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On (1,2)* b-Open Functions and (1,2)* b-Closed Functions In Bitopological Spaces

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Abstract
The main goal of this paper is to create special type of open and closed functions in bitopological spaces namely, quasi (1,2)* b-open functions and quasi (1,2)* b-closed functions. Also, we give some properties and equivalent statements of this concept.

Keywords: (1,2)* b-continuous function, (1,2)* b-irresolute function, contra (1,2)* b-irresolute function, quasi (1,2)* b-open function and quasi (1,2)* b-closed function.

Introduction
The concept of a bitopological space (X,τ_1,τ_2) was first introduced by Kelly [1], where X is a nonempty set and τ_1,τ_2 are topologies on X. Also, the concept of (1,2)* b-open sets was first introduced and studied by Sreeja and Janaki [2]. The purpose of this paper is to give a new type of open and closed functions in bitopological spaces called quasi (1,2)* b-open functions and quasi (1,2)* b-closed functions. Also, we study the relation between the quasi (1,2)* b-open (resp. quasi (1,2)* b-closed) functions and each of the (1,2)* open (resp. (1,2)* closed) functions, (1,2)* b-open (resp. (1,2)* b-closed) functions and pre-(1,2)* b-open (resp. pre-(1,2)* b-closed) functions. Moreover, we study the characterizations and basic properties of quasi (1,2)* b-open functions and quasi (1,2)* b-closed) functions.

Throughout this paper (X,τ_1,τ_2), (Y,σ_1,σ_2) and (Z,η_1,η_2) (or simply X, Y and Z) represent non-empty bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned

1. Preliminaries
First, we recall the following definitions:

(1.1) Definition [3]:
A subset A of a bitopological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-open if $A=U_1\cup U_2$ where $U_1\in\tau_1$ and $U_2\in\tau_2$. The complement of a $\tau_1\tau_2$-open set is called $\tau_1\tau_2$-closed.

Notice that $\tau_1\tau_2$-open sets need not necessarily form a topology [3].

(1.2)Definition [3]:
Let (X,τ_1,τ_2) be a bitopological space and $A\subseteq X$. Then:-

i) The $\tau_1\tau_2$-closure of A, denoted by $\tau_1\tau_2 cl(A)$, is defined by:
$$\tau_1\tau_2 cl(A)=\bigcap\{F: A\subseteq F \,\&\, F \text{ is } \tau_1\tau_2-closed\}$$
ii) The $\tau_1\tau_2$-interior of A, denoted by $\tau_1\tau_2 int(A)$, is defined by:
$$\tau_1\tau_2 int(A)=\bigcup\{U: U\subseteq A \,\&\, U \text{ is } \tau_1\tau_2-open\}.$$

(1.3)Definition [4]:
A subset A of a bitopological space (X,τ_1,τ_2) is called an (1,2)*-neighborhood of a point x in X if there exists a $\tau_1\tau_2$-open set U in X such that $x\in U\subseteq A$.

(1.4)Definition [2]:
A subset A of a bitopological space (X,τ_1,τ_2) is said to be (1,2)* b-open if $A\subseteq\tau_1\tau_2 cl(\tau_1\tau_2 int(A))\cup\tau_1\tau_2 int(\tau_1\tau_2 cl(A))$. The complement of an (1,2)* b-open set is said to be (1,2)* b-closed. The class of all (1,2)* b-open (resp. (1,2)* b-closed) subsets of X is denoted by $(1,2)^*BO(X,\tau_1,\tau_2)$ (resp. $(1,2)^*BC(X,\tau_1,\tau_2)$).

(1.5) Definition:
Let (X,τ_1,τ_2) be a bitopological space and $A\subseteq X$. Then :-

i) The (1,2)* b-closure of A, denoted by $(1,2)^*bcl(A)$, is defined by: $(1,2)^*bcl(A)=\bigcap\{F: A\subseteq F \,\&\, F \text{ is } (1,2)^*b-closed\}$.

ii) The (1,2)* b-interior of A, denoted by $(1,2)^*bint(A)$ is defined by: $(1,2)^*bint(A)=\cup\{U: U\subseteq A \,\&\, U \text{ is } (1,2)^*b\text{-open}\}$.

