The Impact of Switching on the Dynamics of Prey-Predator Model for a Switching Tendency

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Abstract

In this paper, a mathematical model, consists from a stage structured predator interacting with prey which is assumed to live in two distinct habitats, and the predator has the tendency to switch among these habitats, is proposed and analyzed. The existence and uniqueness of the solution of the proposed model are discussed. The existence and the stability analyses of all possible equilibrium points are studied. Because of difficulty of analytical computation the global stability of these equilibrium points and the persistence of the model will be studied numerically.

Keywords**:** equilibrium points, local stability, global stability, persistence.

Introduction

In a natural ecosystem, the interaction between the prey and their predator is a complex dynamical phenomenon. This complexity is attributed largely to the diverse individual behavior of the ecological populations. As far as the predator population is concerned, such behavior manifests itself mainly in the form of predation characteristic which is measured by the predation functional response. Beginning with the pioneering works of Alfred Lotka and Vito Volterra in 1920, a good volume of literature concerns itself with the functional response of the predator, how predators react to different prey densities and how this reaction changes predation rate [1-4]. Tansky [5] investigated mathematical model of one-predator and two –prey system which has switching property of predation. B. Mukhopadhyay and R. Bhattacharyya [6] studied an ecological food chain, where the prey species is segregated into two distinct habitats and the predator has the tendency to switch among these habitats, the authors analyze such a food chain model by study the Hopf-bifurcation of this model. In this paper we will study the model of Tanksy with stage structure for predator.

Mathematical model

In this section, an ecological model of one-predator and two–prey system which has switching property of predation with stage structure for predator is proposed. In order to formulate the dynamic equations for such model the following assumptions are made.

A1) The prey population is assumed to live in two distinct habitats whose population density at time t is denoted by $x(t)$ *and* $y(t)$.

A2) The predator population is divided into two classes, immature predator population, whose population density at time t is denoted by $z_1(t)$, and mature predator population, whose population density at time t is denoted by $z_2(t)$ and the predator consumes prey from these habitats with a switching tendency.

A3) The immature predator population transfer to mature predator population at a rate $E_3 z_1$, where E_3 $(E_3 > 0)$ represents the conversion rate coefficient. Finally, both the immature and mature predator populations decreases due to the natural death rates E_4 $(E_4 > 0)$ and r $(r > 0)$ respectively. Thus, depending on the above assumptions the evolution equations for our model can be written as:

$$\frac{dx_1}{dt} = E_1 x - \frac{a z_2 x^n x}{x^n + y^n} \quad \text{(1a)}$$

$$\frac{dy}{dt} = E_2 y - \frac{b z_2 y^n y}{x^n + y^n} \quad \text{(1b)}$$

$$\frac{dz_1}{dt} = -E_3 z_1 + \frac{a x^n z_2 x}{x^n + y^n} + \frac{b z_2 y^n y}{x^n + y^n} - E_4 z_1 \quad \text{(1c)}$$

$$\frac{dz_2}{dt} = E_4 z_1 - r z_2 \quad \text{.. (1d)}$$

where $E_1 > 0$ *and* $E_2 > 0$ are the growth rates of prey population, the functions

$$(\frac{a}{1+(\frac{y}{x})^n}) \quad and \quad (\frac{b}{1+(\frac{x}{y})^n}),$$

have the characteristic property of switching mechanism. Biologically, these functions signify the fact that the predatory rate, namely the frequency with which an individual of the prey species is attacked by a predator, decreases when the population of that species become rare compared to the population of the other species, a represent the predation coefficient of the first species and b represent the predation coefficient of the second species n (n>1) is the intensity of predator switching.

Then system (1) can be turned into the following dimensionless form:

$$\frac{dy_1}{dT} = y_1 - \frac{y_1^n y_4 y_1}{y_1^n + y_2^n} \quad \text{.. (2a)}$$

$$\frac{dy_2}{dT} = w_1 y_2 - w_2 \frac{y_4 y_2^n y_2}{y_1^n + y_2^n} \quad \text{............................ (2b)}$$

$$\frac{dy_3}{dT} = -w_3 y_3 + w_2 \frac{y_1^n y_4 y_1}{y_1^n + y_2^n} + w_2^2 \frac{y_4 y_2^n y_2}{y_1^n + y_2^n} - w_4 y_3 \quad \text{...................... (2c)}$$

$$\frac{dy_4}{dT} = \frac{w_4}{w_2} y_3 - w_5 y_4 \quad \text{.. (2d)}$$

where $T = E_1 t$, $w_1 = \frac{E_2}{E_1}$; $w_2 = \frac{b}{a}$; $w_3 = \frac{E_3}{E_1}$; $w_4 = \frac{E_4}{E_1}$; and $w_5 = \frac{r}{E_1}$ .are the dimensionless parameters.

