

Laser Acceleration of Electrons in Magnetized Collisionless Plasma

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Abstract

A computational investigation was carried out in the field of laser-plasma interaction to study the acceleration of electrons with non - relativistic velocities in magnetized collisionless plasma. The interaction of a Nd: Yag laser pulse of 25 fs duration and a 5×10^{15} W/cm² intensity with a plasma was studied at a plasma electron density $n_e = 1 \times 10^{18}$ cm⁻³ in the presence of an external magnetic field for the three values of the field strength $B = 60$ MG, 70 MG and 80 MG. It was found that the electron acquires a maximum energy of ~ 19 keV during the interaction when an external magnetic field of strength $B = 80$ MG is applied. The maximum energy of the electron during the interaction reaches ~ 1 keV in the absence of an external magnetic field for the same plasma electron density and laser pulse, after the interaction the maximum energy of the electron reached ~ 15 eV. However, the maximum energy of the electron after the interaction reaches ~ 3 keV when an external magnetic field strength of $B = 70$ MG is applied. This is due to a sustainable generated laser wakefield of $\sim 2 \times 10^9$ V/cm. Thus, it is concluded that an applied external magnetic field assists the acceleration of the electron and can subsidize for a high laser beam intensity.

Keywords: laser acceleration, collisionless plasma, plasma frequency, cyclotron frequency, laser wakefield.

Introduction

It is well known that particle accelerators are important tools in various fields such as particle physics, material science, medical diagnostics and treatment, and manufacturing industry. However, further expansion of the utilization of conventional accelerators has become difficult because such accelerator facilities require large land areas and huge financial resources. Therefore, a laser driven particle accelerator would provide the most promising approach to realizing high performance compact accelerators. The interaction of a laser pulse with a plasma can give rise to a large number of electrons from the accelerator in the plasma in the presence of an external magnetic field. This is due to the huge electric field that can be sustained [1].

Tajima and Dawson [2] originally were the first to propose about three decades ago laser-driven plasma based accelerators.

Dawson was responsible for many of the early developments in this field, including the plasma beat wave accelerators, the laser wakefield accelerators, and the photon accelerators [2].

The computational work in this paper is carried out to investigate the effect of a laser beam on the acceleration of electrons in a homogenous magnetized collisionless plasma for different values of the external magnetic field applied on the plasma.

Basic Equations

A- The Laser Pulse

To characterize the laser intensity, the unitless laser amplitude, a_0 , is introduced, which is defined as [3]:

$$a_0 = 8.65 \times 10^{-10} \lambda (\mu\text{m}) I^{1/2} \left(\frac{\text{W}}{\text{cm}^2} \right) \dots \dots \dots (1)$$

where I is the intensity of the laser pulse and λ is its wavelength. The parameter a_0 is very important for distinguishing the relativistic region ($a_0 > 1$) from the non-relativistic region ($a_0 < 1$), which is assumed in the present work.

The maximum amplitude of the electric field intensity of the laser pulse can be calculated from [4]:

$$E_0 = \frac{2 \pi m_e c^2 a_0}{e \lambda} \dots \dots \dots (2)$$

where m_e is the electron mass, c is the speed of light and e is the electron charge. The shape of the laser pulse used in the present work is represented by a sinusoidal wave of electric field in the x -direction with a certain value of rising time T_r , fall time T_f , and duration time t . Thus, one can write the electric field of the laser pulse as [5]:

$$\vec{E}(t) = \vec{E}_0 \sin(\omega_0 t - kr) \left(1 - \exp\left(\frac{-t}{T_r}\right) \right) \dots (3)$$

for $t \leq T_{off}$

$$\vec{E}(t) = \vec{E}_0 \sin(\omega_0 t - kr) \left(1 - \exp\left(\frac{-t}{T_r}\right) \right) \dots (4)$$

$$\exp\left(-\frac{(t - T_{off})}{T_f}\right) \text{ for } t > T_{off}$$

where T_{off} is the time at which the driving field is switched off, ω_0 is the angular frequency of the laser pulse, k is the wave number, E_0 is the maximum amplitude of the electric field intensity of the laser pulse and r is the distance.

B- The Plasma

The oscillation of electrons around their equilibrium positions takes place with a characteristic frequency known as the plasma frequency, ω_p , given by [6]:

$$\omega_p = \left(\frac{4\pi n_e e^2}{m_e} \right)^{1/2} \dots (5)$$

where n_e is the density of the plasma electrons.

