M3Y-P0 as a Residual Interaction to Study Elastic Magnetic Electron Scattering Form Factors for Ca-41

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Abstract

The studying of elastic magnetic electron scattering form factors has been done for Ca-41with the consideration of $1f_{7/2}$ orbit as a model space and F7MBZ as a model space effective interaction to construct the model space wave functions. Core and higher configurations orbits have been included to the calculations as a first order corrections through the so called core polarization effect with the use of M3Y-P0 as a residual interaction to join the particle hole pair across the model space with 2 ħω excitation energy, with the available experimental data, theoretical results have been compared with it.

Keywords: M3Y-P0, Elastic scattering, Ca-41, Core polarization.

1. Introduction

Elastic electron scattering is the process that the scattered electron will leave the nucleus in its ground state and the major momentum is distributed among the whole nucleus, therefore the initial and the final single particle states is the same for such transition.

^{41}Ca is the simplest and most important nucleus because it serves bench marks for microscopic description of nuclei and gives a chance to measure the radius of doubly closed nuclei in a stretched case (j = $1\mathbf{f}_{7/2}$) neutron orbit through the magnetic elastic electron scattering which may be defined as an elegant technique.

The electron scattering is such a powerful tool for studying nuclear structure because the basic interaction between the electron and the target nucleus is known. The electron interacts with the electromagnetic charge and current density of the nucleus. Since the interaction is relatively weak, of order $\alpha = 1/137$, one can make measurements on the target nucleus without greatly disturbing its structure. This is in contrast to the situation with strongly interacting projectiles where the scattering mechanism cannot be clearly known and separated from 180° electron scattering in the momentum-transfer range 0.9-2.0 fm^{-1}, the elastic magnetic form factors of ^{41}Ca had been determined [1] the data indicated that the amplitudes of the M3 and M5 multipoles were quenched by factors of 0.57±0.16 and 0.68±0.07 relative to the simple shell model. The magnitude of the M7 form factors in contrast, is in a good accord with this model. The elastic magnetic form factors of ^{17}O and ^{41}Ca were calculated [2] including both the 2ħω particle-hole excitations and the Zuker-type multi-particle-multi-hole configuration mixing.

Determination the radii for the $1d_{5/2}$ orbit in ^{17}O and the $1f_{7/2}$ orbit in ^{41}Ca had been carried out [3] which are within 2% less than those deduced from the magnetic electron scattering form factors. The displacement energies are also sensitive to the rms radius of the valence orbits, and the SKX interactions give radii for the $1d_{5/2}$ orbit in ^{17}O and the$1f_{7/2}$ orbit in ^{41}Ca which were within 2% less than those deduced from the magnetic electron scattering form factors.

Through their study [4], on elastic magnetic form factors of exotic nuclei and the magnetic form factors of $^{15,17,19}C$, ^{23}O, ^{17}F and $^{49,59}Ca$ calculated in the relativistic impulse approximation have found great differences in the form factors of the same nucleus with different configurations. Therefore, the elastic magnetic electron scattering can be used to determine the orbital of the last nucleon of odd-A exotic nuclei, their results can provide references for the electron scattering from exotic nuclei in the near future.

A computer program written in FORTRAN 90 language code [5], which calculates the model space form factors (zeroth-order) and the first-order cp effects has been utilized, and a computer program written in FORTRAN 90 language code [6] has been modified to receive the new set of fitting parameters belong to Paris fitting [7].

2. Theory

The reduced matrix element of the electron scattering operator $T\Lambda$ is expressed as the sum of the product of the elements of the one-body density matrix (OBDM) $X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta)$ times the single-particle matrix elements, which is given by[8]:

$$\langle \Gamma_f \| |\hat{T}_\Lambda| \| \Gamma_i \rangle = \sum_{\alpha\beta} X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta) \langle \alpha \| |\hat{T}_\Lambda| \| \beta \rangle \quad \text{.................} (1)$$

where α and β label single-particle states (isospin is included) for the shell model space. The states $|\Gamma_i|)$ and $|\Gamma_f|)$ are described by the model space wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e., $\Gamma_i \equiv J_iT_i$, $\Gamma_f \equiv J_f\,T_f$ and $\Lambda \equiv JT$. According to the first-order perturbation theory, the single-particle matrix element of the one-body operator is given by [8].