System (2) needs to analyzed with a specific initial condition, which may be taken as any point in the region $R_+^4 = \{(y_1, y_2, y_3, y_4) \in R^4 : y_i \geq 0; i = 1,2,3,4\}$.

Existence and stability analysis of system (2)

The stage structured prey-predator model given by system (2) has at most four nonnegative equilibrium points, namely $E_0 = (0,0,0,0)$, $E_1 = (\bar{y}_1, 0, \bar{y}_3, \bar{y}_4)$, $E_2 = (0, \tilde{y}_2, \tilde{y}_3, \tilde{y}_4)$ and $E_3 = (y_1^*, y_2^*, y_3^*, y_4^*)$.

The equilibrium point E_0 always exists, however the equilibrium point E_1 exists in the positive part of $y_1 y_3 y_4 -$surface where

$$\bar{y}_1 = \frac{(w_3 + w_4) w_5}{w_4} \quad \text{.. (3a)}$$

$$\bar{y}_3 = \frac{w_2 w_5}{w_4} \quad \text{.. (3b)}$$

$$\bar{y}_4 = 1 \quad \text{.. (3c)}$$

The equilibrium point E_2 exists in the positive part of $y_2 y_3 y_4 -$surface where

$$\tilde{y}_2 = \frac{(w_3 + w_4) w_5}{w_2 w_4} \quad \text{.. (4a)}$$

$$\tilde{y}_3 = \frac{w_1 w_5}{w_4} \quad \text{.. (4b)}$$

$$\tilde{y}_4 = \frac{w_1}{w_2} \quad \text{.. (4c)}$$

Finally the positive equilibrium point $E_3 = (y_1^*, y_2^*, y_3^*, y_4^*)$,where

$$y_1^* = \frac{w_2 (w_3 + w_4)(1 + \frac{w_2}{w_1}) w_5}{w_1 w_4 \left[\left(\frac{w_2}{w_1} \right)^{\frac{n+1}{n}} + w_2 \right]} \quad \text{............................ (5a)}$$

$$y_2^* = \frac{(w_3 + w_4)(1 + \frac{w_2}{w_1}) w_5}{w_4 \left[\left(\frac{w_2}{w_1} \right)^{\frac{n+1}{n}} + w_2 \right]} \quad \text{.............................. (5b)}$$

$$y_3^* = \frac{w_2 w_5}{w_4} \left(\frac{w_1}{w_2} + 1 \right) \quad \text{.................................... (5c)}$$

$$y_4^* = \frac{w_1}{w_2} + 1 \quad \text{.. (5d)}$$

In the following, the local dynamical behavior of system (2) around each of the above equilibrium points is discussed. First the Jacobian matrix of system (2) at each point is determined and then the eigenvalues for the resulting matrix are computed. The Jacobian matrix of system (2) at the equilibrium point $E_1 = (\bar{y}_1, 0, \bar{y}_3, \bar{y}_4)$ is given by:

$$J(E_1)=\begin{bmatrix}0 & 0 & 0 & -\bar{y}_1\\ 0 & w_1 & 0 & 0\\ w_2\bar{y}_1^n & 0 & -(w_3+w_4) & w_2\bar{y}_1\\ 0 & 0 & \frac{w_4}{w_2} & -w_5\end{bmatrix}$$

Accordingly the characteristic equation of $J(E_1)$ is given by

$$(w_1-\lambda)(\lambda^3+A_1\lambda^2+A_2\lambda+A_3)=0$$

Where

$A_1=-(b_{22}+b_{33})=w_3+w_4+w_5>0,$ .... (6a)

$A_2=b_{22}b_{33}-b_{23}b_{32}=0,$ .... (6b)

$A_3=-(b_{13}b_{21}b_{32})=\dfrac{((w_3+w_4)w_5)^{n+1}}{w_4^n}>0$ .... (6c)

with

$$b_{22}=-(w_3+w_4), b_{33}=-w_5$$

$$b_{23}=w_2\bar{y}_1, b_{32}=\frac{w_4}{w_2}.,$$

$$b_{13}=-\bar{y}_1, and\ b_{21}=w_2\bar{y}_1^n.$$

Clearly from the characteristic polynomial $\lambda_{y_2}=w_1>0$, the eigenvalue of $J(E_1)$ in the $y_2-direction$. Also, we have $\Delta=A_1A_2-A_3=-A_3<0$ and hence E_1 is unstable equilibrium point.