At the critical density, the laser beam cannot propagate into the plasma anymore but decreases exponentially. For a given angular laser frequency, ω_0 , and wavelength, λ , the cut-off takes place at the critical density, n_c , where [7]:

$$\omega_p = \omega_0$$

and

$$n_c (\text{cm}^{-3}) = \frac{1.1 \times 10^{21}}{\lambda^2 (\mu\text{m})} \dots (6)$$

This critical density defines the regimes of underdense and overdense plasmas. When the incident laser frequency is larger than the plasma frequency, i.e.,

$$\omega_0 > \omega_p$$

then,

$$n_c > n_e$$

In this case the electrons respond to the incident field, resulting in transparency to the radiation from the so-called underdense plasma [7], which is assumed in the present work.

When the incident laser frequency is less than the plasma frequency, i.e.,

$$\omega_0 < \omega_p$$

then

$$n_c < n_e$$

In this case the electrons can respond and exclude the incident field, resulting in a reflection of wave energy from the so-called overdense plasma [7].

C- Laser Acceleration of Electrons in a Magnetized Plasma

A laser pulse propagating in a plasma in the presence of an external magnetic field can generate an electrostatic wakefield resulting in electron acceleration in the plasma [8]. The acceleration of a non-relativistic electron in a magnetized plasma is given by [9]:

$$\vec{a}(t) = \frac{e}{m_e} \vec{E}(t) + \omega_c \vec{v}(t) - \omega_p^2 \vec{\rho}(t) \dots (7)$$

where ω_c is the cyclotron frequency, $\vec{v}(t)$ is the velocity of the electron and $\vec{\rho}(t)$ is its displacement. It is assumed here that the electric field is in the x -direction, while the applied magnetic field is in the z -direction. By numerical integration of eq. (7), the velocity, momentum and energy of the electron can be found.

D- The Generation of the Wakefield

The wakefield in a plasma can be generated by injection of an electron beam or laser beam in the plasma region. The mechanism for generating the wakes is by

exciting the electrostatic field of the plasma behind the injected electron beam or laser beam [10,11]. This can be done by transfer of energy from the injected beam to the background plasma. When a laser beam propagates in an underdense plasma, plasma electrons are pushed out due to the ponderomotive force that drives a wakefield in the underdense plasma to accelerate the electrons [12,13]. The laser beam sets the electrons into oscillations with a net gain of energy in the region behind the beam [13] and zero net energy in the region of propagation of the beam. The ponderomotive force and the wakefield in a magnetized plasma can be calculated by substituting eqs. (3) and (4) in eq.(7) to obtain:

$$F_x^p = -m_e \omega_p^2 \rho_x(t) + e E_o \sin(\omega_o t - kx) \left(1 - \exp\left(\frac{-t}{T_r}\right) \right) \cdot F_{off} + m_e \omega_c v_y(t) \dots\dots\dots(8)$$

$$F_y^p = -m_e \omega_p^2 \rho_y(t) - m_e \omega_c v_x(t) \dots\dots\dots(9)$$

$$E_x^w = -\frac{m_e}{e} \omega_p^2 \rho_x(t) + E_o \sin(\omega_o t - kx) \left(1 - \exp\left(\frac{-t}{T_r}\right) \right) \cdot F_{off} + \frac{m_e}{e} \omega_c v_y(t) \dots\dots\dots(10)$$

$$E_y^w = -\frac{m_e}{e} \omega_p^2 \rho_y(t) - \frac{m_e}{e} \omega_c v_x(t) \dots\dots\dots(11)$$

where,

$$F_{off} = \begin{cases} 1 & \text{for } t = T_{off} \\ \exp\left(-\left(\frac{t - t_{off}}{T_f}\right)\right) & \text{for } t > T_{off} \end{cases}$$

F_x^p is the ponderomotive force in the x-direction, F_y^p is the ponderomotive force in the y-direction, E_x^w is the wakefield in the x-direction and E_y^w is the wakefield in the y-direction.

Computations, Results and Discussion

A- Shape of the Laser Pulse

The shape of a Nd: YAG laser pulse was determined using a computer program written in FORTRAN 90 based on eqs.(3) and (4). The input data used in this program are shown in Table (1), where the intensity of the laser pulse was chosen according to the non-relativistic interactions that occur at $a_o < 1$, and the duration time of the laser pulse was calculated from its length.