The following proposition holds. The proof is easy and hence omitted.

(1.6) Proposition:

Let (X,τ_1,τ_2) be a bitopological space and $A\subseteq X$. Then:-

1) The union (resp. intersection) of any family of (1,2)* b-open (resp. (1,2)* b-closed) sets in a ***bitopological space*** (X,τ_1,τ_2) ***is (1,2)* b-open (resp. (1,2)* b-closed).***

2) $A\subseteq(1,2)^*bc(A)$.

3) $(1,2)^*bc(A)$ is an (1,2)* b-closed set in X.

4) A is (1,2)* b-closed in X iff $A=(1,2)^*bc(A)$.

5) $(1,2)^*bint(A)\subseteq A$.

6) $(1,2)^*bint(A)$ is an (1,2)* b-open set in X.

7) A is (1,2)* b-open iff $A=(1,2)^*bint(A)$.

(1.7) Definition [5]:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2)* -continuous if $f^{-1}(V)$ is $\tau_1\tau_2$-open set in X for every $\sigma_1\sigma_2$-open set V in Y.

(1.8) Definition [6]:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2)*-open (resp. (1,2)*-closed) if the image of every $\tau_1\tau_2$-open (resp. $\tau_1\tau_2$-closed) subset of X is a $\sigma_1\sigma_2$-open (resp. $\sigma_1\sigma_2$-closed) set in Y.

(1.9) Definition [2]:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2)* b-irresolute if $f^{-1}(V)$ is (1,2)*b-open set in X for every (1,2)* b-open set V in Y.

(1.10) Proposition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (1,2)* b-irresolute iff $f^{-1}(V)$ is (1,2)* b-closed set in X for every (1,2)* b-closed set V in Y.

(1.11) Definition [7]:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be pre-(1,2)* b-closed (resp. pre-(1,2)* b-open) if the image of every (1,2)* b-closed (resp. (1,2)* b-open) subset of X is an (1,2)* b-closed (resp. (1,2)* b-open) set in Y.

2. Quasi (1,2)* b-open Functions

Now, we introduce the following definitions:

(2.1) Definition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2)* b-open if the image of every $\tau_1\tau_2$-open subset of X is an (1,2)* b-open set in Y.

(2.2) Definition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be quasi (1,2)* b-open if the image of every (1,2)* b-open set in X is $\sigma_1\sigma_2$-open in Y.

(2.3) Proposition:

Every quasi (1,2)* b-open function is (1,2)*-open as well as (1,2)* b-open.

(2.4) Remark:

The converse of (2.3) may not be true in general. Consider the following example.

<u>Example:</u>

Let $X=Y=\{a,b,c\}$, $\tau_1=\{X,\phi,\{a\}\}$, $\tau_2=\{X,\phi\}$, $\sigma_1=\{Y,\phi,\{a\},\{b,c\}\}$ & $\sigma_2=\{Y,\phi\}$. So the sets in $\{X,\phi,\{a\}\}$ are $\tau_1\tau_2$- open in X and the sets in $\{Y,\phi,\{a\},\{b,c\}\}$ are $\sigma_1\sigma_2$-open in Y. Also, $(1,2)^*BO(X,\tau_1,\tau_2)=\{X,\phi,\{a,c\},\{a,b\},\{a\}\}$ & $(1,2)^*BO(Y,\sigma_1,\sigma_2)=\{Y,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$.

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by: $f(a)=a$, $f(b)=b$ & $f(c)=c$. It is clear that f is (1,2)* b-open as well as (1,2)*-open, but f is not quasi (1,2)* b-open, since {a, c} is (1,2)* b-open in (X,τ_1,τ_2), but $f(\{a,c\})=\{a,c\}$ is not $\sigma_1\sigma_2$-open in (Y,σ_1,σ_2).

