Surface Area Evaluation of Cartesian Mirrors with Varying Asphericity Factor

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Abstract

The dependence of a surface depth and a surface area on the asphericity factor of Cartesian surfaces is exhibited. This has been achieved by evaluating the surface areas of Cartesian surfaces with constant radius of curvature and constant aperture diameter and varying asphericity factor. For this purpose a skew ray-tracing code is constructed and a surface area program in MATLAB, for computing the areas, is composed. The evaluation of these areas demanded modifying the equation that describes these surfaces. The modification is summarized by converting the surface equation into a curve equation and the latter is employed to evaluate the areas by making use of a ray-tracing and MATLAB.

Keywords : Asphericity factor, Cartesian surfaces, ray tracing.

Introduction

Surface area of Cartesian surfaces (conic surfaces or quadric surfaces of revolution) is well-known in calculus text books. The importance of these surfaces in optical design is known for their ability to form free-aberrations images; therefore optical elements (mirrors/lenses) of Cartesian surfaces are widely used.

It is known that the reflector mirror is the most important component of an astronomical optical telescope. The telescope efficiency (magnification, resolving power and light-gathering power) is directly related to its area. Increasing the mirror aperture the mirror weight and this means more and more technical and engineering complexities. This, in turn, affects the cost or the budget of any engineering project for building a telescope or an observatory. The mirror weight reduction is a necessary first step.

In the past few decades, mirror weight reduction had been a major research topic for telescope scientists and engineers [1]. A number of techniques developed in this aspect include [2]:

(a) using a thin mirror;
(b) using a honeycomb mirror;
(c) building a multiple-mirror telescope;
(d)building a segmented mirror telescope.
(e) using mirrors made of metal, or carbon fiber reinforced plastic (CFRP) composite.

Regardless the efforts achieved to reduce mirrors densities by producing lightweight mirrors [1] or by using the techniques previously mentioned, the mirror surface area of a telescope is still the determining factor of a telescope project.

This work is motivated by the results of *Ref.* [3]. This Ref. studied the effects of varying the asphericity factor on the performance of the mirrors The results (figures) of *Ref.* [3] exhibit the profiles of the Cartesian surfaces of the same radius of curvature and the same aperture diameter with varying asphericity factor. Those figures show that changing the asphericity factor changes the profile of the mirrors (Cartesian surfaces) and consequently changes the mirrors performance. We can summarize the results of *Ref.* [3] concerning the surfaces profiles as follows. Increasing the aspericity factor higher than 1, led to increase the surfaces' depths, i.e, deeper and deeper surfaces. Besides, the more negative asphericity factor is the more flattened surfaces one can observe in the case of hyperboloids.

In this paper, we introduce a method to evaluate the surface areas of Cartesian surfaces which can be used as telescope monolithic mirrors by means of Skew-ray tracing equation.

Ray-tracing

Ray tracing procedures, in optics and optical design text books [4-8] are a mainstay to exhibit the performance of optical systems and the mathematical tools essential for evaluating rays-aberrations which are necessary for correcting these systems before being constructed.

In symmetrical optical systems there are mainly three different types of ray tracing procedures for the different types of the incoming rays. These procedures are the paraxial, meridianal, and the skew ray tracing [5].

The equation that represents a surface of revolution about the *z*–axis (the optical axis), passing through the origin (passing through the x-y plane that is tangent to the optical element surface and having curvature *C* at that point is [4]:

$$z = \frac{C}{2}\left(x^2 + y^2 + \varepsilon z^2\right) \quad \text{.................................(1)}$$

The parameter ε determines the asphericity factor as follows [4]:

ε > 1 for oblate ellipsoid surfaces,

0 < ε < 1 for prolate ellipsoid surfaces,

ε=1 for spherical surface,

ε =0 for paraboloid surface, and

ε < 0 for hyperboloid surfaces.

The utility of equation (1) is to give a range for asphericities while keeping the paraxial curvature C constant, which is essential in designing conic surfaces [5]. Different surfaces profiles are obtained by varying either the paraxial curvature *C* or the asphericity factor *ε*.

