Plotting the Profiles of Cartesian Surfaces by Ray-tracing

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Abstract

A bundle of light rays, completely parallel to the optical axis, has been used to plot profiles of the Cartesian surfaces with constant paraxial curvature and varying asphericity factor. To accomplish this study, a programming code of skew ray tracing has been constructed to trace rays through all types of Cartesian or quadric surfaces of revolution. The results of this work show a remarkable utility of ray-tracing procedure to plot distinguished Cartesian surfaces profiles.

Keywords: Asphericity factor, Cartesian surfaces, skew ray tracing.

Introduction

Ray tracing procedures, in optics and optical design text books [1-11] are a mainstay to exhibit the performance of optical systems and the mathematical tools essential for rays-aberrations evaluations which are necessary for correcting these systems before being constructed. In symmetrical optical systems [4], there are, mainly, three different types of ray tracing procedures for the different types of the incoming rays. These procedures are the paraxial, meridianal, and the skew ray tracing.

In this paper, the skew ray tracing method has been considered to plot the profiles the profiles of quadric surfaces of revolution or the Cartesian surfaces (Cartesian surfaces are reflecting or refracting surfaces that form perfect images). In general, the surfaces of mirrors or lenses are quadric surfaces of revolution. Different surfaces profiles are obtained by varying either the paraxial curvature *C* or the asphericity factor *ε*. The equation that represents a surface of revolution about the *z*–axis (the optical axis), passing through the origin (passing through the x-y plane that is tangent to the optical element surface and having curvature *C* at that point is [4]:

$$z = \frac{C}{2}\left(x^2 + y^2 + \varepsilon z^2\right) \quad \text{.................................} (1)$$

The parameter ε determines the asphericity factor as follows [4]:

- $\varepsilon > 1$ for oblate ellipsoid surfaces,
- $0 < \varepsilon < 1$ for prolate ellipsoid surfaces,
- $\varepsilon = 1$ for spherical surface,
- $\varepsilon = 0$ for paraboloid surface, and
- $\varepsilon < 0$ for hyperboloid surfaces.

The utility of equation (1) is to give a range for asphericities while keeping the paraxial curvature C constant, which is essential in designing conic surfaces [4].

In this work, the length segment values (Δ values, see Eq.3) in the skew ray tracing equations in *Ref.* [4] are considered to plot the profiles of the Cartesian surfaces (Cartesian surfaces are reflecting or refracting surfaces that form perfect images). The Δ values represent the distance values from the x-y plane that is tangent to the surface of the optical element at its vertex. Thus, Δ values are used as a measure to indicate the effect of ε on the surface shape. To achieve the goal of this work, the paraxial curvature *C* has been considered constant and the aperture diameter to be fixed as far as possible. The appendix in this paper contains a part of the programming code. This part is for tracing rays reflected from an oblate ellipsoid surface.

Skew Ray Tracing

It is considered as the ray tracing method that gives the exact analysis; because it uses solid geometry [4, 5]. The skew ray is the most general case of light rays income an optical system as it is defined as the ray that is not co-planer with the optical axis [5] (geometrically, there is no plane which can contain both the optical axis and the skew ray).

Skew Ray Tracing equations are divided into two sets of equations. The first set is for ray transfer between surfaces and second set is for reflection or refraction.

Transfer between Surfaces

It can be expressed by [4]:

$$\left.\begin{aligned} x_0 &= x_{-1} + \frac{L}{N}\left(d - z_{-1}\right) \\ y_0 &= y_{-1} + \frac{M}{N}\left(d - z_{-1}\right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

L, *M*, and *N* are the direction cosines of the ray along x-axis, y-axis, and z-axis respectively; x_0, and y_0 are the coordinates of ray intersection with the tangent x-y plane; x_{-1} and y_{-1} are the coordinates of coming ray. The ray intersects the optical element surface at coordinates given by [4]:

$$\left.\begin{aligned} x &= x_0 + L\Delta \\ y &= y_0 + M\Delta \\ z &= N\Delta \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where Δ is given by [12]:

$$\Delta = \frac{F}{G + \sqrt{G^2 - CF(1 + (\varepsilon - 1)N^2)}} \quad \ldots\ldots\ldots (4)$$

where *F* and *G* are given by[4]:

$$F = C\left(x_0^2 + y_0^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$G = N - C\left(Lx_0 + My_0\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Reflection (or Refraction) Equations Set