(2.5) Proposition:

Every quasi (1,2)* b-open function is pre-(1,2)* b-open.

(2.6) Remark:

The converse of (2.5) may not be true in general. Consider the following example.

Example:

Let $X=Y=\{a,b,c\}$, $\tau_1=\{X,\phi\}$, $\tau_2=\{X,\phi,\{a,c\}\}$, $\sigma_1=\{Y,\phi,\{a\},\{b\},\{a,b\}\}$ & $\sigma_2=\{Y,\phi,\{a\}\}$. So the sets in $\{X,\phi,\{a,c\}\}$ are $\tau_1\tau_2$-open in X and the sets in $\{Y,\phi,\{a\},b\},\{a,b\}\}$ are $\sigma_1\sigma_2$-open in Y. Also, $(1,2)^*BO(X,\tau_1,\tau_2)=\{X,\phi,\{a\},\{c\},\{a,b\},\{a,c\},\{b,c\}\}$ & $(1,2)^*BO(Y,\sigma_1,\sigma_2)=\{Y,\phi,\{a\},\{b\},\{a,c\},\{a,b\},\{b,c\}\}$.

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by: $f(a)=a, f(b)=c$ & $f(c)=b$. It is clear that f is pre-(1,2)* b-open,but f is not quasi (1,2)* b-open, since $\{a,b\}$ is (1,2)* b-open in (X,τ_1,τ_2), but $f(\{a,b\})=\{a,c\}$ is not $\sigma_1\sigma_2$-open in (Y,σ_1,σ_2).

Thus we have the following diagram:

quasi (1,2)* b-open ⇒ pre-(1,2)* b-open

⇓ ⇓

(1,2)* -open ⇒ (1,2)* b-open

(2.7) Theorem:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is quasi (1,2)* b-open iff $f((1,2)^*bint(U))\subseteq\sigma_1\sigma_2\,int(f(U))$ for every subset U of X.

Proof: ⇒

Let f be a quasi (1,2)* b-open function. To prove that $f((1,2)^*bint(U))\subseteq\sigma_1\sigma_2\,int(f(U))$ for every subset U of X.

By (1.6) no. 5, $(1,2)^*bint(U)\subseteq U \Rightarrow f((1,2)^*bint(U))\subseteq f(U)$. Since $(1,2)^*bint(U)$ is an (1,2)* b-open set in X and f is quasi (1,2)* b-open, then $f((1,2)^*bint(U))$ is $\sigma_1\sigma_2$-open in Y. Thus $f((1,2)^*bint(U))\subseteq\sigma_1\sigma_2\,int(f(U))$.

Conversely Suppose that $f((1,2)^*bint(U))\subseteq\sigma_1\sigma_2\,int(f(U))$ for every subset U of X. To prove that f is quasi (1,2)* b-open. Let U be an (1,2)* b-open set in X. Then by (1.6) no.7, $U=(1,2)^*bint(U) \Rightarrow f(U)=f((1,2)^*bint(U))\subseteq\sigma_1\sigma_2\,int(f(U))$. But $\sigma_1\sigma_2\,int(f(U))\subseteq f(U)$. Consequently $\sigma_1\sigma_2\,int(f(U))=f(U)\Rightarrow f(U)$ is a $\sigma_1\sigma_2$-open set in Y. Hence f is a quasi (1,2)* b-open function.

(2.8) Theorem:

If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is quasi (1,2)* b-open, then $(1,2)^*bint(f^{-1}(U))\subseteq f^{-1}(\sigma_1\sigma_2\,int(U))$ for every subset U of Y.

Proof:

Let U be any arbitrary subset of Y. Then, $f^{-1}(U)$ is a subset of X. Since f is quasi (1,2)* b-open, then by (2.7) $f((1,2)^*bint(f^{-1}(U)))\subseteq\sigma_1\sigma_2\,int(f(f^{-1}(U)))\subseteq\sigma_1\sigma_2\,int(U)$. Thus $(1,2)^*bint(f^{-1}(U))\subseteq f^{-1}(\sigma_1\sigma_2\,int(U))$ for every subset U of Y.

