Develop a Nonlinear Model for the Conditional Expectation of the Bayesian Probability Distribution (Gamma – Gamma)

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Abstract

In this paper a method has been suggested to describe the conditional expectation of Bayesian probability distribution (Gamma-Gamma) by nonlinear regression model and using power transformation for the observations of the predictor variables in the observable distribution to get the best possible fitting to the model of the posterior conditional expectation. The parameters of the described model have been estimated by depending on experimental data which has been generated using different values for the parameters of conditional probability distribution. The best estimation of the power parameter of the described model was found by using Draper & Smith method which gave best fitting of the suggested model and best estimate for the conditional expectation of the Bayesian Probability Distribution (Gamma–Gamma).

Keywords**:** *Conditional Expectation CE, Gamma Distribution, Bayesian Probability Distribution (Gamma-Gamma), Linear Regression Model, Power Transformation.*

Introduction

This research interested in the recruitment regression model to express the CE equation when a theoretical equation of this expectation- derived from the conditional distribution- unable in recruitment data to describe the relation, through the use of power transformation applied it in a linear regression model to construct a nonlinear model succeed in describing the relation.

In the experimental aspect, CE equation that has been selected, has nonlinear formula – of the Bayesian probability distribution (Gamma-Gamma) to develop a model of nonlinear regression that represents this expectation. To achieve possible better fitting for this model, power transformation to the response variable in regression linear model to transform it into a nonlinear model has been used. The research included the theoretical framework of the probability distribution is assumed as well as the theoretical framework for the application of the proposed method for this distribution. It also includes an experimental part of the application of the proposed method.

The research aims to propose a method to describe the efficiently nonlinear model satisfy best fitting to the CE function of the Bayesian probability distribution (Gamma-Gamma) by linking conditional expectation Equation. $E(Y|x,\theta)$ derived from the distribution function with nonlinear regressionmodel $E(Y|\psi^{\lambda}x;\theta)$ using power transformation.

Bayesian Probability Distribution (Gamma-Gamma)

Assuming that γ is a random variable distributed according to gamma distribution with the following Probability function,

$$f_{y_i}(y_i;y,\delta)=\frac{1}{\Gamma(y)\delta^{y}}\,y_i^{y-1}e^{-y/\delta}$$
$$,y_i \rangle 0, y, \delta \rangle 0$$
$$i=1,2,......,n \quad .... (1)$$

Where n is the number of values of the random variable Y. Assuming that the values of the random variable Y represent the scale parameters of the probability density functions for n samples of the random variable X selected from population distributed according to gamma distribution with following probability functions,

$$f_{x_{ji}/y_i}(x_{ji}/y_i;\theta_i) = \frac{1}{\Gamma(\theta_i) y_i^{\theta_i}} x_{ji}^{\theta_i - 1} e^{-x_{ji}/y_i}$$

$x_{ji} > 0 \quad ,\theta_i > 0$

$j = 1, 2, \dots \dots, m$ .... (2)

Where m is the number of values of the random variable X in each selected sample from the population. Using Bayes theorem of conditional probability defined according to the following equation,

$$f_{Y i/x_{ji}}(y_i/x_{ji};\theta_i) = \frac{f_{x_{ji}/y_i}(x_{ji}/y_i;\theta_i) f_{Y_i}(y_i)}{\int f_{x_{ji}/y_i}(x_{ji}/y_i;\theta_i) f_{Y_i}(y_i) dy_i} \quad \text{................ (3)}$$

Then the conditional distribution of the random variable Y relative to the random variables X_j in the i^th^ sample defined according to the following equation,

$$f_{Y i/x_{ji}}(y_i/x_{ji}) = \frac{(x_{ji} + (1/\delta_i))^{\gamma_i + \theta_i}}{\Gamma(\gamma_i + \theta_i)} y_i^{(\gamma_i + \theta_i) - 1} e^{-(x_{ji} + (1/\delta_i)) y_i}$$