Skew Ray Tracing

It is considered as the ray tracing method that gives the exact analysis; because it uses solid geometry [5, 6]. The skew ray is the most general case of light rays income an optical system as it is defined as the ray that is not co-planer with the optical axis [6]. Skew Ray Tracing equations are divided into two sets of equations. The first set is for ray transfer between surfaces and second set is for reflection or refraction.

1. Transfer between Surfaces

It can be expressed by [5]:

$$\left.\begin{aligned} x_0 &= x_{-1} + \frac{L}{N}\left(d - z_{-1}\right) \\ y_0 &= y_{-1} + \frac{M}{N}\left(d - z_{-1}\right) \end{aligned}\right\} \quad \text{................................(2)}$$

L, *M*, and *N* are the direction cosines of the ray along x-axis, y-axis, and z-axis respectively; x_o, and y_o are the coordinates of ray intersection with the tangent x-y plane; x_{-1} *and* y_{-1} are the coordinates of coming ray. The ray intersects the optical element surface at coordinates given by [5]:

$$\left.\begin{aligned} x &= x_0 + L\Delta \\ y &= y_0 + M\Delta \\ z &= N\Delta \end{aligned}\right\} \quad \text{...(3)}$$

where Δ is given by [9]:

$$\Delta = \frac{F}{G + \sqrt{G^2 - CF(1 + (\varepsilon - 1)N^2)}} \quad \text{................(4)}$$

where *F* and *G* are given by[4]:

$$F = C\left(x_0^2 + y_0^2\right) \text{...(5)}$$

$$G = N - C\left(Lx_0 + My_0\right) \text{.................................(6)}$$

2. Reflection/ Refraction Equations Set

To obtain reflection or refraction equations through a surface, we start with determining the components of the unit normal (*α*, *β*, *γ*) as [9]:

$$\left.\begin{aligned} \alpha &= \frac{-Cx}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \\ \beta &= \frac{-Cy}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \\ \gamma &= \frac{1 - C\varepsilon z}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \end{aligned}\right\} \quad \text{..........(7)}$$

The cosine of the angle of incidence cos*I* can be obtained by the scalar multiplication with the direction cosines of the ray tracing. It can be expressed as [9]:

$$\cos I = \frac{N - C(Lx + My + N\varepsilon z)}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \quad \text{.......(8)}$$

The angle of reflection or refraction can be obtained by [5]:

$$n'\cos I' = \sqrt{(n')^2 - n^2(1 - \cos^2 I)} \quad \text{................(9)}$$

The non-primed parameters are those of the previous surface. The new ray direction cosines in order to complete the set of equations for transfer between surfaces are given by [5]:

$$\left.\begin{aligned} n'L' - nL &= k\alpha \\ n'M' - nM &= k\beta \\ n'N' - nN &= k\gamma \end{aligned}\right\} \quad \text{......................................(10)}$$

where

$k = n'\cos I' - n\cos I$ ……………………..(11)

The direction cosines should be checked in order to assert the tracing validity. This can be done by [5]:

$(L')^2 + (M')^2 + (N')^2 = 1$ ....(12)

Surface Area Evaluation

Equation (1) gives the z-axis (optical axis) coordinate of any point on a quadric surface of revolution.

$$z = \frac{C}{2}\left(x^2 + y^2 + \varepsilon z^2\right)$$

Setting either *x* or *y* equal to zero the resultant equation represents a curve. Setting *x* equal to zero and writing *y* as a function of the optical axis, Eq. (1) becomes:

$y = \sqrt{2rz - \varepsilon z^2}$ ....(13)

where $r=1/C$.

Eq. (13) is numerically differentiated and squared by the program written in MATLAB. The areas of mirrors are computed by using the well-known calculus equation of surface area by rotating a curve expressed as [10]:

$Area = 2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dz}\right)^2}\,dz$ ....(14)

The lower limit *a* =0 (the origin of the z-axis). The upper limit *b* is the surface depth (the z-coordinate of the mirror margin) which is obtained from the Eq. (3), specifically, $z=\Delta N$.