To obtain reflection or refraction equations through a surface, we start with determining the components of the unit normal (α, β, γ) as [12]:

$$\left.\begin{aligned} \alpha &= \frac{-Cx}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \\ \beta &= \frac{-Cy}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \\ \gamma &= \frac{1 - C\varepsilon z}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \end{aligned}\right\} \quad \ldots\ldots\ldots (7)$$

The cosine of the angle of incidence cos*I* can be obtained by the scalar multiplication with the direction cosines of the ray tracing. It can be expressed as [12]:

$$\cos I = \frac{N - C(Lx + My + N\varepsilon z)}{\sqrt{1 - 2C(\varepsilon - 1)z + C^2\varepsilon(\varepsilon - 1)z^2}} \quad \ldots\ldots (8)$$

The angle of reflection or refraction can be obtained by [4]:

$$n'\cos I' = \sqrt{(n')^2 - n^2(1 - \cos^2 I)} \quad \ldots\ldots\ldots\ldots (9)$$

The non-primed parameters are those of the previous surface. The new ray direction cosines in order to complete the set of equations for transfer between surfaces are given by [4]:

$$\left.\begin{aligned} n'L' - nL &= k\alpha \\ n'M' - nM &= k\beta \\ n'N' - nN &= k\gamma \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

where

$$k = n'\cos I' - n\cos I \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The direction cosines should be checked in order to assert the tracing validity. This can be done by [1]:

$$(L')^2 + (M')^2 + (N')^2 = 1 \quad \ldots\ldots\ldots\ldots\ldots (12)$$

Results and Discussion

The results of varying asphericitry factor (ε) on the profiles of the Cartesian surfaces with constant radius of curvature (*r=5m*), i.e, constant curvature (for *C*=1/r) are listed in Tables (1) and (2).

Table (1) gives the values of Δ(cm) as a function of 1*m* aperture diameter size (the ray height *y* is ranging from -50cm to 50cm) for the case of oblate ellipsoids (ε>1), where ε has the values of 20, 40, 60, and 80. Table (2) gives the values of Δ (cm) as a function of 9m aperture diameter size (the ray height y is ranging from -450cm to 450cm) for the other cases of the Cartesian surfaces; the values of *ε* for each case are indicated in Table (2).

The chosen aperture diameter of 1m for the oblate ellipsoids differs from that of other cases where the aperture diameter is 9m. This is because, in the case of oblate ellipsoids with *r*=5m, the ray tracing procedure couldn't be proceeded for light ray heights beyond an aperture of 1m diameter (i.e., beyond the interval 50cm ≤ *y* ≤ -50cm). This, indeed, emphases that the oblate ellipsoids become more and more deep dishes (surfaces) such that light ray of heights beyond the interval indicated above wouldn't intersect these surface with r=5m and this in turn suspended the ray tracing procedure.

To represent the profiles of the Cartesian surfaces graphically, the values of Δ for each case of those surfaces given in Tables (1) and (2) to be plotted versus the aperture diameter. The graphical representation of the profiles of the Cartesian surfaces is exhibited in Figs. (1-4). These figures exhibit the profiles of oblate ellipsoids, spherical surface and paraboloid, prolate ellipsoids, and hyperboloids respectively.

Fig.(1) shows that the profiles of oblate ellipsoids surfaces which are plotted against aperture diameter of 1m for the reason explained formerly. Figs. (2, 3) and (4) show the profiles of Cartesian surfaces with r=5m against 9m aperture diameter. The difference in the aperture diameters in the cases of $\varepsilon>1$ and other cases, with constant r, emphases that the Cartesian surfaces profiles become more sensitive and highly responding to ε when $\varepsilon>1$ than other cases.