$\gamma_i, \delta_i, \theta_i > 0$ .... (4)

The CE for Y_i relative to X_{ji} defined according to the following equation,

$$E(Y_i/x_{ji}) = \frac{\gamma_i + \theta_i}{x_{ji} + (1/\delta_i)} \quad \text{.................... (5)}$$

To transform CE equation (5) to univariat statistical model requires dealing with a function in terms of samples of a random variable X; it was assumed that this function is the mean of the samples. It is known that the probability distribution of the arithmetic means of a random variable distributed according to the gamma distribution cannot be changed and defined according to the following Gamma function equation,

$$f_{\bar{X}_i}(\bar{x}_i/y_i;\theta_i) = \frac{1}{\Gamma(n\theta_i)(y_i/n)^{n\theta_i}} \bar{x}_i^{(n\theta_i - 1)} e^{-n\bar{x}_i/y_i}$$

$\bar{x}_i > 0$ .... (6)

And the CE for $\bar{X}_i$ relative to y_i defined according to the following equation,

$$E(\bar{X}_i/y_i) = \theta_i y_i$$

Using equation (3), the posterior distribution of the variable Y can be defined according to the following Gamma function equation,

$$f_{Y_i}(y_i/\bar{x}_i) = \frac{(n\bar{x}_i + (1/\delta_i))^{n\theta_i + \gamma_i}}{\Gamma(n\theta_i + \gamma_i)} y_i^{(n\theta_i - \gamma_i) - 1} e^{-(n\bar{x}_i + (1/\delta_i)) y_i}$$

.... (8)

And the CE for Y relative to $\bar{X}$ defined according to the following equation,

$$E(Y_i/\bar{x}_i) = \frac{n\theta_i + \gamma_i}{n\bar{x}_i + (1/\delta_i)} \quad \text{..................... (9)}$$

The main idea of the research is to develop the CE equation (9) to efficiently model to estimate the conditional expectation at their best for fitting the requirements.

Proposed Application Method for the Develop a Nonlinear Model for the CE of Posterior Distribution

The aim of this research is to propose a methodology of application to develop a nonlinear regression model for the CE equation $E_i(Y_i/\bar{x}_i)$ which is derived from the posterior distribution and defined according to the equation (9) to an efficient model by re-estimating a parameter values θ_i in the linear regression model for the CE equation $E(\bar{X}_i/y_i)$ which is derived from the observable distribution and is defined according to the equation (7) and compensated in the equation (9). According to the preceding ideas, the researchers have set the research hypothesis in such a way to reach in the first step at to transferring the functions of the index Y_i from prior variable in the prior distribution as in equation (1) to the scale parameter in observable distribution as in equation (2), and then to explanatory variable in a system of linear equations as in system (7), and then to posterior variable in a posterior distribution as in equation (8), as well as getting access to transfer the index θ_i from location parameters as in the set of functions (2) to a set of slops in a system of linear equations as in the system (7). The transitions of Y_i and θ_i are were the base of the proposal research which develops the posterior

CE model (9) into a model based on the re-estimating the slops in the system (7) according the reality of its relationship with the prior variable Y_i. This means that, a new nonlinear model of CE of posterior variable is obtained in the end in terms of the estimation of the prior variable because it represented the parameter of observable distribution, or briefly the reason is due to the use the CE equation of the observable distribution to develop the CE equation for posterior distribution.

According to the preceding ideas, the proposed methodology focuses on re-estimating the parameter θ_i and assumes the parameters γ_i and δ_i are known by developing a nonlinear relationship linking θ_i and Y_i with $\bar{X}_i$. The basic idea of re-estimating was shifted the data response variable Y_i using power parameter λ to develop the regression model $E_i(\bar{X}_i/\phi(y_i) = y_i^{\lambda})$ which describes the nonlinear relationship between $\bar{X}_i$ and the original values for Y_i and describes the linear relationship between $\bar{X}_i$ and the transformed values (y_i). In the end, the best estimated values for the parameter θ_i are the values resulting from the equality of the linear model $E_i(\bar{X}_i/y_i)$ with the linear regression model $E_i(\bar{X}_i/\phi(y_i))$.