Results

The surface areas of 1m aperture diameter, (area of a unit circle), in order to compare the affect the asphericity factor on the Cartesian surfaces, are evaluated as explained above. All considered surfaces are of 5m radius of curvature with varying asphericity factor ε(ε is ranging from -1000 to 100).

Our result are tabulated in a table and exhibited in figures [1-3]. The table, indicates that varying ε produces surfaces with varying depths, the more negative ε is the more flattened surfaces (surfaces depths decreases) and ε>1 produces more deep surfaces (surfaces depth increases). Fig.(1) shows how the ε varies the surfaces depths. So, it is clear that as ε goes to infinity the surface depth goes to zero which means that surface becomes a plane. The table, also, shows the areas decrease with degreasing ε. Fig.(2) shows how ε changes the values of the surfaces' areas. The values of relative areas tabulated are determined by dividing the areas of the surfaces by the area of the unit circle to obtain normalized areas values as a function of the ε and Fig.(3) exhibits this case. So, it is obvious that when ε goes to infinity the relative area goes to unity which is the case of the area of a unit circle and this confirms these results and justifies the use of normalized areas of the Cartesian surfaces. Fig. (3) shows that the variation in the areas is very little when ε < 0. Although the variation in the relative areas seems negligible when ε>1 but the areas of the surfaces becomes very sensitive to ε.

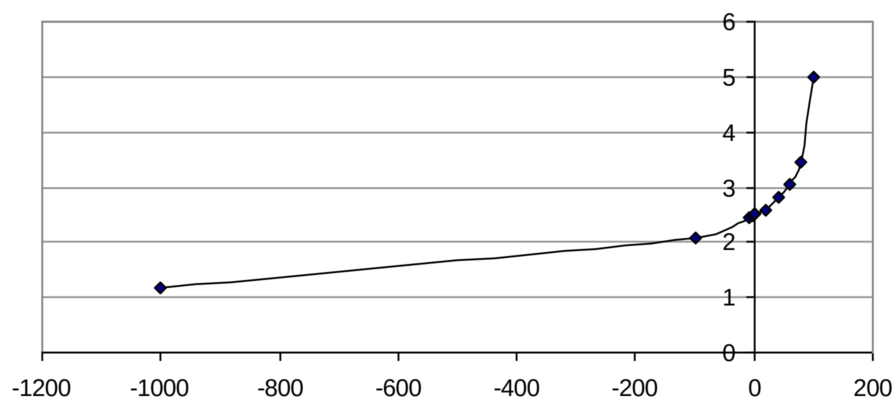
Conclusions

These results show the usefulness of ray tracing evaluate the areas of Cartesian surfaces. It is concluded that ε goes to infinity the Cartesian surface becomes a plane. When ε goes to infinity the relative area gives the value of a unit circle. The results of this paper are not only in a good agreement with those of Ref. [3], but they complement each other. We stress the importance of Fig.(3). Where even these changes in areas seems little, the variation in mirrors' weights is high for the high mass densities of the telescope mirrors. From this figure it is concluded that the variation of the weight of mirrors having the same aperture diameter and the same radius of curvature is similar to the relation of this figure. Finally, it is concluded that this work can be used to evaluate the weight of monolithic mirrors precisely.

Table (1)
The results of Cartesian Surfaces of 1m aperture diameter 5m radius of curvature.

	ε	*surface depth (z -axis)*	*surface area (cm^2)*	*Relative area*
Hyperboloids (ε<0)	-1000	1.158312	7856.966	1.00038
	-100	2.071068	7866.020	1.00153
	-10	2.440442	7872.370	1.00234
	-1	2.493781	7873.454	1.00247
	-0.01	2.499938	7873.582	1.00249
Paraboloid (ε=0)	0	2.5	7873.583	1.002495
Prolate ellipsoids (0< ε<1)	0.2	2.501251	7873.610	1.002499
	0.4	2.502505	7873.636	1.002502
	0.6	2.502505	7873.662	1.002505
	0.8	2.505020	7873.689	1.002509
Sphere (ε=1)	1	2.506281	7873.715	1.0025092
Oblate ellipsoid ε>*1*	20	2.565835	7874.994	1.00267
	40	2.817542	7881.113	1.00345
	60	3.062871	7888.363	1.00437
	80	3.454915	7903.349	1.00629
	100	5	8090.253	1.03008

