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Nuclear Level Density Parameter of $^{161-168}$Er and $^{204-210}$Bi Deforme Nuclei

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Abstract

In present work we calculate the Level density Parameter (LDP) of $^{161-168}$Er and $^{204-210}$Bi isotopes deformated nuclei using collective enhancement including rotation ground state modes at neutron binding energy and obtained by equidistant and non equidistance method for each nucleus and compared the results with several methods and expermintal one.

Keywords: Deform nuclei, level density parameter, Isotope.

Introduction

Nuclear level densities are very useful to understand properties of excited nuclei and to describe fission dynamics also relevant in transport theories and in astrophysical applications. Theoretical calculations within the shell model [1] and the Monte Carlo [2] methods, which generally include pairing correlations, the influence of spin and parity, have been quite successful in this context.The agreement of theoretical predictions with the experimental data constitutes in general a stringent tests.We performed such an investigation in the framework of the macroscopic–microscopic model searching to establish an analytical expression of the level density parameter depending on mass number A, isospin parameter and nuclear deformation like in Ref. [3].

Calculations of all parameters of fission and fusion and other nuclear technology fields depend strongly on cross-section data. And, it is well known that nuclear level density parameters are very important for the cross-section.

However, models used to estimate production cross-sections is still far from the performance required for technical applications. Nuclear reactions calculations based on standard nuclear reaction models play an important role in determining the accuracy of various parameters of theoretical models and experimental measurements. Especially, the calculations of nuclear level density parameters for the isotopes can be helpful in the investigation of reaction cross-sections. In this manner, many theoretical approaches have been developed to estimate total level densities of atomic nuclei, especially in the region of deformed heavy and light nuclei.

The analytical expressions used for the nuclear level density calculations [4, 5, 6] are based on the Fermi gas model. The most widely used description of the nuclear level density is Bethe formula, based on the framework of non interacting particles of the Fermi gas. The traditional Bethe theory of the nuclear level density calculation, which uses the assumption that the individual neutrons and protons occupy a set of low energy levels in the ground state and fill up the higher individual states at any excitation energy, has been successfully used so far, with different contributions made to this model in the form of shell , pairing , deformation effects [5,7,8], finite size effects [12],and thermal and quantal effects [10], as well as improvements in the determination of the spin cut-off factors [11]. However, such contributions do not take into account the collective effects, which may play a basic role in describing the nuclear level density of some deformed nuclides.

In the presents study, the nuclear level density parameters of deformed $^{161-168}$Er and $^{204-210}$Bi target isotopes have been calculated by using collective excitation mode of the nuclear spectra near the neutron binding energy and a simple model introduced in works [6,13], in which the collective character of the nuclear excitations is available. The results are comparing with different methods.

Nuclear Level Density

Bethe theory gives dependence of the nuclear level density on the total angular momentum *J* of the nucleus idepending on ESM model. The expression used for the observable nuclear level density at any excitation energy *U* and momentum *J* can be written as [4]:

$$\rho(U,I)=\frac{\sqrt{\pi}\ \exp\left(2\sqrt{aU}\right)}{12\ a^{1/4}\ U^{5/4}}\frac{(2I+1)\ \exp[-(I+(1/2))^2/2\sigma^2]}{2\sqrt{2\pi}\sigma^3} \quad \text{.......(1)}$$

where *I* is the total spin, *a* and *σ* are the level density parameter and spin distribution parameter, respectively, and defined by [5]:

$$a=\frac{\pi^2}{6}g(\varepsilon_F) \quad \text{.......(2)}$$

$$\sigma^2=g(\varepsilon_F)\langle m^2\rangle t \quad \text{.......(3)}$$

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Here, the parameter $g(\varepsilon_F)$ is the sum of the neutron and proton single-particle states density at the Fermi energy(ε_F),$\langle m^2\rangle$ is the mean square magnetic quantum number for single-particle states, and *t*-is the nuclear thermodynamic temperature of an excited nucleus in the Fermi gas model. These factors are expressed as follows:

$$\left.\begin{aligned} g(\varepsilon_F)&=\frac{3}{2}\frac{A}{\varepsilon_F}\\ \langle m^2\rangle&=0.146A^{2/3}\\ t^2&=\frac{U}{a}\end{aligned}\right\} \quad \text{.......(4)}$$

where which *A* is the mass number of a nucleus.