The proposals to re-estimate the parameter θ_i by equating the two previous models represent exactly the process of estimating the polynomial model $E_i(\theta_i/\varphi(\phi(y_i)),\omega(y_i))$ where $\varphi(\phi(y_i))$ represents a function of the transformed variable y_i^{λ} and $\omega(y_i)$ represents a function of the original variable y_i.

Returning to the equation (8) of CE of $\bar{X}_i$ that relative to Y and to get the best estimate of the expectation, the following nonlinear regression model can be assumed,

$$E(\bar{X}_i/y_i) = \begin{cases} \alpha + \beta Y_i^{\lambda} + \epsilon_i & , \quad \lambda \neq 0 \\ \alpha + \beta \ln Y_i + \epsilon_i & , \quad \lambda = 0 \end{cases} \qquad \text{.................................... (10)}$$

Where α and β represent the model parameters and ϵ_i represent the random error and λ represent the parameter of the power transformation to transform the data of the random variable Y to get the best fitting of the model. Equation (10) represents a nonlinear model to describe the relation between the variables Y and $\bar{x}_i$. On the other hand, it is a linear model to describe the relation between $\phi(y) = Y_i^{\lambda}$ and $\bar{x}_i$ when $\lambda \neq 0$ and between $\phi(y) = \ln Y_i$ and $\bar{x}_i$ when $\lambda = 0$. Finney [1971] first proposed this transformation at the end of the forties of the last century to treat the lack of fitting the biological experiments models through the transformation of dose variable data (explanatory variable) according to the equation η (x) = xλ when λ≠0 and η (x)=ln x when λ = 0. A large number of transformations were developed by several researchers, such as Tukey's transformation [1949] and [1957] and Box-Cox transformation [1964] as a common transformation that has become known as the "Box Cox Family". For more see, Weisberg [2005], and Klein, Entink, W. Linden and Fox [2009].

There are many ways to estimate the parameter of the power transformation, such as the method of Box & Cox [1964] which estimates the model parameters and power parameter by using maximum likelihood method in an iterative manner. In [1985] Breiman & Freidman develop a way known algorithm "ACE" (Alternating Conditional Expectation) which are summarized as iterative method and are also to estimate the parameter of power transformation and other model parameters according to the base's decision summarized by reducing the unexplained variance in the multiple linear regression model, see Wang & Murphy, [2004]. Agarwal & Freidman in [2009] develop another way called power transformation weighting in least squares analysis in the case of inhomogeneity of variance. In this research, a method developed by Draper & Smith in [1998] has been used. The method has been summed up in use as an iterative way to work out a number of default values for transformation parameter that is chosen from a certain range, which is usually a closed interval $[-2,2]$. In each experiment, after working out a default value of λ, other model parameters are estimated by the Ordinary Least Squares method and in the end a best value of λ is selected according to the decision rule (either maximizing the value if we want to maximize the value of the coefficient of determination or decreasing the value if we want to minimize the mean square error MSE of the model).

Equating the CE equation (7) to the estimation of the regression model (10), the following n of new functions to estimate θ_k is obtained,

$$\theta_i^{\sim} = \begin{cases} \Psi\left(y_i, y_i^{\lambda^\wedge}\right) & , \quad \lambda^\wedge \neq 0 \\ \Psi(y_i, \ln y_i) & , \quad \lambda^\wedge = 0 \end{cases}$$

$$\lambda^\wedge \epsilon\, [-2,2] \quad , i = 1, 2, \ldots \ldots, n \quad \ldots\ldots\, (11)$$