The Jacobian matrix of the system (2) at the positive equilibrium point $E_2=(0,\tilde{y}_2,\tilde{y}_3,\tilde{y}_4)$ can be written as:

$$J(E_2)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & w_1 & 0 & -w_2\tilde{y}_2\\ 0 & w_1w_2 & -(w_3+w_4) & w_2^2\tilde{y}\\ 0 & 0 & \frac{w_4}{w_2} & -w_5\end{bmatrix}$$

Accordingly the characteristic equation of $J(E_2)$ is given by

$$(1-\lambda)(\lambda^3+A_1\lambda^2+A_2\lambda+A_3)=0$$

where

$A_1=-(c_{11}+c_{22}+c_{33})=w_3+w_4+w_5-w_1$ .... (7a)

$$A_2=c_{11}c_{22}+c_{22}c_{33}-c_{23}c_{32}+c_{11}c_{33}$$
$$=-w_1(w_3+w_4+w_5)<0.$$

.... (7b)

$$A_3=-c_{11}c_{22}c_{33}+c_{11}c_{23}c_{32}-c_{13}c_{21}c_{32}$$
$$=\ w_1(w_3+w_4)>0$$

.... (7c)

With

$$c_{11}=w_1,\ \ c_{13}=\frac{-(w_3+w_4)w_5}{w_4},$$

$$c_{21}=w_1w_2,\ c_{22}=-(w_3+w_4)$$

$$c_{23}=\frac{w_2w_5(w_3+w_4)}{w_4},\ c_{32}=\frac{w_4}{w_2},$$

$$and\ \ c_{33}=-w_5.$$

Note that, due to Routh-Hurwitz criterion, the necessary and sufficient conditions for E_2 to be locally asymptotically stable in the $Int.R_+^4$, are $A_1>0$, $A_3>0$ and $\Delta=A_1A_2-A_3>0$.

Straightforward computation shows that, if the following condition holds

$w_1<w_3+w_4+w_5$ .... (8a)

Then we obtain $A_1>0$. But condition (8a) implies that

$$\Delta=A_1A_2-A_3$$
$$=-w_1(w_3+w_4)[w_3+w_4+w_5-w_1+1]<0$$

That is E_2 is unstable equilibrium point.

Finally, the Jacobian matrix of the system (2) at the positive equilibrium point $E_3=(y_1^*,y_2^*,y_3^*,y_4^*)$, where $y_i^*\ \ for\ i=1,2,3,4$ are given in equations(5a-5d) can be written as:

$$J(E_2)=(a_{ij})_{4\times4}$$

where

$$a_{11}=1-\frac{(y_1^*)^n y_4^*}{A}\left(1+\frac{n(y_2^*)}{A}\right),\ A=(y_1^*)^n+(y_2^*)^n$$

$$a_{12}=\frac{n(y_1^*)^{n+1}(y_2^*)^{n-1}y_4^*}{A^2}>0, a_{13}=0, a_{14}=-\frac{(y_1^*)^{n+1}}{A}<0$$

$$a_{21}=\frac{nw_2(y_1^*)^{n-1}(y_2^*)^{n+1}y_4^*}{A^2}>0, a_{22}=w_1-\frac{w_2(y_2^*)^n y_4^*}{A}(1+\frac{n(y_1^*)^n}{A})$$