In the cases when $\varepsilon<1$, the profiles of the Cartesian surfaces become more and more flattened as ε decreases. This is indicated in the results of Tables (1) and (2) and Figs. (1) to (4) illustrate this fact graphically. Fig.(4), the case of hyperboloid surfaces, emphases that the surfaces become more and more flattened and shows that as the absolute value of ε goes to infinity the Cartesian surface becomes a plane.

Conclusions

First, In addition to the utilities of ray tracing in optical design, this work exhibits another utility of ray-tracing which is the ability of ray tracing procedure to plot distinguished profiles of optical Cartesian surfaces of the same paraxial curvature with varying asphericity factor ε.

Second, from the case of oblate ellipsoids, the paraxial curvature C of an optical element, in comparison with the asphericity factor ε, is the dominant parameter in shaping the profiles of the Cartesian surfaces.

Third, in the case of hyperboloid surfaces, when the asphericity factor goes to infinity the surface becomes plane.

Fourth, Different surfaces' shapes, absolutely, have different surfaces' performance. So, varying ε leads to change the surface performance consequently.

Appendix

The programming language is QBASIC

```
Open "file path\ file name" for output  as # 1
r = - 500  :  C=1/r : n(1) = 1 :n(2) = -1  :  n(3) = 1
 input  e
For h= - 50 t0 50 step 5
If h=0 Goto 7
NN =1 : MM= 0 :  LL = 0
Yp=h
F=C*(X^2+Y^2)
G= NN-C*(LL*Xp+MM*Yp))
Delta=F/(G+(G^2-C*F*(1+(e-1)*NN^2))^-.5)
X=Xp+LL*Delta
Y=Yp+MM*Delta
Z=NN*Delta
denom = ( 1 – 2* (e-1)* C*Z + e( e-1)* C^2 *Z^2 )
cosi =( NN – C*( X*LL + Y*MM+
Z*NN))/denom
K=( n(2)*cosi – n(1)* cosi )
alpha = - (C*X) / denom
beta = - (C*Y) / denom
gamma = ( 1 - C*e*Z ) / denom
LL = (K*alpha + LL*n(1)) / n(2)
MM = (K*beta + MM*n(1)) / n(2)
NN = (K*gamma + NN*n(1)) / n(2)
Print as # 1,   Y,Z
7 Next
Close #1
```

Table (1)

Δ values as a function of 1m aperture diameter forming oblate ellipsoids with r=5m.

	Δ(cm)				
y(cm)	***ε=10***	***ε=20***	***ε=40***	***ε=60***	***ε=80***
50	2.565835	2.63932	2.817542	3.062871	3.454915
40	1.626454	1.654765	1.718071	1.792861	1.883938
30	0.9082492	0.9168109	0.9349665	0.9546855	0.9762443
20	0.4016129	0.4032522	0.4066134	0.4100904	0.4136913
10	0.1001002	0.1002008	0.1004032	0.1006073	0.100813
-10	0.1001002	0.1002008	0.1004032	0.1006073	0.100813
-20	0.4016129	0.4032522	0.4066134	0.4100904	0.4136913
-30	0.9082492	0.9168109	0.9349665	0.9546855	0.9762443
-40	1.626454	1.654765	1.718071	1.792861	1.883938
-50	2.565835	2.63932	2.817542	3.062871	3.454915

Table (2)

Δ values as a function of 9m aperture forming hyperboloids, paraboloid, prolate ellipsoids, and spherical with r=5m.