$$\langle \alpha \| |\hat{T}_\Lambda| \| \beta \rangle = \langle \alpha \| |\hat{T}_\Lambda| \| \beta \rangle + \left\langle \alpha \left| \left\| \hat{T}_\Lambda \frac{Q}{E_i - H_O} V_{res} \right\| \right| \beta \right\rangle + \left\langle \alpha \left| \left\| V_{res} \frac{Q}{E_f - H_O} \hat{T}_\Lambda \right\| \right| \beta \right\rangle \quad \text{............................} (2)$$

The first term is the zero-order contribution. The second and third terms are the first-order contributions which give the higher-energy configurations (hec). The operator Q is the projection operator onto the space outside the model space.

The hec terms given in Eq. (2) are written as [8]

$$\sum_{\alpha_1\alpha_2\Gamma} \frac{(-1)^{\beta+\alpha_2+\Gamma}}{e_\beta - e_\alpha - e_{\alpha_1} + e_{\alpha_2}} (2\Gamma+1) \begin{Bmatrix} \alpha & \beta & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{\alpha_1\alpha_2})(1+\delta_{\alpha_2\beta})} \times \langle \alpha\alpha_1 | V_{res} | \beta\alpha_2 \rangle \langle \alpha_2 \| |\hat{T}_\Lambda| \| \alpha_1 \rangle + \mathit{term\ with\ } \alpha_1 \mathit{\ and\ } \alpha_2 \mathit{\ exchanged\ with\ an\ over\ all\ minus\ sign} \quad \text{..................} (3)$$

where the index α_1 runs over particle states and α_2 over hole states and e_i is the single-particle energy, which is calculated according to [8] as:

$$e_{nlj} = (2n + l - \tfrac{1}{2})\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r) \rangle_{nl}, & \text{for } j = l - \frac{1}{2} \\ \frac{1}{2} l \langle f(r) \rangle_{nl}, & \text{for } j = l + \frac{1}{2} \end{cases} \quad \text{...} (4)$$

with:

$$\langle f(r) \rangle_{nl} \approx -20A^{-2/3} MeV \quad \text{..........................} (5)$$

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$$

The single particle matrix elements reduced in both spin and isospin, are written in terms of the single-particle matrix elements reduced in spin only [8]

$$\langle \alpha \| |\hat{T}_\Lambda| \| \beta \rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \langle j_2 \| |\hat{T}_\Lambda| \| j_1 \rangle \quad \text{...} (6)$$

with:

$$I_T(t_z) = \begin{cases} 1 & for\ T = 0 \\ (-1)^{\frac{1}{2}-t_z} & for\ \ T = 1 \end{cases} \quad \text{...............} (7)$$

where $t_z = \frac{1}{2}$, for proton and –1/2 for neutron.

Higher energy configurations are taken into consideration through 1p-1h excitations from the model space orbits into higher orbits. All excitations are considered with 2ħω excitations.

For the residual two-body interaction *V*res, the M3Y-P0 interaction [7] is adopted. The form of the potential is defined in Eqs. (1)-(3) in Ref. [9]. The parameters of ‘Elliot’ are used which are given in Table 1 of the mentioned reference. A transformation between *LS* and *jj* is used to get the relation between the two-body shell model matrix elements and the relative and the center of mass coordinates, using the harmonic oscillator radial wave functions with Talmi-Moshinsky transformation.