Fig.(1) Surface depth vs. the asphericity factor (ε)



Fig(2) Surface area (cm^2) of 1m aperture vs. the asphericity factor (ε)

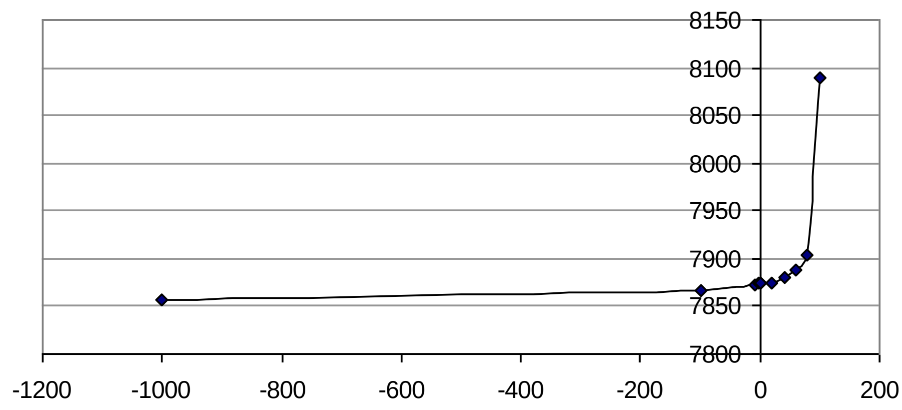
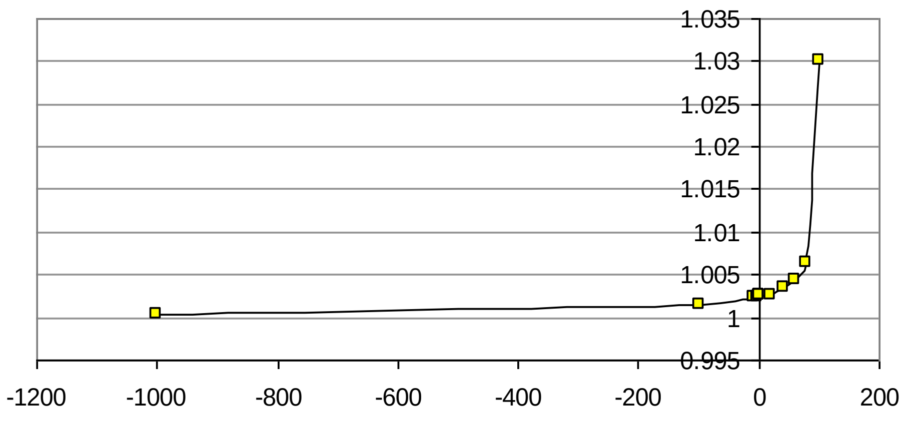


Fig.(3) The relative area vs. the asphericty factor(ε)



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الخلاصة

نعرض في هذا العمل إعتماد كل من عمق السطح والمساحة السطحية للسطوح الكارتيزية على عامل اللاتكور. و قد تم ذلك بإحتساب المساحة السطحية لسطوح كارتيزية ذات نصف قطرتقوس ثابت و قطرفتحة منفذ ثابتين وعامل لاتكورمتغير. ولتحقيق هذا الهدف فقد تم بناء برنامج لإقتفاء أثرالشعاع وكتابة برنامج بلغة MATLAB لحساب المساحات. لقد تطلب إحتساب المساحات تغييرالمعادلة التي تصف هذه السطوح. يتلخص التغيير بتحوير معادلة السطح الى معادلة منحني وقد وظفت هذه الأخيرة لإحتساب المساحات بأستخدام برنامج إقتفاء أثر الشعاع و MATLAB.