After obtaining the estimated value $\theta_k^{\sim}$ to be compensated in the CE equation (9) to obtain the proposed nonlinear model of estimating the CE for the Bayesian probability distribution (Gamma–Gamma),

$$E_i(Y_i/\bar{x}_i) = \begin{cases} \dfrac{n\,\Psi\left(y_i, y_i^{\lambda}\right) + \gamma}{n\,\bar{x}_i + (1/\delta)} & , \lambda \neq 0 \\ \dfrac{n\,\Psi(y_i, \ln y_i) + \gamma}{n\,\bar{x}_i + (1/\delta)} & , \lambda \neq 0 \end{cases}$$

$$\lambda^\wedge \epsilon\, [-2,2] \quad , i = 1, 2, \ldots \ldots, n \quad \ldots\ldots\ldots\ldots\, (12)$$

That means that, and as explained previously, the estimation of the CE of the posterior distribution of the Bayesian probability distribution (Gamma–Gamma) depends on estimated functions arising from the relation between the prior distribution and observable distribution. This relation was produced the functions $\Psi\left(y_i, y_i^{\lambda^\wedge}\right)$ and $\Psi(y_i, \ln y_i)$ in the shape of a new nonlinear model.

Estimate the Parameters

As explained in the previous section, the proposed methodology, as shown in the target model (9), assumes that the values of the parameters θ_i are unknown and all of the other parameters are constant. The method of moments to estimate represents an alternative way for the maximum likelihood method. The first and second moments are equal,

$$E(X_{ji}) = \frac{\sum x_{ji}}{m} \quad , \quad E(X_{ji}{}^2) = \frac{\sum x_{ji}^2}{m}$$

Because,

$$E(X_{ji}) = \frac{\theta_i}{y_i} \quad , \quad E(X_{ji}{}^2) = \frac{\theta_i(\theta_i + 1)}{y_i{}^2}$$

Then,

$$\theta_i^\wedge = \frac{\bar{x}_i^2}{\frac{\sum x_{ji}^2}{n} - \bar{x}_i^2} , y_i^\wedge = \frac{\bar{x}_i}{\frac{\sum x_{ji}^2}{n} - \bar{x}_i^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\, (13)$$

Simulation Experiment

In this section, “Minitab Program” is used to generate the observable distribution data which follows the gamma distribution of two parameters (γ, δ) in three experiments and assumed it was equal to the following different values of sample sizes, $n = 10, 20, 30$. And then was used all the values of the variable y were used as a scale parameter to generate a sample of prior distribution variable Xi, which takes the same sample size used in the observable distribution.Therefore, we have $(10, 20, 30)$ samples of prior distribution , $f_{X_{ji}/y_i}(x_{ji}/y_i; \theta_i)$ which are distributed according gamma distribution also with parameters θ_i and y_i so that θ was assumed to equal different values for each vvalueof γ and δ are used to generate y. In this step of proposed application steps, the researchers have felt the need to deal with one observation of X against one observation of Y. To deal with this problem, the researchers have chosen an arithmetic mean to represent each sample of the prior distribution which is distributed according to the gamma distribution, and also with two parameters $n\theta$ and y/n. The steps of the proposed application method are shown in the following steps. Tables (1) and (2) describes one case of the assumption that $n = 10$, and $\gamma = 3.5$, $\delta = 0.5$, $\theta = 1$.

Step 1: Compensation value of the assumed parameters $\gamma = 3.5$, $\delta = 0.5$, $\theta = 1$ for which data have been generated on its basis in the CE equation of the posterior distribution get the true conditional expectation values ui,

$$u_i = E_i(Y_i/\bar{x}_i; \gamma, \delta, \theta) = \frac{n\theta + \gamma}{n\bar{x}_i + (1/\delta)}$$

$$i = 1, 2, \ldots \ldots, n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\, (14)$$

To get the first column of the Table (1).