Electron scattering form factors involving angular momentum *J* and momentum transfer *q*, between initial and final nuclear shell model states of spin $J_{i,f}$ and isospin $T_{i,f}$ are [10]:

$$\left|F_J^{\eta}(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}\left|\sum_{T=0,1}(-1)^{T_f-T_{zf}}\begin{pmatrix} T_f & T & T_i \\ -T_{Zf} & M_T & T_{Zi}\end{pmatrix}\left\langle \Gamma_f \left\| T_{J,T}^{\eta}(q) \right\| \Gamma_i \right\rangle\right|^2 \times \left|F_{c.m}(q)\right|^2 \times \left|F_{f.s}(q)\right|^2 \quad \text{.................. (8)}$$

$$v_{12}(M3Y-P0) = v_{12}^{(c)} + v_{12}^{(LS)} + v_{12}^{(TN)} \quad \text{.......... (9)}$$

The three potentials are expressed as [9]:

$$v_{12}^{(c)} = \sum_n (t_n^{(SE)} P_{SE} + t_n^{TE} P_{TE} + t_n^{(SO)} P_{SO} + t_n^{(TO)} P_{TO}) f_n^{(C)}(r_{12})$$
$$v_{12}^{(LS)} = \sum_n (t_n^{(LSE)} P_{TE} + t_n^{(LSO)} P_{TO}) f_n^{(LS)}(r_{12}) L_{12}\cdot(\vec{s}_1 + \vec{s}_2)$$
$$v_{12}^{(TN)} = \sum_n (t_n^{(TNE)} P_{TE} + t_n^{(TNO)} P_{TO}) f_n^{(TN)}(r_{12}) r_{12}^2 S_{12} \quad \text{.................... (10)}$$

The values of the best fit to the potential parameters ($t_n^{(SE)}$, $t_n^{(SO)}$, $t_n^{(TO)}$, $t_n^{(TE)}$, $t_n^{(LSE)}$, $t_n^{(LSO)}$, $t_n^{(TNE)}$, $t_n^{(TNO)}$) for (M3Y-P0) in Table (2), [7].

Results and Discussion

Identify the orbital of the valence nucleon (s) of exotic nuclei is an important problem. The elastic magnetic electron scattering is an excellent probe to determine the valence structure of odd-A nuclei [4]. The shell model theory with core polarization has been successfully applied to study of the elastic magnetic electron scattering form factors, the allowed values of the angular momenta are ($|J_i - J_f| \leq J \leq J_i + J_f$) so that the multipolarity for $J_i = J_f = 7/2$ are (J = 0, 1, 2, 3, 4, 5, 6, 7) and the isospin values (T = 0, 1), Figs. (1) and (2) show the total form factors (TFF = M1+M3+M5+M7) with respect to the experimental one, it is clear that the total form factors is in an excellent agreement with the experimental one.

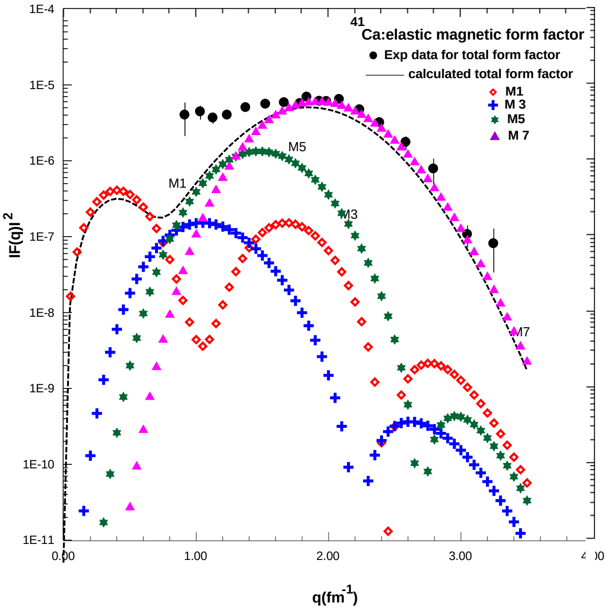

Fig.(1) The Total elastic magnetic form factors, model space contribution and core polarization parts of the total elastic magnetic form factors in 1f$_{7/2}$ model space calculation and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