Table (1)

Input data for the computer program used to compute the shape of the laser pulse.

I	5.0×10^{15} (W/ cm ²)
t	25×10^{-15} (sec)
T _r	0.3×10^{-14} (sec)
T _f	0.3×10^{-14} (sec)
T _{off}	1.7×10^{-14} (sec)
x	0
λ	1.06 (μm)
E _o	2.0×10^9 (V/ cm)

The output data from the computer program are illustrated in Fig.(1), which shows the shape of a Nd: YAG laser pulse that has maximum electric field amplitude $E_o = 2.0 \times 10^9$ V/cm and intensity $I = 5.0 \times 10^{15}$ W/cm² that is directly proportional to the unitless laser amplitude $a_o \sim 7.0 \times 10^{-2}$.

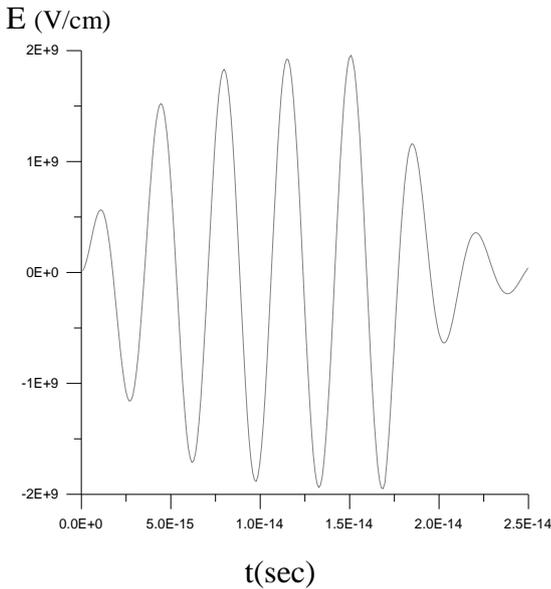


Fig. (1) Electric field strength for the Nd:YAG laser pulse used in the present work as a function of time for $I=5.0 \times 10^{15}$ (W/cm^2).

B-Computation of the Acceleration of Electrons in a Magnetized Plasma

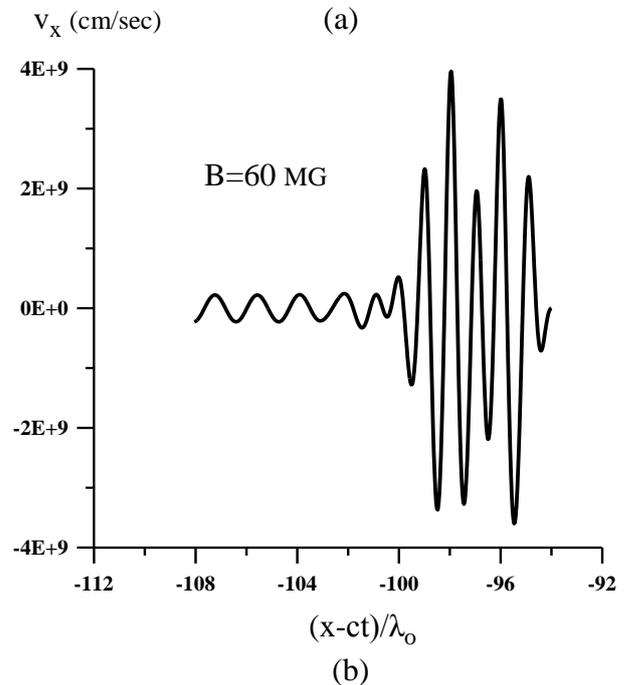
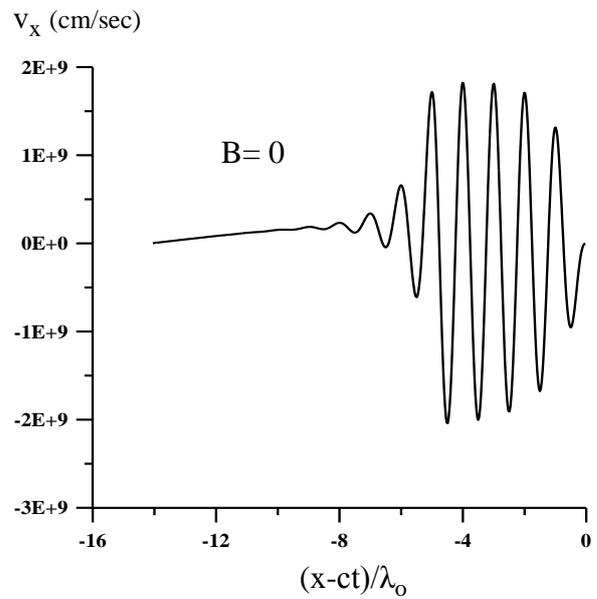
The acceleration of an electron from initial velocity ($v_0=0$) at initial position ($r=0$) in a magnetized plasma is determined using a computer program written in FORTRAN 90 to numerically integrate eq.(7) to find the velocity and energy of the electron with the input data shown in Table (2). The critical density of the plasma was determined using eq. (6), to be $1.0 \times 10^{21}cm^{-3}$. In the present work, the plasma is assumed to be underdense, therefore, one can choose the value of the plasma density $n_e= 1.0 \times 10^{18} cm^{-3}$.