Step 2: Using the method of moments, the estimated values of the parameters θ_i of the distribution functions $f_{x_{ji}/y_i}(x_{ji}/y_i;\theta_i)$, are defined in accordance to the equation,

$$\theta_i^{\wedge} = \frac{\bar{x}_i^2}{\frac{\Sigma x_{ji}^2}{n} - \bar{x}_i^2} \quad ....(15)$$

These estimated values are shown in the second column of the Table (1). And compensation values of this column in the equation of CE for posterior distribution are defined by the following equation, get the values of z_i shown in the third column, which represents conditional expectation values estimated from the data,

$$z_i = E_i(Y/\bar{x}_i;\theta_i^{\wedge}) = \frac{n\theta_i^{\wedge} + \gamma}{n\bar{x}_i + (1/\delta)}$$

$$\gamma = 3.5, \delta = 0.5, i = 1, 2, \dots\dots, n \quad .....(16)$$

Step 3: Estimate MSE_1 using the following equation,

$$MSE_1 = \frac{\sum(z_i - u_i)^2}{n} \quad ....(17)$$

Step 4: Estimate the following regression equation,

$$\bar{X}_i = \begin{cases} \alpha + \beta Y_i^{\lambda} + \epsilon_i & , \quad \lambda \neq 0 \\ \alpha + \beta \ \ln Y_i + \epsilon_i & , \quad \lambda = 0 \end{cases} \quad ....(18)$$

By selecting the best estimator of λ through compensation, the assumed values are usually taken from a range$(-2,2)$ [see table (1)]. The best estimated value of the parameter λ is those that maximize the value of R^2. The previous studies show that the relationship curve between the values of the coefficient of determination and power parameter have single summit, see Box. & Cox [1964] and Wang & Michael [2005]. In the end get,

$$E_i(\bar{X}_i/y_i) = \begin{cases} \alpha^{\wedge} + \beta^{\wedge} y_i^{\lambda^{\wedge}} \\ \alpha^{\wedge} + \beta^{\wedge} \ln y_i \end{cases} \quad , \lambda^{\wedge} \epsilon [-2,2]$$

$$i = 1, 2, \dots\dots, n \quad ....(19)$$

Step 5: Return to the CE for the prior distribution of $f_{\bar{x}_i}(\bar{x}_i/y_i;\theta_i)$ defined according to the following linear equation,

$$E(\bar{X}_i/y_i) = \theta_i y_i \quad ....(20)$$

and equating the two previous expectation equations (24) and (25), get,

$$\theta_i^{\sim} = \begin{cases} \Psi(y_i, y_i^{\lambda^{\wedge}}) = \alpha^{\wedge} y_i^{-1} + \beta^{\wedge} Y^{\lambda^{\wedge}-1} \\ \Psi(y_i, \ln y_i) = \alpha^{\wedge} y_i^{-1} + \beta^{\wedge} y_i^{-1} \ln y_i \end{cases}$$

$$\lambda^{\wedge} \epsilon [-2,2] \quad , i = 1, 2, \dots\dots, n \quad ....(21)$$

and from equation, estimates of $\theta^{\sim}$ can be obtained and described in the fourth column of Table (1). They are then compensated in the CE equation (12) to obtain new values for that expectation is defined as the following equation and shown in the fifth column of the table (1),

$$h_i = E_i(Y/\bar{x}_i;\theta_i^{\sim})$$

$$= \begin{cases} \frac{n(\alpha^{\wedge} y_i^{-1} + \beta^{\wedge} y_i^{\lambda^{\wedge}-1}) + \gamma}{n\,\bar{x}_i + (1/\delta)} & \lambda \neq 0 \\ \frac{n(\alpha^{\wedge} y_i^{-1} + \beta^{\wedge} y_i^{-1} \ln y_i) + \gamma}{n\,\bar{x}_i + (1/\delta)} & \lambda \neq 0 \end{cases}$$

$$................................(22)$$

Step 6: Estimate MSE_2 using h_i values according to the following equation,

Table (1)