$$a_{23}=0, a_{24}=\frac{-w_2(y_2^*)^{n=1}}{A}<0,$$

$$a_{31}=\frac{w_2y_4^*}{A^2}[(y_1^*)^{n-1}(y_2^*)^n((n+1)y_1^*-w_2ny_2^*)+(y_1^*)^{2n}]$$
$$=\frac{w_2y_4^*(y_1^*)^n}{A^2}\left[A-n(y_2^*)^n\left(\frac{w_2y_2^*}{y_1^*}-1\right)\right]$$
$$a_{32}=\frac{w_2y_4^*}{A^2}[(y_1^*)^n(y_2^*)^{n-1}(w_2(n+1)y_2^*-ny_1^*)+(y_2^*)^{2n}]$$
$$=\frac{w_2^2y_4^*y_2^*}{A^2}\left[A+n(y_1^*)^n\left(1-\frac{y_1^*}{w_2y_2^*}\right)\right]$$
$$a_{33}=-(w_3+w_4)<0, a_{34}=\frac{w_2}{A}((y_1^*)^{n+1}+w_2(y_2^*)^{n+1})>0$$
$$a_{41}=0,\ a_{42}=0,\ a_{43}=\frac{w_4}{w_2}>0,\ a_{44}=-w_5<0.$$

Accordingly the characteristic equation of $J(E_3)$ is given by

$$\lambda^4+A_1\lambda^3+A_2\lambda^2+A_3\lambda+A_4=0$$

where

$A_1=-(\gamma_1+\gamma_2)$ .... (9a)

$A_2=-\gamma_3+\gamma_1\gamma_2+\gamma_4$ .... (9b)

$A_3=\gamma_2\gamma_3-\gamma_1\gamma_4-a_{43}\gamma_5$ .... (9c)

$A_4=-\gamma_3\gamma_4-a_{31}a_{43}\gamma_6-a_{32}a_{43}\gamma_7$ .... (9d)

with

$\gamma_1=a_{11}+a_{22}$, $\gamma_2=a_{33}+a_{44}<0$,

$\gamma_3=a_{12}a_{21}-a_{11}a_{22}$, $\gamma_4=a_{33}a_{44}-a_{34}a_{43}$,

$\gamma_5=a_{14}a_{31}+a_{24}a_{32}$, $\gamma_6=a_{12}a_{24}-a_{14}a_{22}$,

$\gamma_7=a_{14}a_{21}-a_{11}a_{24}$.

Note that, due to Routh-Hurwitz criterion, the necessary and sufficient conditions for E_2 to be locally asymptotically stable in the $Int.R_+^4$, are $A_i>0$ *for* $i=1,2,3,4.$, and $\Delta=A_1A_2A_3-A_3^2-A_1^2A_4>0$.

Straightforward computation shows that, if the following condition holds

$a_{11}<0$ *iff*

$w_2^n(w_1^n+w_2^n)y_4^*+nw_1^nw_2^ny_4^*>(w_1^n+w_2^n)^2$

.....................................(10a)

$a_{22}<0$ *iff*

$w_1^nw_2(w_1^n+w_2^n)y_4^*+nw_1^nw_2^{n+1}y_4^*>w_1(w_1^n+w_2^n)^2$

.... (10b)

$a_{31}>0$ *and* $a_{32}>0$ *iff* $\frac{w_2y_2^*}{y_1^*}>1$ ....(10c)

$w_1(w_1^n+w_2^n)[(w_1^n+w_2^n)+w_1^{n-1}w_2y_4^*+w_2^ny_4^*)+R_1>R_2$

Where

$R_1=w_1^nw_2^ny_4^*(w_1^n+w_2^n)^2(w_1^n+w_2^n)(n-1),$

$R_2=nw_1^nw_2^n(y_4^*)(w_1^n+w_2^n)(w_2^{n+1}+w_1^nw_2)$

.... (10d)

$w_2^nw_5(w_3+w_4)(w_1^n+w_2^n)>w_4w_2^ny_1^*(w_2^n+w_1^{n+1})$

..................................(10e)

Conditions (10a)-(10e) guarantees' that $\gamma_i<0$ *for* $i=1,3,5,6,7$ *and* $\gamma_4>0$.

And hence $A_i>0$ *for* $i=1,2,3,4$.