Table (2)

Input data for the computer program used for the computation of electron acceleration.

I	5.0×10^{15} (W/ cm^2)
t	25×10^{-15} (sec)
T_r	0.3×10^{-14} (sec)
T_f	0.3×10^{-14} (sec)
T_{off}	1.7×10^{-14} (sec)
n_e	1.0×10^{18} (cm^{-3})
B	60 (MG), 70 (MG), 80 (MG)

The output data from this computer program are illustrated in Figs. (2), (3) and (4) for the three values of magnetic field strength B. In this case, the electron is accelerated according to the Lorentz equation that includes electric and magnetic forces in addition to the oscillation force of the electron in the plasma. From Fig.(2), it is observed that the velocity of the electron in the x-direction in a magnetized plasma increases when the value of external magnetic field increases. This is due to the increase in the displacement of the electron which leads to the increase of its acceleration, velocity and energy. The maximum values of the velocity of the electron in a magnetized plasma are shown in Table (3) for magnetic field strength B= 60 MG, 70 MG and 80MG.



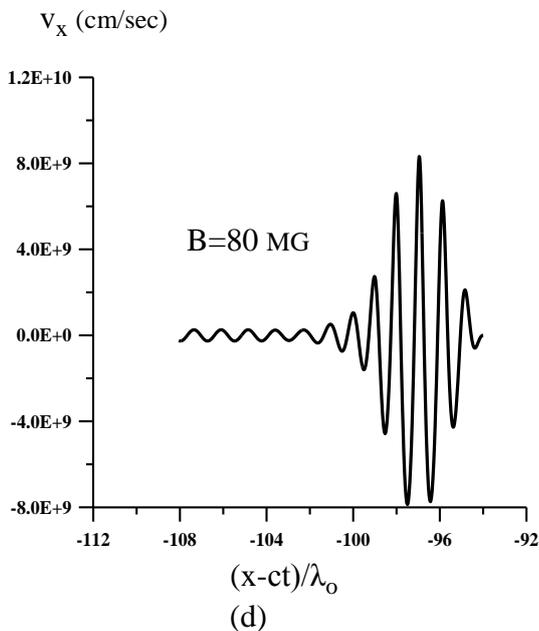
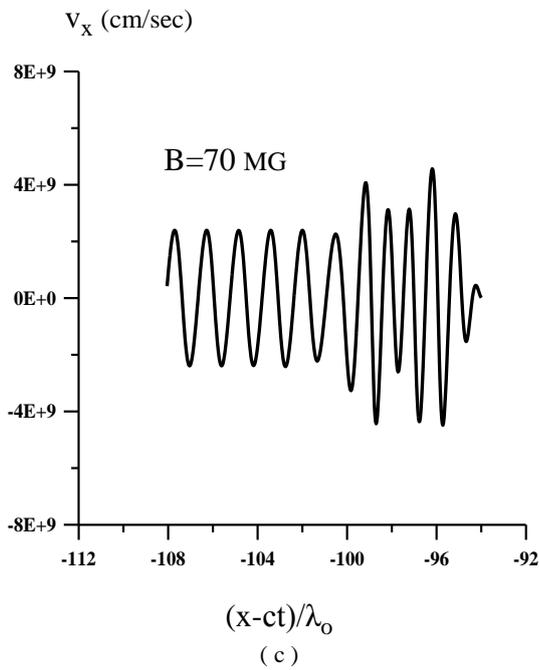
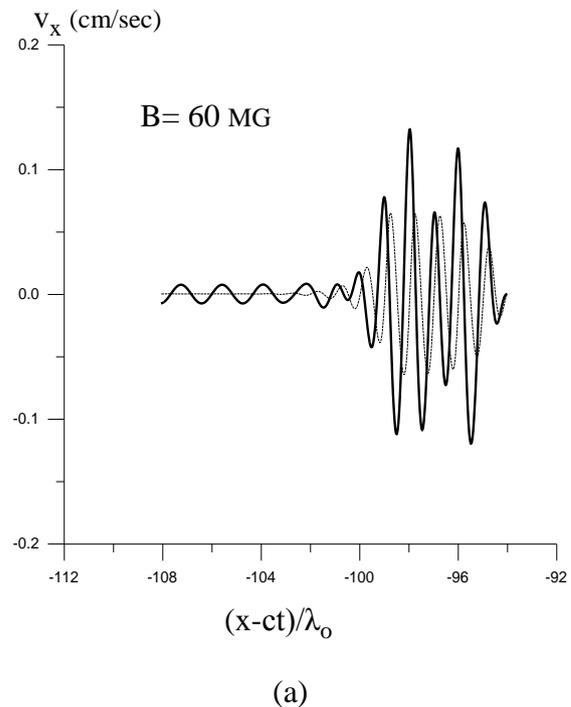