(2.9)Definition:

A subset A of a bitopological space (X,τ_1,τ_2) is said to be an (1,2)* b-neighborhood of a point x in X if there exists an (1,2)* b-open set U in X such that $x\in U\subseteq A$.

(2.10) Theorem:

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function from a bitopological space X into a bitopological space Y. Then the following are equivalent:-

i) f is quasi (1,2)* b-open.

ii) $f((1,2)^*bint(U))\subseteq\sigma_1\sigma_2\,int(f(U))$ for each subset U of X.

iii) for each $x\in X$ and each (1,2)* b-neighborhood U of x in X, there exists an (1,2)*-neighborhood V of f(x) in Y such that $V\subseteq f(U)$.

Proof:

$(i)\to(ii)$.It follows from theorem (2.7)

$(ii)\to(iii)$.

Let $x\in X$ and U be an arbitrary (1,2)* b-neighborhood of x in X, then by (2.9) there exists an (1,2)* b-open set V in X such that $x\in V\subseteq U$. Since V is (1,2)* b-open in X, then by (1.6) no.7, $V=(1,2)^*bint(V)$. By (ii) , we have $f(V)=f((1,2)^*bint(V))\subseteq\sigma_1\sigma_2\,int(f(V))\Rightarrow f(V)\subseteq\sigma_1\sigma_2\,int(f(V))$. Since $\sigma_1\sigma_2\,int(f(V))\subseteq f(V) \Rightarrow$

$f(V) = \sigma_1\sigma_2 \operatorname{int}(f(V)) \Rightarrow$ f(V) is $\sigma_1\sigma_2$-open in Y such that $f(x) \in f(V) \subseteq f(U)$.

$(iii) \rightarrow (i)$.

Let U be an arbitrary (1,2)* b-open set in X. Then for each $y \in f(U)$ there exists $x \in U$ such that $f(x) = y$. By (iii) there exists an (1,2)*-neighborhood V_y of y in Y such that $V_y \subseteq f(U)$. Since V_y is an (1,2)*-neighborhood of y , then there exists a $\sigma_1\sigma_2$-open set W_y in Y such that $y \in W_y \subseteq V_y$. Thus $f(U) = \bigcup_{y \in f(U)} W_y$ which is a $\sigma_1\sigma_2$-open set in Y. This implies that f is quasi (1,2)* b-open function.

(2.11) Theorem:

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is quasi (1,2)* b-open iff for any subset B of Y and for any (1,2)* b-closed set F of X containing $f^{-1}(B)$, there exists a $\sigma_1\sigma_2$-closed set G of Y containing B such that $f^{-1}(G) \subseteq F$.

<u>Proof:</u> $\Rightarrow$

Suppose that f is quasi (1,2)* b-open. Let $B \subseteq Y$ and F be an (1,2)* b-closed subset of X such that $f^{-1}(B) \subseteq F$. Now, put $G = Y - f(X - F)$. Since $f^{-1}(B) \subseteq F$ $\Rightarrow X - F \subseteq f^{-1}(B^c) \Rightarrow$ $f(X - F) \subseteq f(f^{-1}(B^c)) \subseteq B^c \Rightarrow$ $B \subseteq Y - f(X - F) \Rightarrow B \subseteq G$. Since f is quasi (1,2)* b-open, then G is a $\sigma_1\sigma_2$-closed subset of Y. Moreover, we have $f^{-1}(G) \subseteq F$.