u_i	$\theta_i^{\wedge}$	z_i	$\theta_i^{\sim}$	h_i
1.19	1.00	1.19	1.05	1.23
0.47	0.75	0.38	0.87	0.43
1.23	1.39	1.58	1.16	1.37
0.70	0.56	0.47	1.06	0.73
0.61	1.52	0.84	0.86	0.54
1.12	1.4	1.49	1.21	1.30
0.98	1.32	1.21	0.99	0.97
1.26	0.94	1.20	1.24	1.48
0.52	1.11	0.57	0.94	0.50
0.50	1.76	0.79	0.92	0.47
		$MSE_1 = 0.05$		$MSE_2 = 0.01$

$$MSE_2 = \frac{\sum(h_i - u_i)^2}{n} \quad \text{..............................} (23)$$

The Results and Conclusions

Table (1) illustrates the status of one of the first experiment cases of the assumption that n = 10, and $\gamma = 3.5, \delta = 0.5, \theta = 1$. The results of the remaining cases are shown in Table (3).

As known in the first step of the application of the proposed method, u_i represents the values of CE calculated from the original data after compensation default values for the parameters. On re-estimating parameter θ using moments method, estimator values z_i were obtained for the CE, either when estimating the distribution parameter according to the research suggestion by developing the parameter in the location of response variable in the nonlinear regression model, estimator values h_i of CE were obtained and it is quite clear that it is better than its predecessor, "using the traditional estimators of the distribution parameters" based on the values of mean square errors "MSE_1 and MSE_2".

Estimated values for the parameter θ by moments and proposed methods and the values of conditional expectation values when $n = 10$ and $\gamma = 3.5$, $\delta = 0.5, \theta = 1$.

Table (2) and Fig.(1) illustrates the behavior of the relation between the default values of the power parameter and the estimated values of the determination coefficient. The best estimated value for the power parameter is equal to "0.7" corresponds to the unique highest value of the coefficient of determination which is equal "71.89" .

Table (2)

Default values for the power parameter λ and determination coefficient values when $n = 10$ ***and*** $\gamma = 3.5$, $\delta = 0.5$, $\theta = 1$.

λ	R^2	λ	R^2	λ	R^2
-1	62.10	0.1	70.62	1.1	71.17
-0.9	63.08	0.2	71.03	1.2	70.81
-0.8	64.04	0.3	71.36	1.3	70.38
-0.7	64.97	0.4	71.62	1.4	69.89
-0.6	65.86	0.5	71.79	1.5	69.35
-0.5	66.71	0.6	71.88	1.6	68.76
-0.4	67.51	0.7	71.89	1.7	68.12
-0.3	68.26	0.8	71.83	1.8	67.43
-0.2	68.95	0.9	71.68	1.9	66.71
-0.1	69.58	1	71.46	2	65.96

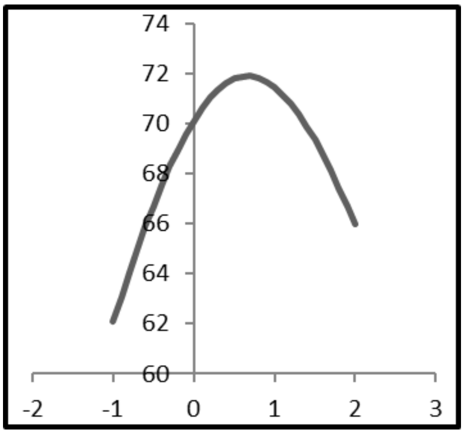

Fig. (1) The relation curve between the parameter transformation values And the determination coefficient values R^2 ***for the first experiment.***

Table in the appendix shows the results of other experiments that have been conducted to different sample sizes and different parameter values. The first and third numbers in each cell of the fourth column refer to the lower and upper limits of the set of default values of λ, while the second number refers to the value of λ that correspond to the largest value of the determination coefficient – that it represent $\lambda^{\wedge}$ in equation (19), i.e. the estimation of the power parameter λ.