Fig. (2) The velocity of the electron in x-direction in a magnetized plasma, during and after the interaction, for intensity of laser pulse $I=5.0 \times 10^{15}$ (W/cm^2) and for magnetic field strength (a) $B=0$, (b) $B=60$ MG, (c) $B=70$ MG and (d) $B=80$ MG.

Table (3)
Maximum values of the velocity of the electron for laser pulse intensity $I= 5.0 \times 10^{15}$ W/cm^2 and magnetic field strengths $B= 0, 60$ MG, 70 MG, 80 MG.

B (MG)	v_x (cm/sec)
0	2.3×10^8
60	3.2×10^8
70	3.4×10^9
80	2.6×10^8

The calculations for the velocity of the electron in the y-direction indicated that it is nearly the same as the velocity of the electron in the x- direction and has the same behaviour. Fig.(3) shows the propagation of the laser pulse and the acceleration of the electron in a magnetized plasma during and after the interaction.



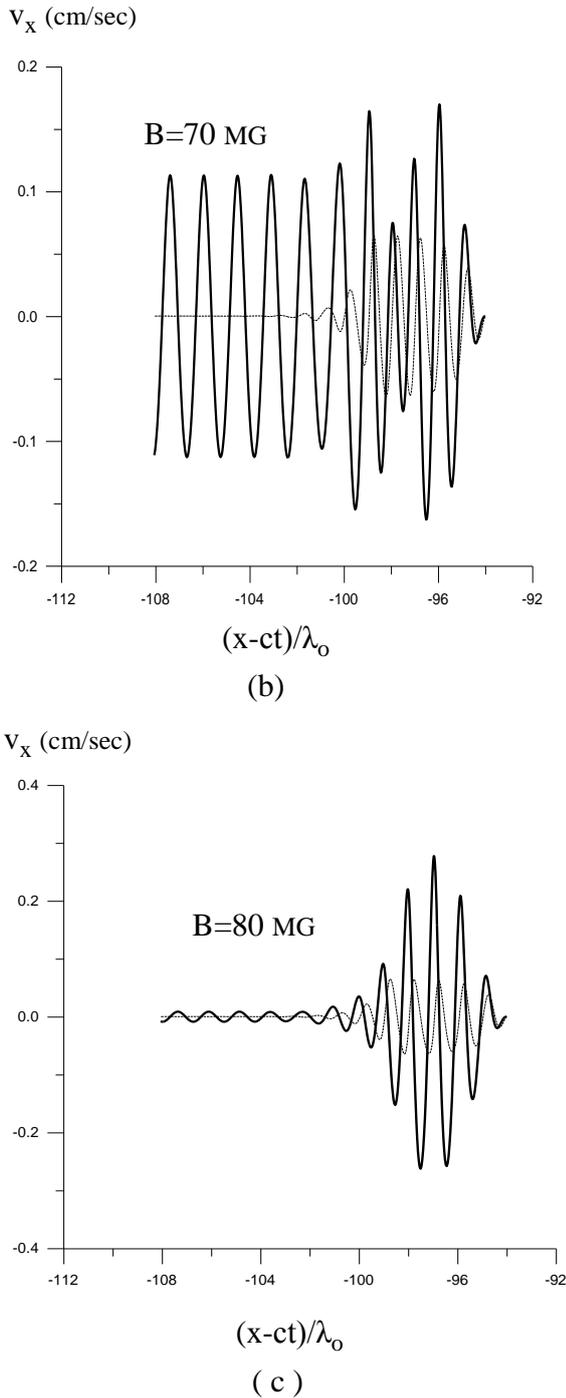
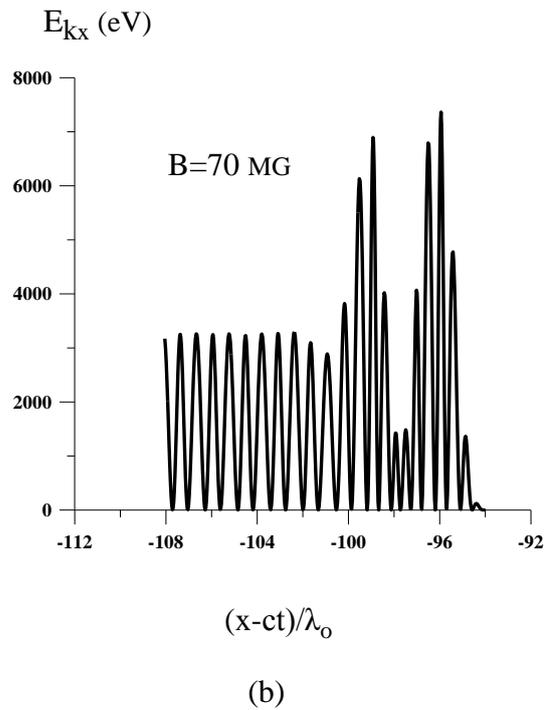
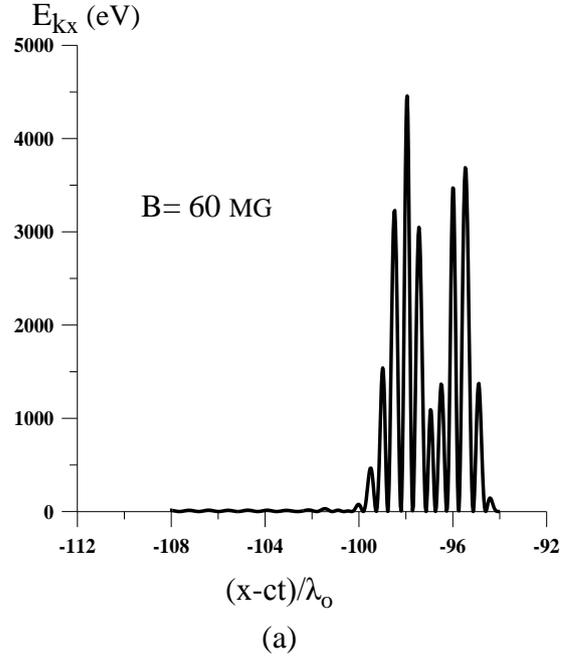


Fig. (3) The laser pulse (dashed line), the electron momentum (continues line), where the x-axis is normalized by λ_0 and the y-axis is the unitless amplitude, a , for the laser pulse intensity $I=5.0 \times 10^{15}$ (W/cm²) and momentum β of the electron for magnetic field strengths (a) $B=60$ MG, (b) $B=70$ MG and (c) $B=80$ MG.

Fig.(4) shows that when the magnetic field strength increases by about a factor of 10 the kinetic energy of the electron increases by about a factor of 100 due to the increase in the velocity of the electron by about a factor of 10 as shown in Table (4).