Conversely, let U be an (1,2)* b-open set in X. To prove that $f(U)$ is a $\sigma_1\sigma_2$-open set in Y. Put $B = Y - f(U)$, then $X - U$ is an (1,2)* b-closed set in X such that $f^{-1}(B) \subseteq X - U$. By hypothesis, there exists a $\sigma_1\sigma_2$-closed subset F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq X - U$. Hence, we obtain $f(U) \subseteq Y - F$. On the other hand, since $B \subseteq F$ $\Rightarrow Y - F \subseteq Y - B = f(U) \Rightarrow$ $Y - F \subseteq f(U)$. Thus $f(U) = Y - F$ which is $\sigma_1\sigma_2$-open and hence f is a quasi (1,2)* b-open function.

(2.12) Theorem:

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is quasi (1,2)* b-open iff $f^{-1}(\sigma_1\sigma_2 cl(B)) \subseteq (1,2)^* bcl(f^{-1}(B))$ for every subset B of Y.

<u>Proof:</u> $\Rightarrow$

Suppose that f is quasi (1,2)* b-open. To prove that $f^{-1}(\sigma_1\sigma_2 cl(B)) \subseteq (1,2)^* bcl(f^{-1}(B))$ for every subset B of Y. Since $f^{-1}(B) \subseteq (1,2)^* bcl(f^{-1}(B))$ for any subset B of Y, then by (2.11) there exists a $\sigma_1\sigma_2$-closed set F in Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq (1,2)^* bcl(f^{-1}B))$. Since $B \subseteq F \Rightarrow$ $\sigma_1\sigma_2 cl(B) \subseteq \sigma_1\sigma_2 cl(F) = F$. Therefore, we obtain $f^{-1}(\sigma_1\sigma_2 cl(B)) \subseteq f^{-1}(F) \subseteq (1,2)^* bcl(f^{-1}(B))$. Thus $f^{-1}(\sigma_1\sigma_2 cl(B)) \subseteq (1,2)^* bcl(f^{-1}(B))$ for every subset B of Y.

Conversely, let $B \subseteq Y$ and F be an (1,2)* b-closed subset of X such that $f^{-1}(B) \subseteq F$. Put $W = \sigma_1\sigma_2 cl(B)$, then we have $B \subseteq W$ and $f^{-1}(W) = f^{-1}(\sigma_1\sigma_2 cl(B)) \subseteq (1,2)^* bcl(f^{-1}(B))$ $\subseteq (1,2)^* bcl(F) = F$. Then by theorem (2.11) f is a quasi (1,2)* b-open function.

However the following theorem holds. The proof is easy and hence omitted.

(2.13) Theorem:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ be two functions. Then:-

1) If f and g are quasi (1,2)* b-open, then $g \circ f$ is quasi (1,2)* b-open.

2) If f and g are quasi (1,2)* b-open, then $g \circ f$ is pre-(1,2)* b-open.

3) If f is quasi (1,2)* b-open and g is (1,2)*-open, then $g \circ f$ is quasi (1,2)* b-open.

4) If f is quasi (1,2)* b-open and g is (1,2)* b-open, then $g \circ f$ is pre-(1,2)* b-open.

5) If f is quasi (1,2)* b-open and g is pre-(1,2)* b-open, then $g \circ f$ is pre-(1,2)* b-open

6) If f is (1,2)* b-open and g is quasi (1,2)* b-open, then $g \circ f$ is (1,2)*-open.

7) If f is pre-(1,2)* b-open and g is quasi (1,2)* b-open, then $g \circ f$ is quasi (1,2)* b-open.

8) If f is (1,2)*-open and g is quasi (1,2)* b-open, then $g \circ f$ is (1,2)*-open.

(2.14)Definition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2)* b-continuous if $f^{-1}(V)$ is (1,2)* b-open set in X for every $\sigma_1\sigma_2$-open set V in Y.

(2.15)Proposition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is (1,2)* b-continuous iff $f^{-1}(V)$ is (1,2)* b-closed set in X for every $\sigma_1\sigma_2$- closed set V in Y.

(2.16)Definition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be contra (1,2)* b-irresolute if $f^{-1}(V)$ is (1,2)* b-closed set in X for every (1,2)* b-open set V in Y.