Finally, substituting the values of A_i for $i=1,2,3$ in $\Delta=A_1A_2A_3-A_3^2-A_1^2A_4$ and then simplifying the resulting term we get that

$$\Delta=(\gamma_1\gamma_4-\gamma_2\gamma_3+a_{43}\gamma_5)((\gamma_1+\gamma_2)(\gamma_1\gamma_2)-a_{43}\gamma_5)+(\gamma_3+\gamma_4)^2\gamma_1\gamma_2+a_{43}[(a_{31}\gamma_6+a_{32}\gamma_7)(\gamma_1+\gamma_2)^2+(-\gamma_1\gamma_3+\gamma_2\gamma_4)\gamma_5]$$

obviously $\Delta>0$ if and only if in addition to conditions (10a)-(10e) the following two conditions holds:

$(\gamma_1+\gamma_2)\gamma_1\gamma_2>a_{43}\gamma_5$ ....(10f)

$\frac{(-\gamma_1\gamma_3+\gamma_2\gamma_4)\gamma_5}{(\gamma_1+\gamma_2)^2}>a_{31}\gamma_6+a_{32}\gamma_7$ .... (10g)

Consequently the following theorem for locally stability of E_3 can be proved easily.

Theorem 2:

Assume that the positive equilibrium point E_3 of system (2) exists. Then E_3 is locally asymptotically stable in the $Int.R_+^4$ if the conditions (10a)-(10e), (10f) and (10g) are satisfied.

In the following section the dynamical behavior of system (2) will be studied numerically because of difficulty of analytical computations.

Numerical analysis

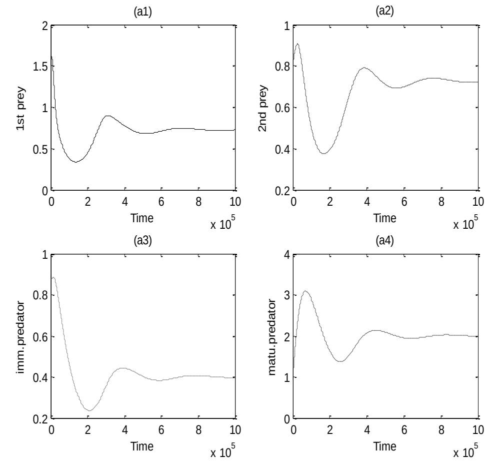
In this section the global dynamics of system (2), for n=2,n=3 and n=4 is studied numerically. System (2) is solved numerically for different sets of parameters and for the

initial point (1.5,0.8,0.9,0.9), and then their time series are drown.
For the following set of parameters

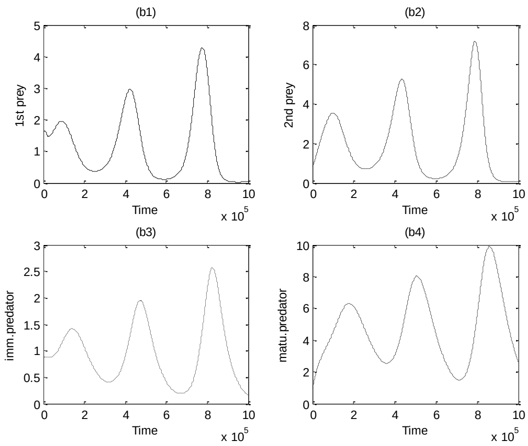
$$w_1 = 0.1,\ w_2 = 0.1\ w_3 = 0.1,\ w_4 = 0.1,\ w_5 = 0.2 \quad \text{..........} \ (11)$$

The time series of system (2) are drown in Fig.(1) (a)-(f) for n=2, Fig.(2) (a)-(f) for n=3 and Fig.(3)(a)-(f) for n=4.

Fig.(1):(a)-(f) The time series of the attractor initiated at (1.5,0.8,0.9,0.9) for the intensity of predator switching n=2.

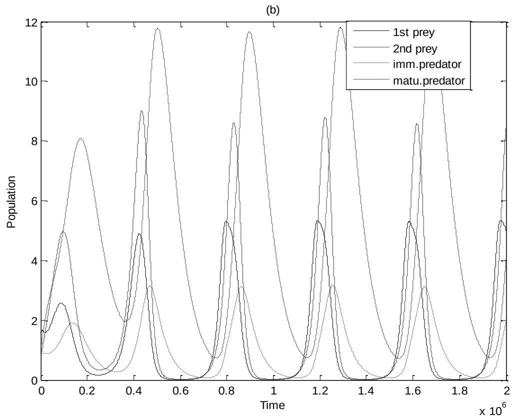


(a1) The trajectory of y_1 as a function of time.
(a2) The trajectory of y_2 as a function of time.
(a3) The trajectory of y_3 as a function of time.
(a4) The trajectory of y_4 as a function of time.
Fig.(a1)-(a4) Time series of the attractor initiated at (1.5,0.8,0.9,0.9) for the set of parameters values (11) which shows that the solution of system (2) approaches to stable point $E_3 = (0.72,0.72,0.39,1.99)$.