	Δ (cm)										
	Hyperboloids					***Paraboloid***	***Prolate ellipsoids***				***Spherical***
y(cm)	***ε = -1000***	***ε = -100***	***ε = -10***	***ε = -1***	***ε = -0.01***	***ε=0***	***ε= 0.2***	***ε= 0.4***	***ε= 0.6***	***ε= 0.8***	***ε=1***
450	**13.73903**	**40.27693**	**100.831**	**172.6812**	**202.0916**	**202.5**	**211.4415**	**222.2598**	**235.8851**	**254.1901**	**282.0551**
350	**-10.57926**	**30.35534**	**71.44958**	**110.3278**	**122.3503**	**122.5**	**125.658**	**129.1744**	**133.135**	**137.6603**	**142.9286**
250	**7.42149**	**20.4951**	**43.54144**	**59.017**	**62.46099**	**62.5**	**63.30142**	**64.14588**	**65.03796**	**65.98301**	**66.9873**
150	**4.269696**	**10.81139**	**18.92024**	**22.01533**	**22.49494**	**22.5**	**22.60217**	**22.70623**	**22.81224**	**22.92027**	**23.0304**
50	**1.158312**	**2.071068**	**2.440442**	**2.493781**	**2.499938**	**2.5**	**2.501251**	**2.502505**	**2.503761**	**2.50502**	**2.506281**
-50	**1.158312**	**2.071068**	**2.440442**	**2.493781**	**2.499938**	**2.5**	**2.501251**	**2.502505**	**2.503761**	**2.50502**	**2.506281**
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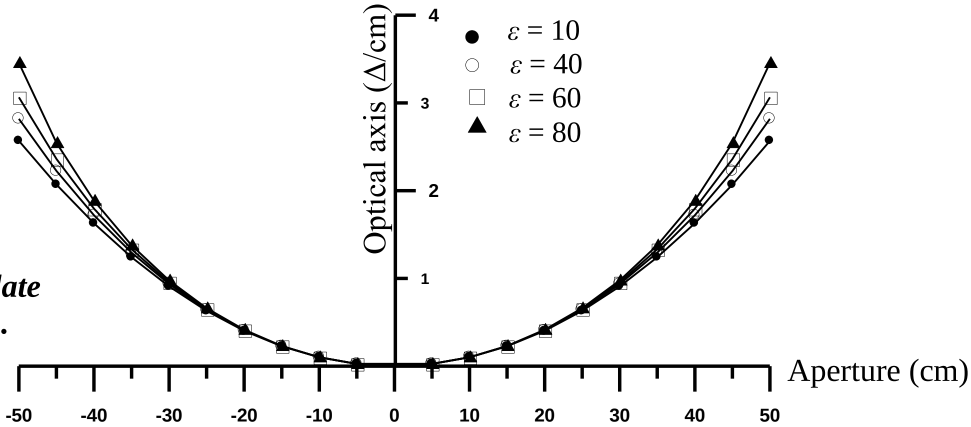

Fig. (1) Oblate ellipsoids.

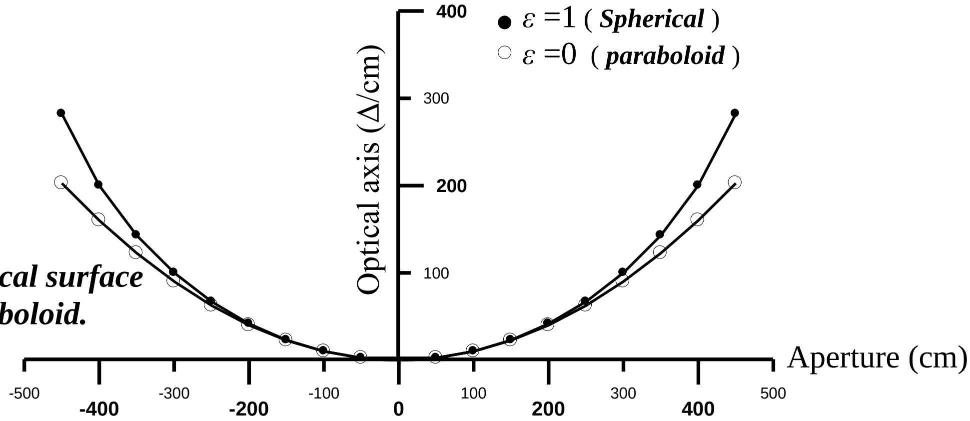

Fig.(2) Spherical surface and Paraboloid.