(2.17)Theorem:

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two functions. Then:-

1) If $g \circ f$ is quasi (1,2)* b-open and g is (1,2)*-continuous and one-to-one, then f is quasi (1,2)* b-open.

2) If $g \circ f$ is quasi (1,2)* b-open and g is (1,2)* b-continuous and one-to-one, then f is pre-(1,2)* b-open.

3) If $g \circ f$ is contra (1,2)* b-irresolute and g is quasi (1,2)* b-open and one-to-one, then f is contra (1,2)* b-irresolute.

4) If $g \circ f$ is quasi (1,2)* b-open and f is (1,2)* b-irresolute and onto, then g is (1,2)*-open.

Proof:

1) To prove that $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is quasi (1,2)* b-open. Let U be an (1,2)* b-open subset of X,since $g \circ f$ is quasi (1,2)* b-open, then $(g\circ f)(U)$ is $\eta_1\eta_2$-open in Z. Since g is (1,2)*-continuous, then $g^{-1}(g\circ f(U))=(g^{-1}\circ g)(f(U))$ is $\sigma_1\sigma_2$-open in Y. Since g is one-to-one, then $f(U)$ is $\sigma_1\sigma_2$-open in Y. Thus $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a quasi (1,2)* b-open function.

2) To prove that $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is pre-(1,2)* b-open. Let U be an (1,2)* b-open subset of X, since $g\circ f$ is quasi (1,2)* b-open, then $(g\circ f)(U)$ is $\eta_1\eta_2$-open in Z. Since g is (1,2)* b-continuous, then $g^{-1}(g\circ f(U))=(g^{-1}\circ g)(f(U))$ is (1,2)* b-open in Y. Since g is one-to-one, then $f(U)$ is (1,2)* b-open in Y. Thus $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is a pre-(1,2)* b-open function.

3) To prove that $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is contra (1,2)* b-irresolute. Let U be an (1,2)* b-open subset of Y, since g is quasi (1,2)* b-open, then $g(U)$ is $\eta_1\eta_2$-open in Z.

Since every $\eta_1\eta_2$-open set is (1,2)* b-open, then $g(U)$ is (1,2)* b-open in Z. Since $g\circ f$ is contra (1,2)* b-irresolute, then $(g\circ f)^{-1}(g(U))$ is (1,2)* b-closed in X, since g is one-one, then $(g\circ f)^{-1}(g(U))=f^{-1}(g^{-1}\circ g)(U))=f^{-1}(U)$ is an (1,2)* b-closed set in X, hence $f^{-1}(U)$ is (1,2)* b-closed in X. Thus $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is contra (1,2)* b-irresolute.

4) To prove that $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is (1,2)* -open. Let U be a $\sigma_1\sigma_2$-open subset of Y, then U is an (1,2)* b-open subset of Y, since f is (1,2)* b-irresolute, then $f^{-1}(U)$ is an (1,2)* b-open set in X, since $g\circ f$ is quasi (1,2)* b-open, then $(g\circ f)(f^{-1}(U))=g(f\circ f^{-1}(U))$ is $\eta_1\eta_2$-open in Z. Since f is onto, then $g(U)$ is $\eta_1\eta_2$-open in Z. Thus $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ is an (1,2)* -open function.

3. Quasi (1,2)* b-closed Functions

(3.1) Definition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be (1,2)* b-closed if the image of every $\tau_1\tau_2$-closed subset of X is (1,2)* b-closed set in Y.

(3.2) Definition:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is said to be quasi (1,2)* b-closed if the image of every (1,2)* b-closed set in X is $\sigma_1\sigma_2$-closed set in Y.

(3.3) Proposition:

Every quasi (1,2)* b-closed function is (1,2)*-closed as well as (1,2)* b-closed.

(3.4) Remark:

The converse of (3.3) may not be true in general. Consider the following example.