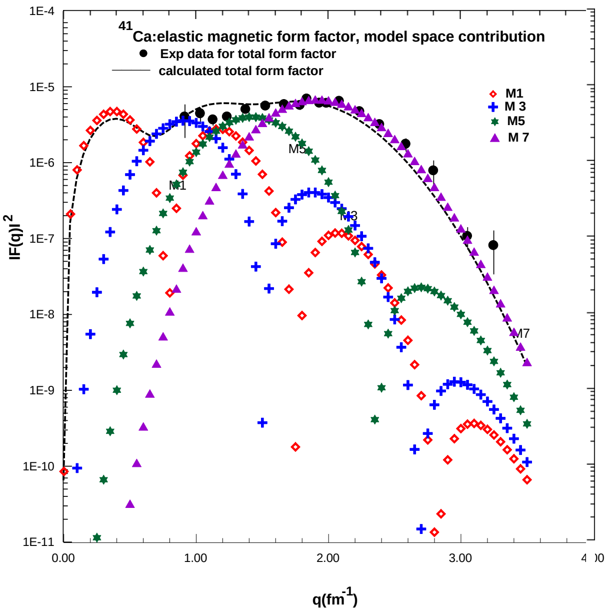

Fig.(2) The Total elastic magnetic form factors, and the individual total magnetic form factors multipolarities in 1f$_{7/2}$ model space calculation and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

The distribution of the individual total form factors of different multipolarities indicates that the distribution of form factors with respect to the (q-values) are, therefore shifting toward the increasing of the q-values with the increasing of the (J-values), it is caused by the tensor part of the residual interaction and the radial wave function inherent with the radial integral of the transition matrix elements, this is quite clear from Fig.(3).

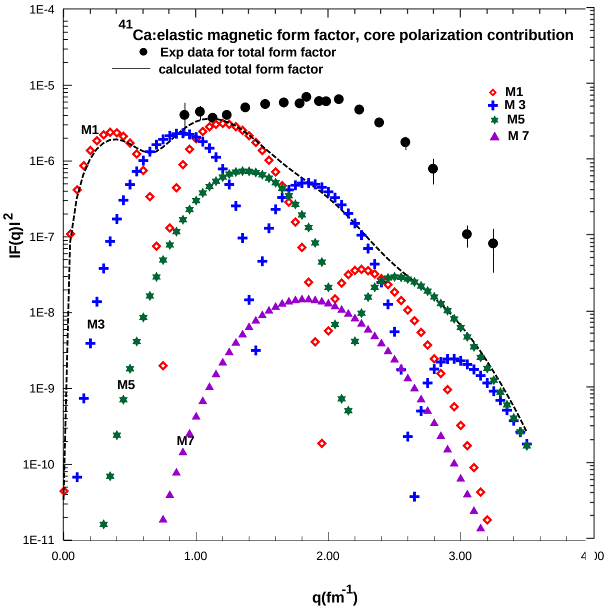

Fig.(3) The individual model space contribution in elastic magnetic form factors for different multipolarities in 1f$_{7/2}$ model space and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

For M1 form factors where the tensor part of the residual interaction is not active and vanished (J = 1), but the core contribution is shifted in the direction of increasing of q-values, this is caused by the radial wave function of single particle, the major contribution to the form factors comes from the model spaces.

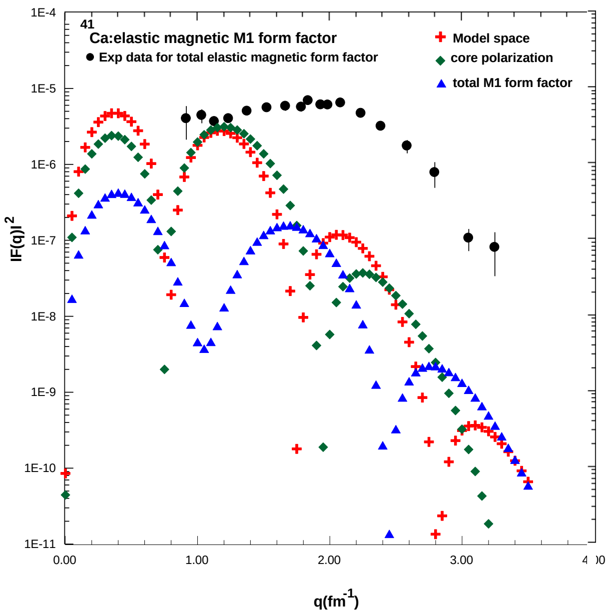

Fig.(4) The Total M1 elastic magnetic form factors, and the model space and core polarization in 1f$_{7/2}$ model space calculation and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