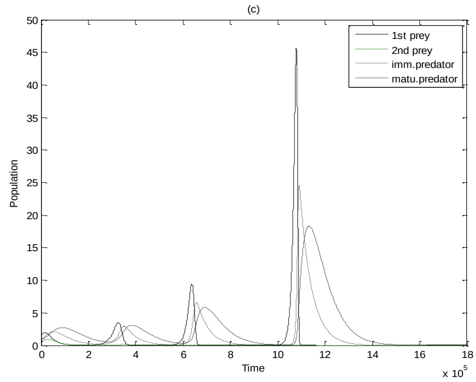


(b1) The trajectory of y_1 as a function of time.
(b2) The trajectory of y_2 as a function of time.
(b3) The trajectory of y_3 as a function of time.
(b4) The trajectory of y_4 as a function of time.

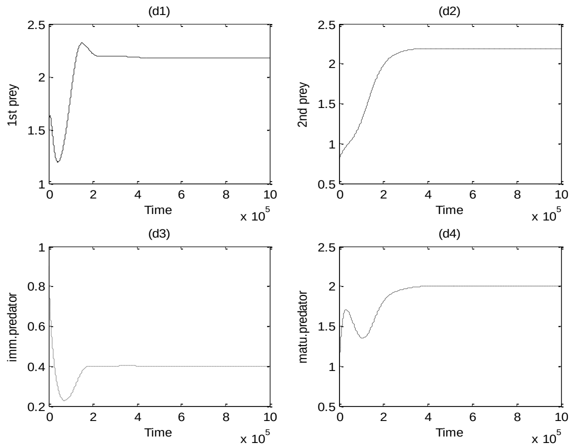
Fig.(b1)-(b4) shows that system (2) has periodic dynamic in the $Int.R_+^4$ for the data given in (11) with w_1=0.35**.**



(b) Periodic attractor in the $Int.R_+^4$. for the set of parameters values (11) with w_1=0.4 .

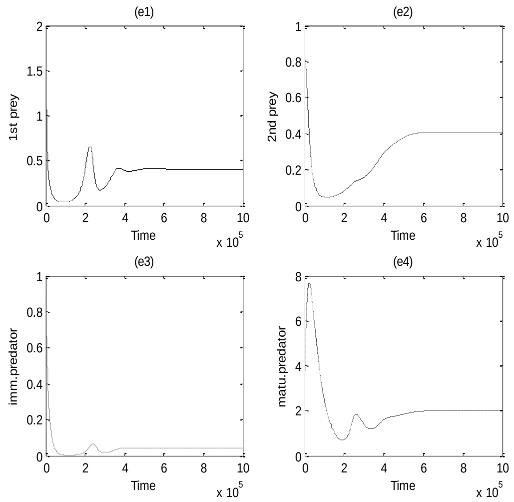


(c) Time series of the attractor initiated at (1.5,0.8,0.9,0.9) for the set of parameters values (11) with w_2=0.30 which shows that the solution of system (2) approaches to stable point $E_0 = (0,0,0,0)$.



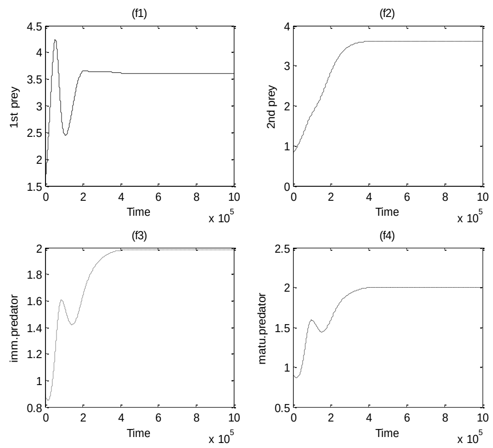
(d1) The trajectory of y_1 as a function of time.
(d2) The trajectory of y_2 as a function of time.
(d3) The trajectory of y_3 as a function of time.
(d4) The trajectory of y_4 as a function of time.

Fig.(d1)-(d4) Time series of the attractor initiated at (1.5,0.8,0.9,0.9) for the set of parameters values (11) with w_3=0.5 shows that the solution of system (2) approaches to stable point $E_3 = (2.182, 2.181, 0.400009, 1.9997)$.