<u>Example:</u>

Let $X=Y=\{a,b,c\}$ & $\tau_1=\{X,\phi,\{a,c\}\}$, $\tau_2=\{X,\phi\}$, $\sigma_1=\{Y,\phi,\{a,c\}\}$ & $\sigma_2=\{Y,\phi,\{a\}\}$. So the sets in $\{X,\phi,\{b\}\}$ are $\tau_1\tau_2$-closed in X and the sets in $\{Y,\phi,\{b\},\{b,c\}\}$ are $\sigma_1\sigma_2$-closed in Y.

Also, $(1,2)^*BC(X,\tau_1,\tau_2)=\{X,\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\}\}$ & $(1,2)^*BC(Y,\sigma_1,\sigma_2)=\{Y,\phi,\{b\},\{c\},\{b,c\}\}$.

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by: $f(a)=a, f(b)=b$ & $f(c)=c$. It is clear that f is (1,2)* b-closed as well as (1,2)*-closed, but f is not quasi (1,2)* b-closed, since $\{a\}$ is (1,2)* b-closed in X, but $f(\{a\})=\{a\}$ is not $\sigma_1\sigma_2$-closed in Y.

(3.5) Proposition:

Every quasi (1,2)* b-closed function is pre-(1,2)* b-closed.

(3.6) Remark:

The converse of (3.5) may not be true in general. In (2.6), f is pre-(1,2)* b-closed, but f is not quasi (1,2)* b-closed, since $\{a\}$ is (1,2)* b-closed in X, but $f(\{a\})=\{a\}$ is not $\sigma_1\sigma_2$-closed in Y.

Thus we have the following diagram:

quasi (1,2)* b-closed ⇒ **pre-(1,2)* b-closed**
⇓ ⇓
(1,2)* -closed ⇒ **(1,2)* b-closed**

(3.7) Theorem:

A bijective function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is quasi (1,2)* b-closed iff it is quasi (1,2)* b-open.

<u>Proof:</u>

Let f be a quasi (1,2)* b-closed function. To prove that f is a quasi (1,2)* b-open function.Let U be an (1,2)* b-open set in $X \Rightarrow U^c$ is (1,2)* b-closed in X. Since f is quasi (1,2)* b-closed, then $f(U^c)$ is $\sigma_1\sigma_2$-closed in Y. Therefore $(f(U^c))^c$ is $\sigma_1\sigma_2$-open in Y. Since f is a bijective function, then $(f(U^c))^c=f(U) \Rightarrow f(U)$ is $\sigma_1\sigma_2$-open in Y. Thus $f:X\to Y$ is a quasi (1,2)* b-open function.

Conversely, Suppose that $f:X\to Y$ is quasi (1,2)* b-open. To prove that f is quasi (1,2)* b-closed. Let F be an (1,2)* b-closed set in X $\Rightarrow F^c$ is (1,2)* b-open in X. Since f is quasi (1,2)* b-open,then $f(F^c)$ is $\sigma_1\sigma_2$-open in Y. Therefore $(f(F^c))^c$ is $\sigma_1\sigma_2$-closed in Y. Since f is a bijective function, then $(f(F^c))^c=f(F) \Rightarrow f(F)$ is $\sigma_1\sigma_2$-closed in Y. Thus $f:X\to Y$ is a quasi (1,2)* b-closed function.

(3.8) Theorem:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ from a bitopological space X into a bitopological space Y is quasi (1,2)* b-closed iff $\sigma_1\sigma_2cl(f(F))\subseteq f((1,2)^*bc(F))$ for every subset F of X.

<u>Proof:</u>

$\Rightarrow$ Let f be a quasi (1,2)* b-closed function . To prove that $\sigma_1\sigma_2cl(f(F))\subseteq f((1,2)^*bc(F))$ for every subset F of X. By (1.6) no.2, $F\subseteq(1,2)^*bc(F) \Rightarrow f(F)\subseteq f((1,2)^*bc(F))$. Since $(1,2)^*bc(F)$ is an (1,2)* b-closed set in X and f is quasi (1,2)* b-closed, then $f((1,2)^*bc(F))$ is $\sigma_1\sigma_2$-closed in Y. Thus $\sigma_1\sigma_2cl(f(F))\subseteq f((1,2)^*bc(F))$ for every subset F of X.