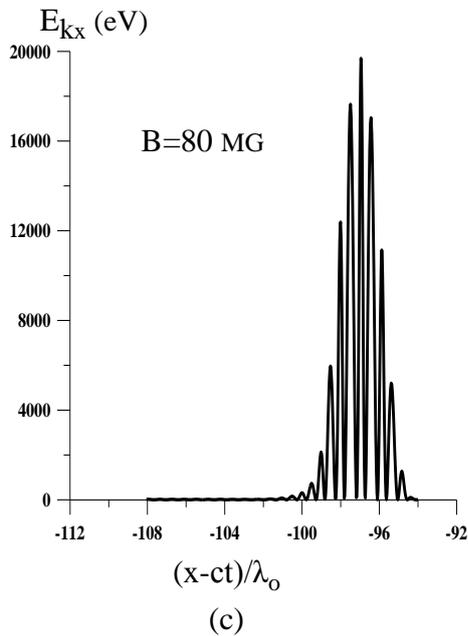


Fig. (4) The kinetic energy of an electron in a magnetized plasma during and after the interaction for intensity of laser pulse $I=5 \times 10^{15}$ (W/cm²) and for magnetic field strengths (a) $B=60$ MG, (b) $B=70$ MG and (c) $B=80$ MG.

Table (4)

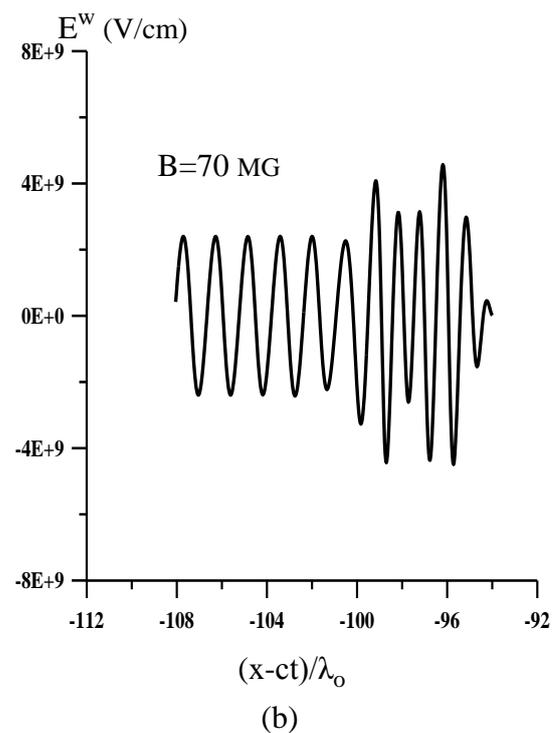
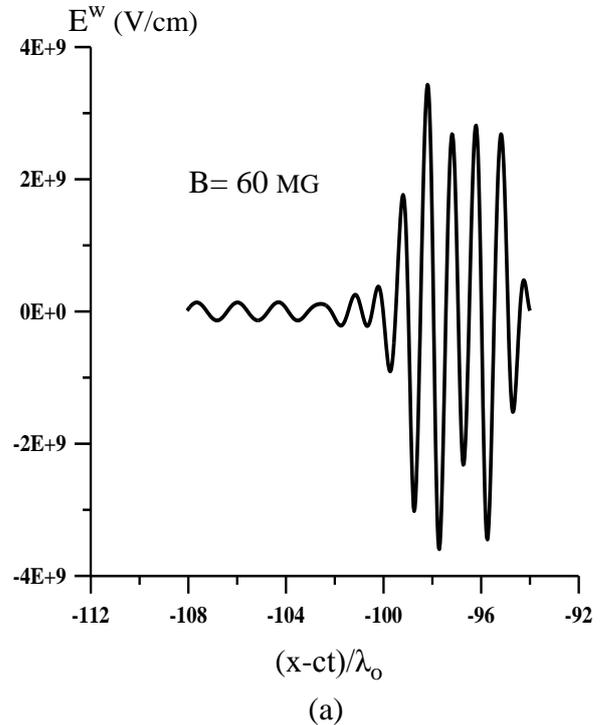
Maximum values of the kinetic energy of an electron for laser pulse intensity $I= 5.0 \times 10^{15}$ W/cm² and magnetic field strengths $B=0, 60$ MG, 70 MG, 80 MG.

B (MG)	E_k (eV)
0	15.1
60	30.6
70	3261
80	20.2

C- Computation of the Laser Wakefield in a Magnetized Plasma

Using a computer program written in FORTRAN 90 and based on eqs.(8), (9), (10) and (11) with the input data shown in Table (2), the laser wakefield resulting from the interaction was calculated. Fig.(5) shows the behaviour of this wakefield for laser pulse intensity $I= 5.0 \times 10^{15}$ W/ cm² and magnetic field strengths $B= 0, 60$ MG, 70 MG, 80 MG. It can be noticed, from the behaviour presented in Fig. (5), that the wakefield after the interaction decreases in value for magnetic field strength $B= 80$ MG where saturation is

observed. Therefore, in the present work, the magnetic field strength $B= 70$ MG can be considered as the best value externally that can be applied on a plasma since it produces the highest wakefield, hence, the highest electron acceleration as shown in Table (5).