(e1) The trajectory of y_1 as a function of time.
(e2) The trajectory of y_2 as a function of time.
(e3) The trajectory of y_3 as a function of time.
(e4) The trajectory of y_4 as a function of time.

Fig.(e1)-(e4) Time series of the attractor initiated at (1.5,0.8,0.9,0.9) for the set of parameters values (11) with w_4=0.9 shows that the solution of system (2) approaches to stable point
$E_3 = (0.40036, 0.40037, 0.0404043, 20001)$ *in* $Int.R^4_+$.



(f1) The trajectory of y_1 as a function of time.
(f2) The trajectory of y_2 as a function of time.
(f3) The trajectory of y_3 as a function of time.
(f4) The trajectory of y_4 as a function of time.

Fig.(f1)-(f4) Time series of the attractor initiated at (1.5,0.8,0.9,0.9) for the set of parameters values (11) with w_5=0.99 shows that the solution of system (2) approaches to stable point.
$E_3 = (3.6, 3.6, 1.98, 2)$ *in* $Int.R^4_+$.

According to the above, the effect of the other parameters on the dynamics of system (2) is also studied in case of varying the parameters and obtained results are summarized in the following tables

Table (1)
Numerical behaviors and persistence of system (2) as varying in some parameters keeping the rest of parameters fixed as in eq. (11) for the initial point (1.5,0.8,0.9,0.9) and n=2.

Parameters varied in system(2)	*Numerical behavior of system (2)*	*Persistence of system (2)*
$0.01 \le w_1 \le 0.02$	Approaches to periodic dynamics in Int. R^4_+	Persists
$0.02 < w_1 \le 0.15$	Approaches to stable point in Int. R^4_+	Persists
$0.15 < w_1 \le 0.9$	Approaches to periodic dynamics in Int. R^4_+	Persists
$0.9 < w_1 \le 2.4$	Approaches to stable point $E_0 = (0,0,0,0)$	Not persist
$0.01 \le w_2 \le 0.21$	Approaches to stable point in Int. R^4_+	Persists
$0.21 < w_2 \le 0.26$	Approaches to periodic dynamics in Int. R^4_+	Persists
$0.27 \le w_2 < 6$	Approaches to stable point $E_0 = (0,0,0,0)$	Not persist
$0.1 \le w_3 < 0.99$	Approaches to stable point in Int. R^4_+.	Persists
$0.01 \le w_4 \le 0.99$	Approaches to stable point in Int. R^4_+.	Persists
$0.1 \le w_5 \le 0.99$	Approaches to stable point in Int. R^4_+.	Persists

In the following the comparison between, the effect of the other parameters on the dynamics of system (2) for n=2, n=3 and n=4 in case of varying the parameters and obtained results are summarized in the following table.

Table (2)

In the following the comparison between, the effect of the other parameters on the dynamics of system (2) for n=2, n=3 and =4 is studied in case of varying the parameters and obtained results are summarized in the following table.

Parameters varied in system (2) for $n=2$	*Parameters varied in system (2) for* $n=3$	*Parameters varied in system (2) for* $n=4$	*Numericasl behavior of system (2)*
$0.01 \le w_1 \le 0.02$	$0.01 \le w_1 \le 0.02$	$0.01 \le w_1 \le 0.02$	*Approach to periodic dynamicsin* $Int.R^4_+$.
$0.02 < w_1 \le 0.15$	$0.02 < w_1 \le 0.14$	$0.02 < w_1 \le 0.11$	*Approach to stable point in* $Int.R^4_+$.
$0.15 < w_1 \le 0.9$	$0.14 < w_1 \le 1.83$	$0.11 < w_1 \le 1.70$	*Approach to periodic dynamicsin* $Int.R^4_+$.
$0.9 < w_1 \le 2.4$	$1.83 < w_1 \le 2.4$	$1.70 < w_1 \le 2.4$	*Approach to stable point* $E_0 = (0,0,0,0)$.
$0.01 \le w_2 \le 0.21$	$0.01 \le w_2 < 0.20$	$0.01 \le w_2 < 0.20$	*Approach to stable point in* $Int.R^4_+$.
$0.21 < w_2 \le 0.26$	$0.20 \le w_2 \le 0.26$	$0.20 \le w_2 \le 0.26$	*Approach to periodic dynamicsin* $Int.R^4_+$.
$0.27 \le w_2 < 6$	$0.26 < w_2 < 6$	$0.26 < w_2 < 6$	*Approach to stable point* $E_0 = (0,0,0,0)$.
$0.1 \le w_3 < 0.99$	$0.1 \le w_3 < 0.99$	$0.1 \le w_3 < 0.99$	*Approach to stable point in* $Int.R^4_+$.
$0.1 \le w_4 < 0.99$	$0.1 \le w_4 < 0.99$	$0.1 \le w_4 < 0.99$	*Approach to stable point in* $Int.R^4_+$.
$0.1 \le w_5 < 0.99$	$0.1 \le w_5 < 0.99$	$0.1 \le w_5 < 0.99$	*Approach to stable point in* $Int.R^4_+$.