Conversely:

Suppose that $\sigma_1\sigma_2cl(f(F))\subseteq f((1,2)^*bc(F))$ for every subset F of X. To prove that f is quasi (1,2)* b-closed. Let F be an (1,2)* b-closed set in X. Then by (1.6) no. 4, $F=(1,2)^*bc(F) \Rightarrow f(F)=f((1,2)^*bc(F))$. By hypothesis $\sigma_1\sigma_2cl(f(F))\subseteq f((1,2)^*bc(F))$ $\Rightarrow \sigma_1\sigma_2cl(f(F))\subseteq f((1,2)^*bc(F))=f(F)$. But $f(F)\subseteq\sigma_1\sigma_2cl(f(F))$. Consequently $f(F)=\sigma_1\sigma_2cl(f(F)) \Rightarrow f(F)$ is a $\sigma_1\sigma_2$-

closed set in Y. Thus f is a quasi (1,2)* b-closed function.

(3.9) Theorem:

A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is quasi (1,2)* b-closed iff for any subset B of Y and for any (1,2)* b-open set G of X containing $f^{-1}(B)$, there exists a $\sigma_1\sigma_2$-open set U of Y containing B such that $f^{-1}(U)\subseteq G$.

Proof:

This proof is similar to that of theorem (2.11).

However the following theorem holds. The proof is easy and hence omitted.

(3.10) Theorem:

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two functions. Then:-

1) If f and g are quasi (1,2)* b-closed, then $g\circ f$ is quasi (1,2)* b-closed.

2) If f and g are quasi (1,2)* b-closed, then $g\circ f$ is pre-(1,2)* b-closed.

3) If f is quasi (1,2)* b-closed and g is (1,2)*-closed, then $g\circ f$ is quasi (1,2)* b-closed.

4) If f is quasi (1,2)* b-closed and g is (1,2)* b-closed, then $g\circ f$ is pre-(1,2)* b-closed.

5) If f is quasi (1,2)* b-closed and g is pre-(1,2)* b-closed, then $g\circ f$ is pre-(1,2)* b-closed.

6) If f is (1,2)* b-closed and g is quasi (1,2)* b-closed, then $g\circ f$ is (1,2)*-closed.

7) If f is pre-(1,2)* b-closed and g is quasi (1,2)* b-closed, then $g\circ f$ is quasi (1,2)* b-closed.

8) If f is (1,2)* -closed and g is quasi (1,2)* b-closed, then $g\circ f$ is (1,2)*-closed.

(3.11) Theorem:

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two functions Then:-

1) If $g\circ f$ is quasi (1,2)* b-closed and g is (1,2)*-continuous and one-to-one, then f is quasi (1,2)* b-closed.

2) If $g\circ f$ is quasi (1,2)* b-closed and g is (1,2)* b-continuous and one-to-one, then f is pre-(1,2)* b-closed.

3) If $g\circ f$ is quasi (1,2)* b-closed and f is (1,2)* b-irresolute and onto, then g is (1,2)*-closed.

4) If $g\circ f$ is contra (1,2)* b-irresolute and f is quasi (1,2)* b-closed and onto, then g is contra (1,2)* b-irresolute.

Proof:

This proof is similar to that of theorem (2.17).

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الخلاصة

إن الهدف الرئيسي من هذا البحث هو تقديم أنواع خاصة من الدوال المفتوحة والدوال المغلقة في الفضاءات التبولوجيه الثنائية أسميناها بالدوال الكوازى المفتوحة- (quasi (1,2)* b - (1,2)* b-open functions) و الدوال الكوازي المغلقة - (quasi (1,2)* b-closed functions.)(1,2)* b كذلك نحن درسنا المكافئات والخواص الأساسية للدوال الكوازى المفتوحة-b (1,2)* و الدوال الكوازي المغلقة -b (1,2)*.