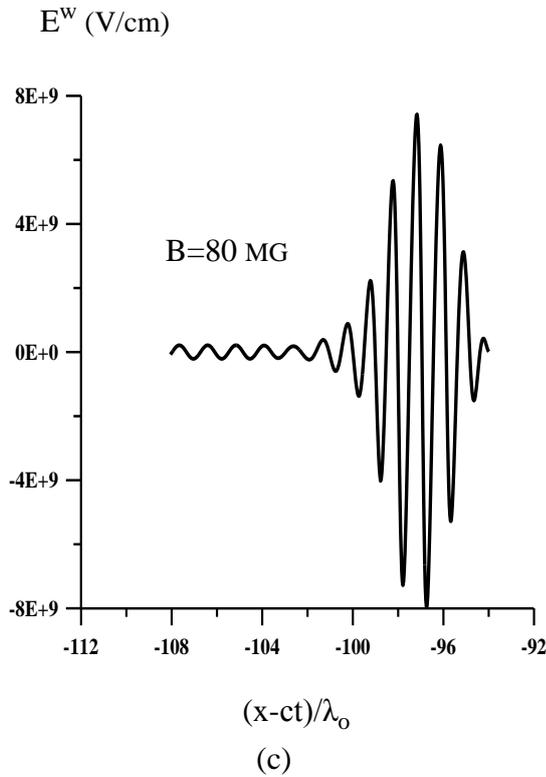


Fig. (5) The wakefield of a laser pulse in a magnetized plasma as a function of distance normalized by λ_0 for intensity of laser pulse $I=5.0 \times 10^{15}$ (W/cm²), magnetic field strengths (a) $B=60$ MG, (b) $B=70$ MG and (c) $B=80$ MG.

Table (5)

Maximum values of the wakefield for laser pulse intensity $I= 5.0 \times 10^{15}$ W/ cm² and magnetic field strengths $B= 0, 60$ MG, 70 MG and 80 MG.

B (MG)	E_x^w (V/cm)
0	4.0×10^7
60	1.0×10^8
70	2.0×10^9
80	2.0×10^8

Conclusions

The computations in the present work lead to the following conclusions:

A high intensity for the laser pulse can be supplemented by applying an external magnetic field on the plasma to attain high electron acceleration. This can be considered as one advantage of using an external magnetic field in acceleration of electrons in a

laser- plasma interaction. The velocity, kinetic energy and wakefield in a magnetized plasma can be sustained after the interaction. This can be considered as another advantage of using an external magnetic field in acceleration of electrons in a laser-plasma interaction.

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الخلاصة

اجريت دراسة نظرية وحاسوبية في مجال تفاعل الليزر مع البلازما باستعمال طريقة الفروق المحددة لفهم ميكانيكية تعجيل الالكترونات عند السرعة اللانسيبية في البلازما اللاتصادمية. تم دراسة تفاعل نبضة ليزر (نيديميوم-ياك) ذات فترة زمنية (٢٥ فيمتوثانية) وشدة (٥ x ١٠^{١٥} واط/سم^٢) مع بلازما ذات كثافة الكترونية (١ x ١٠^{١٨} سم^{-٣}) وبتسليط مجال مغناطيسي خارجي على البلازما وبثلاث قيم لشدة المجال هي (٦٠،٧٠،٨٠ ميكا كاوس) حيث لوحظ تعجيل الالكترون ووصوله الى طاقة بحدود (١٩ كيلو الكترون فولط) عند شدة مجال مغناطيسي (٨٠ ميكا كاوس) اثناء التفاعل في حين وصلت طاقة الالكترون الى (٣ كيلو الكترون فولط) تقريبا عند شدة مجال مغناطيسي مسط قيمته (٧٠ ميكا كاوس) بعد التفاعل وبعد تكون المجال الناهض لليزر بحدود (٢ x ١٠^٩ واط/سم). نستنتج من ذلك ان استخدام المجال المغناطيسي يساعد في الحصول على تعجيل عالي للالكترون والذي يمكن ان يعوض عن استخدام شدة نبضة عالية لليزر.