{\sigma,\tau}$

$+ [\tau(u), \tau(r)]v$

$=v\, \sigma([u,r])+\tau([u,r])v+([v,r]_{\sigma,\tau},u)_{\sigma,\tau}$

$= v\sigma([u,r])+\tau([u,r])v$.

Since $U\subset C_{\sigma,\tau}$, then we have

$2v\sigma([u,r]) =0$ for all $u,v\in U$, $r\in R$.

Also, we have R has a characteristic not 2 , then $v\sigma([u,r])=0$ for all $u,v\in U$, $r\in R$.Therefore, $U\subset Z(R)$. Then we get $\sigma(u)+\tau(u)\in Z(R)$ for all $u\in U$.

Theorem (2.4):

Let $d(U)=0$, $d\tau=\tau d$ and $d\sigma = \sigma d$.

Then $\sigma(u)+\tau(u)\in Z(R)$ for all $u\in U$.

Proof:

By the hypothesis $d(U)=0$, we have for all $u\in U$, $x\in R$. $0 = d((x,u)_{\sigma,\tau})=d(x\sigma(u)+ \tau(u)x)= d(x)\sigma(u)+ xd(\sigma(u))+d(\tau(u))x + \tau(u)d(x)$.Since $d\tau=\tau d$ and $d\sigma=\sigma d$, we get

$d(x)\sigma(u) + \tau(u)d(x)=0$ for all $u\in U$, $x\in R$. That is $(d(x),u)_{\sigma,\tau} =0$ for all $u\in U$, $x\in R$.So, we replace x by vx , $v\in U$ .So, we have

$0=(d(vx),u)_{\sigma,\tau} =(d(v)x+vd(x),u)_{\sigma,\tau}$

$=(d(v)x,u)_{\sigma,\tau}+(vd(x),u)_{\sigma,\tau}=(vd(x),u)_{\sigma,\tau}$

$=v(d(x),u)_{\sigma,\tau} -[v,\tau(u)]d(x)$ for all

u, $v\in U$, $x\in R$ .Then , we get

$0=[v,\tau(u)]\, d(x)$ for all $u,v\in U$, $x\in R$.

Replace x by xy ,$y\in R$.So, we have

$0=[v,\tau(u)]d(xy)=[v,\tau(u)]d(x)y+$

$[v,\tau(u)]xd(y)$ for all $u,v\in U$, $x,y\in R$.

Then $[v,\tau(u)]xd(y)=0$ for all $u,v\in U$, $x,y\in R$. Thus $[v,\tau(u)]Rd(y)=0$.So, by a primeness of R and a non zero derivation d we get

$[v,\tau(u)]=0$ for all $u,v\in U$….(1).

For another side

$0=(d(xv),u)_{\sigma,\tau} =(d(x)v+xd(v),u)_{\sigma,\tau}$

$=(d(x)v,u)_{\sigma,\tau}+(xd(v),u)_{\sigma,\tau}=(d(x)v,u)_{\sigma,\tau}$

$= d(x)[v, \sigma(u)]+(d(x),u)_{\sigma,\tau}\, v$.

$= d(x)\, [v,\sigma(u)]$.

So by the same way we get

$[v,\sigma(u)]=0$ for all $u,v\in U$…(2).

By the adding these relations (1) and (2) , we get $[v, \sigma(u)+ \tau(u)]=0$ for all $u,v\in U$. That is mean $\sigma(u)+ \tau(u)$ in the center of U. But the center of U is subset of center of R , then $\sigma(u)+ \tau(u) \in Z(R)$,for all $u\in U$.

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الخلاصة

فـــي هذا البحث تمكنا من برهنة النتائج التالية . ليكـــن R حلقة أوليـــة ، ولتكـــــــــن U (σ,τ) – مثالي جوردان يساري في R . ولتكن $(\sigma,\tau): R \to R$ دوال متشاكلة فـــي R، d مشتقة غير صفرية في R .(1) أذا كان $(R,a)_{\sigma,\tau}=0$ ، فانـــــه $a \in Z(R)$.(2) أذا كان $aU=0$ (او $Ua=0$) ، فانــــــــه $a=0$ او $U \subset Z(R)$. (3) أذا كان مميز الحلقة لايساوي 2 و $U \subset C_{\sigma,\tau}$ ، فانه $\sigma(u)+\tau(u) \in Z(R)$ لكـــــل $u \in U$. (4) اذا كان $d(U)=0$ وانه $d\tau=\tau d$, $d\sigma=\sigma d$ فان $\sigma(u)+\tau(u) \in Z(R)$ ، لكـــل $u \in U$.