Table (3)

In the following the comparison between, the effect of the other parameters on the dynamics of system (2) for n=2 and n=10 is also studied in case of varying the parameters and obtained results are summarized in the following table.

Parameters varied in system (2) for $n=2$	*Numericasl behavior of system (2)*	*Parameters varied in system (2) for* $n=10$	*Numericasl behavior of system (2)*
$0.01 \le w_1 \le 0.02$	*Approach to periodic dynamicsin* $Int.R^4_+$.	$0.01 \le w_1 < 0.97$	*Approach to periodic dynamicsin* $Int.R^4_+$.
$0.02 < w_1 \le 0.15$	*Approach to stable point in* $Int.R^4_+$.	$0.97 \le w_1 \le 0.5$	*Approach to stable point* $E_0 = (0,0,0,0)$.
$0.15 < w_1 \le 0.9$	*Approach to periodic dynamicsin* $Int.R^4_+$.		
$0.9 < w_1 \le 2.4$	*Approach to stable point* $E_0 = (0,0,0,0)$.	$0.5 < w_1 \le 2.4$	*Approach to periodic dynamicsin* $Int.R^4_+$.
$0.01 \le w_2 \le 0.21$	*Approach to stable point in* $Int.R^4_+$.	$0.01 \le w_2 \le 0.07$	*Approach to stable point in* $Int.R^4_+$.
$0.21 < w_2 \le 0.26$	*Approach to periodic dynamicsin* $Int.R^4_+$.	$0.07 < w_2 \le 0.26$	*Approach to periodic dynamicsin* $Int.R^4_+$.
$0.27 \le w_2 < 6$	*Approach to stable point* $E_0 = (0,0,0,0)$.	$0.27 \le w_2 < 6$	*Approach to stable point* $E_0 = (0,0,0,0)$.
$0.1 \le w_3 < 0.99$	*Approach to stable point in* $Int.R^4_+$.	$0.1 \le w_3 < 0.3$	*Approach to periodic dynamicsin* $Int.R^4_+$.
		$0.4 \le w_3 \le 0.99$	*Approach to stable point in* $Int.R^4_+$.
$0.01 \le w_4 \le 0.99$	*Approach to stable point in* $Int.R^4_+$.	$0.01 \le w_4 \le 0.6$	*Approach to periodic dynamicsin* $Int.R^4_+$.
		$0.7 < w_4 \le 0.99$	*Approach to stable point in* $Int.R^4_+$.
$0.01 \le w_5 \le 0.99$	*Approach to stable point in* $Int.R^4_+$.	$0.01 \le w_5 \le 0.6$	*Approach to periodic dynamicsin* $Int.R^4_+$.
		$0.7 < w_5 \le 0.99$	*Approach to stable point in* $Int.R^4_+$.

Conclusion

In this paper, a mathematical model has been proposed and analyzed to study the impact of switching on the dynamics behavior of a stage structured predator interacting with prey which assumed to live in two distinct habitats, the dynamical behavior of system (2) has been investigated locally, but because of difficulty of analytical computations, the global stability and persistence of system (2) studied numerically.

Now, Table (2) and Table (3) illustrates the effect of changing the parameters on the dynamics of system (2). We observe that when the intensity of predator switching increases w_1 and w_2 which represent the growth rate of second prey and the predation coefficient respectively play a vital role in persistence of system (2), see Table (3), but when the intensity of predator switching decreases the persistence of system (2) valid in the same varying of parameters w_i , $for\, i = 1,2,...,5$,see Table (2).

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الخلاصة

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