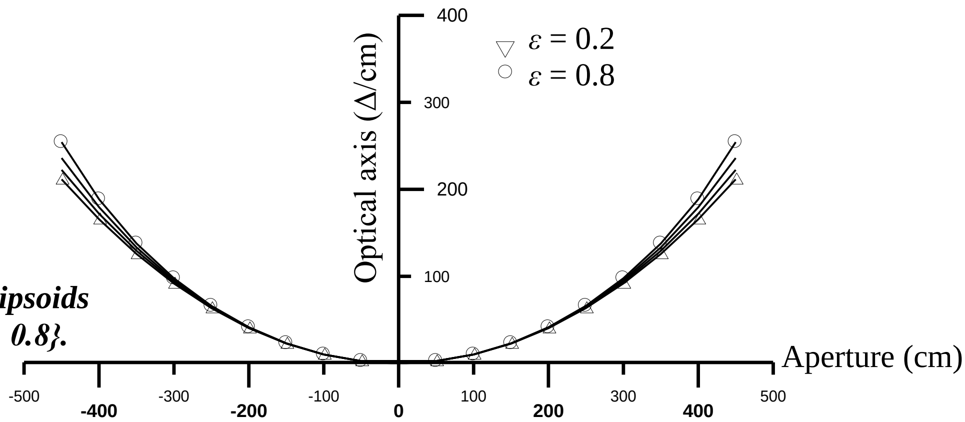

Fig.(3) Prolate ellipsoids $\varepsilon = \{0.2, 0.4, 0.6, 0.8\}$.

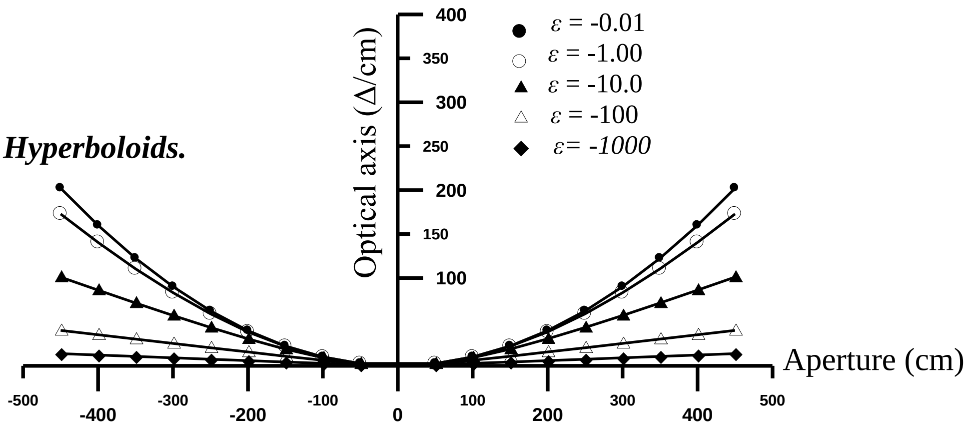

Fig.(4) Hyperboloids.

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الخلاصة

أستخدمت حزمة أشعة ضوئية، موازية للمحور البصري تماماً، لعرض مقاطع سطوح عناصر بصرية (عدسات أو مرايا) كارتيزية (أي لها مقطع مخروطي) والتي لها إنحناء (curvature) ثابت وعامل تكوّرمتغير. لإنجاز هذا العمل، تم بناء شفرة برمجية لأقتفاء الأشعة (skew ray-tracing) وذلك لأقتفاء الأشعة البصرية عبر جميع انواع السطوح الكارتيزية او السطوح الناتجة عن تدوير منحنٍ ذي معادلة من الدرجة الثانية (quadric surface of revolution). تظهر نتائج هذا العمل، إنّ لعملية أقتفاء الأشعة البصرية فائدة ملحوظة لرسم مقاطع متمايزة (الفرق بينها واضحٌ و جليّ) من السطوح الكارتيزية.