Return to the amplitudes of different multipolarities it is clear that there is a fluctuation in its contributions and it is easy to say that this fluctuation comes from the fact that the strong (stretched) M7 multipole contribution is quenched by the weak contribution of the core, that is the particle hole state and the probability to generate this type of (J) is somehow weak and M3, M5 have the same reasons, Figs. (5), (6), and (7) show this idea for the core and model space contributions of the individual multipolarities and it is clear from the Fig.(5) (core contribution) that the higher the multipole transition (J) the higher is the shifting toward the increased q caused by the enlargement of tensor component, and the higher is the increasing of the width of diffraction pattern and the smaller of intensity, meaning that the particle hole configuration contribution is decreased with the increase of the value of (J).

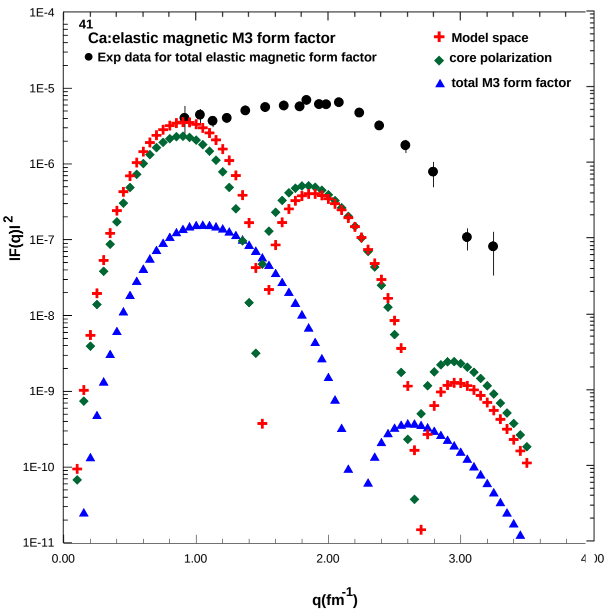

Fig.(5) The Total M3 elastic magnetic form factors, and the model space and core polarization in $1f_{7/2}$ model space calculation and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

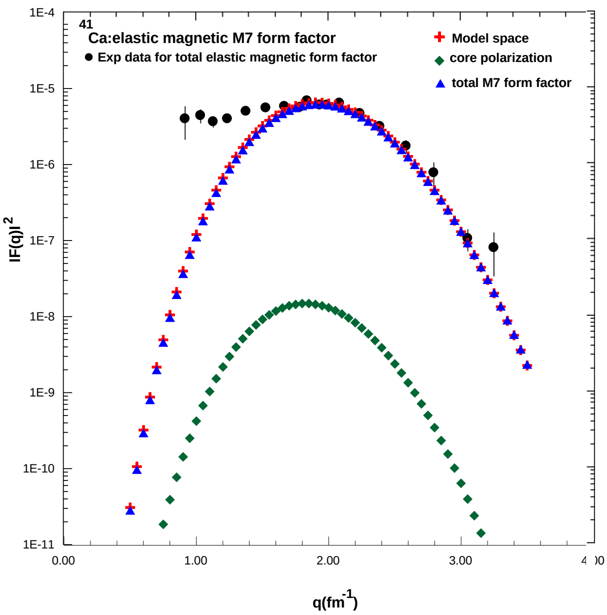

Fig.(7) The Total M5 elastic magnetic form factors, and the model space and core polarization in $1f_{7/2}$ model space calculation and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