The first and third numbers in each cell of the fifth column representing upper and lower limits of the set of values of the determination coefficient for the estimated regression models (19)-which have got a result of using a set of default values of λ to transform the response variable-while the second number represents the unique higher value of the coefficient of determination, which represents the base of decision rule in choosing the best estimate of the power parameter.

The obtained values of MSE_1 and MSE_2 show the success of the proposed application methodology to achieve the aim of the research. It can be observed that the estimated power parameters are close to "one" when the sample size grows up, because the distribution of these samples begins to approach the normal distribution and does not need to using a power parameter stray too far from the "one" to transform the response variable. This exactly corresponds to the statistical theory which means that the proposed methodology is not out of the framework of the statistical logic.

Appendix

Values of MSE_1 and MSE_2 for some experiments using different parameter.

Exp't	*Default sample size*	*Default values of the parameters*	*(1) λ*	*(2) R^2*	MSE_1	MSE_2
1	n = 10	$\gamma = 2 \quad \delta = 3 \quad \theta = 4$	-1	52.40	0.12351	0.00548
			0.9	82.21		
			2	73.30		
2	n = 10	$\gamma = 25 \quad \delta = 3.8 \quad \theta = 20$	-1	82.41	0.00004	0.00000
			0.7	87.74		
			2	83.90		
3	n = 10	$\gamma = 4 \quad \delta = 4 \quad \theta = 4$	-1	73.48	0.03326	0.00013
			0.7	87.39		
			2	81.90		
4	n = 10	$\gamma = 5 \quad \delta = 3 \quad \theta = 9$	-1	62.20	0.00897	0.00001
			0.9	97.29		
			2	89.06		
5	n = 20	$\gamma = 3.5 \quad \delta = 0.5 \quad \theta = 1$	-1	52.23	0.0611	0.0012
			1.4	84.21		
			2	82.99		
6	n = 20	$\gamma = 2 \quad \delta = 3 \quad \theta = 4$	-1	55.96	0.0118	0.0028
			1.1	94.86		
			2	88.06		
7	n = 20	$\gamma = 25 \quad \delta = 3.8 \quad \theta = 20$	-1	89.41	0.0000	0.0000
			0.5	92.60		
			2	89.62		
8	n = 20	$\gamma = 4 \quad \delta = 4 \quad \theta = 4$	-1	68.88	0.0022	0.0000
			1.2	96.72		
			2	92.65		
9	n = 30	$\gamma = 3.5 \quad \delta = 0.5 \quad \theta = 1$	-1	67.67	0.0880	0.0127
			0.7	96.18		
			2	88.46		
10	n = 30	$\gamma = 2 \quad \delta = 3 \quad \theta = 4$	-1	58.59	0.0075	0.0000
			1	97.10		
			2	92.65		
11	n = 30	$\gamma = 25 \quad \delta = 3.8 \quad \theta = 20$	-1	86.51	0.00001	0.00000
			1.1	97.64		
			2	92.02		
12	n = 30	$\gamma = 4 \quad \delta = 4 \quad \theta = 4$	-1	50.565	0.00041	0.00029
			1.1	98.34		
			2	94.70		

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الخلاصة

في هذا البحث اقترحت طريقة لوصف التوقع الشرطي للتوزيع الاحتمالي البيزي (كاما – كاما) بانموذج انحدار لاخطي من خلال استخدام تحويل القوى لمشاهدات متغيرات الاستجابة في التوزيع المشاهد للحصول على افضل مطابقة ممكنة لانموذج التوقع الشرطي اللاحق. معلمات الانموذج الهدف تم تقديرها بالاعتماد بيانات تجريبية ولدت باستخدام قيم مختلفة لمعلمات التوزيع الاحتمالي الشرطي. تم ايجاد افضل تقدير لمعلمة القوى في الانموذج الهدف باستخدام طريقة Draper & Smith والتي يسرت للباحثين الحصول على افضل مطابقة للانموذج المقترح وافضل تقدير للتوقع الشرطي للتوزيع الاحتمالي البيزي (كاما – كاما)، مقارنة بطريقة العزوم.