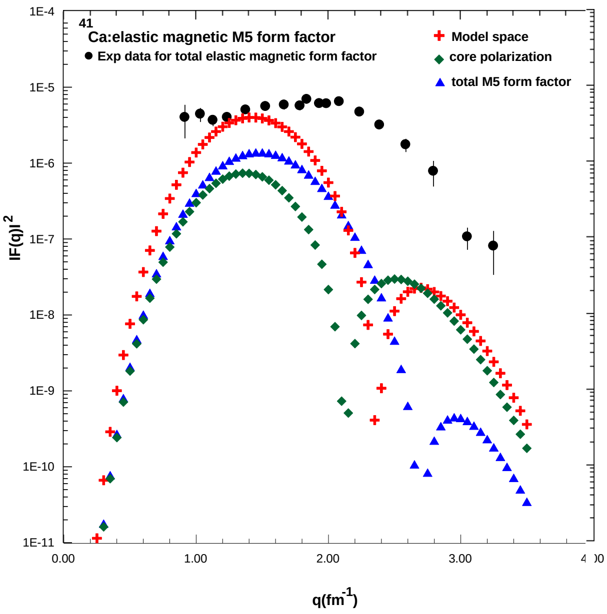

Fig.(6) The Total M5 elastic magnetic form factors, and the model space and core polarization in $1f_{7/2}$ model space calculation and F7MBZ as a model space effective interaction [11] in ^{41}Ca. The experimental data are taken from ref. [1].

And Table (1) shows the values of (OBDM) for the different multipolarities. Final it is shown that the core contribution is positive with respect to the model space one. The experimental data are taken from ref. [1].

Table (1)
The values of (OBDM) for elastic magnetic form factor in ^{41}Ca.

MJ	Ji	Jf	OBDM ($\Delta T=0$)	OBDM ($\Delta T=1$)
M1	7/2	7/2	1	1
M3	7/2	7/2	1	1
M5	7/2	7/2	1	1
M7	7/2	7/2	1	1

Table (2)
PARIS fitting parameters for M3Y interaction.

	R_1=0.25 fm	***R_2=0.40 fm***	***R_3=1.414 fm***
Oscillator matrix elements (Channel)	t_1	t_2	t_3
Centeral Singlet-	11466	–3556	–10.463
Central Triplet-Even (TE)	13967	–4594	–10.463
Central Singlet-Odd (SO)	–1418	950	31.389
Central Triplet-Odd (TO)	11345	–1900	3.488
Tensor-Even (TNE)	0.0	–171.7	–78.03
Tensor-Odd (TNO)	0.0	283.0	13.62
Spin-Orbit Even (LSE)	0.0	–813.0	0.0
Spin-Orbit Odd (LSO)	–2672	–620.0	0.0

Conclusions

The study of elastic magnetic form factors in ^{41}Ca might be considered as a starting point to measure the single particle effects in a model space fp shell and then generalizing the behavior over the many body system, M3Y-P0 prove it self through its modified effects on the total form factors, and it is as same as that when we use the realistic interaction M3Y and there is no doubt that the simple structure of ^{41}Ca shows that it is a hard core of ^{41}Ca plus one active neutron in the model space fp.

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الخلاصة

تمت دراسة عوامل التشكل للاستطاره الالكترونية المغناطيسية المرنة لنواة (aC-٤١) بأستخدام نظرية القشرة النووية على أعتبار الغلاف (١F٧/٢) كأنموذج فضاء و أعتماد التفاعل (F٧MBZ) المؤثر كتفاعل مؤثر لأنموذج الفضاء المذكور لغرض توليد الدوال الموجية اللازمة للحسابات اللاحقه و تمت عملية تضمين مدارات القلب الخامل و المدارات العليا للحسابات من خلال ما يسمى بعملية أستقطاب القلب و بمرتبة التهيج الاولى وبطاقة تهيج مقدارها (٢ħω) لجسيمات القلب نحو المدارات العليا عبر أغلفه أنموذج الفضاء و أعتماد التفاعل الواقعي (M٣Y-P٠) المتطور كتفاعل بقيه لربط الزوج (جسيم فجوه) مع جسيمات أنموذج الفضاء. تمت مقارنة النتائج النظرية مع ما متوفر من معطيات عملية.