The experimental observations cannot determine the different orientation of nuclear angular momentum *J* .Therefore, it is useful to obtain the observable level density, which has the form [4, 5]:

$$\rho(U)=\sum_J\rho(U,I)=\frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{aU}}{a^{1/4}U^{5/4}}\frac{1}{\sqrt{2\pi\sigma}} \quad \text{.......(5)}$$

Hence, substitute eq's.(2)–(4) into eq.(5) to find the observable level density as :

$$\rho(U)=\frac{a}{12\sqrt{2}}\ \frac{\exp(2\sqrt{aU})}{0.298A^{1/3}(aU)^{3/2}} \quad \text{.......(6)}$$

The level density parameters of the Bethe theory have been well established in a number of studies [2,15,16] on the neutron resonance for different mass nuclei. However, this theory does not take into account the collective effects of the nuclear particles in the excitation of the nuclei. On the other hand, the excited states and the magnetic and quadrupol moments are the results of collective motion of many nucleons, not just of those nucleons that are outside the closed shell. The collective motion of the nucleons may be described as a vibrational motion about the equilibrium position and a rotational motion that maintains the deformed shape of the nucleus.

Al-Quraishi [21] suggestion two systematic equations to calculated LDP, first one drived from the semi-empirical formula for determination of the masses of nuclei without the pairing term and given as :

$$a=\alpha A\ \exp[-\gamma(Z-Z_o)^2] \quad \text{.......(7)}$$

Where $\alpha=0.1068$ and $\gamma=0.0389$ and $Z_o=\frac{0.5042\ A}{1+0.0073\ A^{2/3}}$ is the approximate atomic number for beta-stable nucleus of mass number A. Other equation accounts for the effect of isospin on the level density

$$a=\alpha A\ \exp[-\beta(N-Z)^2] \quad \text{.......(8)}$$

where $\alpha=01062$ and $\beta=0.00051$.While Rohr [16] systematic equation which using the structures in the level density parameter as a function of the mass number allow the determination of the hierarchy of the compound states and given by :

$$\left.\begin{aligned} a&=0.071\,A+V\\ \text{Where } V&=1.64 \quad \text{for } A\le 38\\ V&=3.74 \quad \text{for } 38 < A\le 69\\ V&=6.78 \quad \text{for } 69 < A\le 94\\ V&=8.65 \quad \text{for } 94 < A\le 170\\ a&=0.108\,A+2.4 \quad \text{for } A\ge 170\end{aligned}\right\} \text{.......(9)}$$

Collective Excitation Modes

As protons or neutrons are added to a closed‒shell nucleus, the nucleus may shift to oblate, prolate or triaxial shape. In these instances, the spherical shell model is in adequate and collective models , such as that proposed by Bohr and Mottelson [12], must instead be brought to bear and for nuclei that are in regions of strong deformation, these models have a wide array of successes.

The existence of collective energy level bands of rotational and vibrational types can now easily be identified from nuclear spectra data [17] of many deformed nuclei. In the studies [16,6] the contribution of collective motion of nucleons to the energy level density has been considered. However, these studies involve confusion equations and make complex for calculation of the nuclear level density parameters of deformed nuclei. A simpler description of collective model was first suggested by Rainwater [18] who made the relationship between the motion of individual nuclear particles and the collective nuclear deformation. Later a quantitative development of the nuclear collective model taking into consideration the collective motion of the nuclear particles was given by [12,19].

The rotational bands in deformed nuclei and their electromagnetic transitions are fundamental manifistations of nuclear collective modes. The traditional approach to study them is via the geometrical model of Bohr and Mottelson [12]. It is also well known that nuclei are not rigid. This can be seen from the fact that as the spin momentum *J* becomes higher, the nuclear moment of inertia will generally increase as well. This fact is known as stretching.

Recently in considerable studies it has been attempted to identify the nuclear level density parameters in the region of some light and large deformed nuclei by the use of a simple model of nuclear collective excitation mechanism. Almost all data on the estimated level density parameters of these deformed nuclei are well identified on a base of collective rotational and collective vibrational bands such as ground state band, β band (ellipsoidal deformation), octupole band, γ–band (axial deformation), and so forth.

The nuclear level density formula introduced depending on the excitation energy *U* and energy unit ε_o for the *i-th* excitation band of deformed nuclei can be represented as:

$$\rho_i(U,\varepsilon_{oi}) = \frac{\pi^2 a_{oi}}{24\sqrt{3}(a_{oi}U)^{3/2}} \exp\left(2\sqrt{a_{oi}U}\right) \quad ....(10)$$

which are simple and contain only one parameter a_{oi} defined as:

$$a_{oi} = \frac{\pi^2}{6\varepsilon_{oi}} \quad ....(11)$$

and represents a collective level density parameter corresponding to the *i-th* band with the unit energy ε_{oi} is the quasi-particle energy of each single-particle orbital can be obtained by

$$\varepsilon_i = \left[(E_i - \varepsilon_F)^2 + \Delta^2\right]^{1/2}$$

where E_i is single particle energy calculated from[24], ε_F is the Fermi level energy and Δ is the pairing gap energy which is chosen to be $12A^{-1/2}$ MeV [25]. The unite energies are $\varepsilon_{oGS} = E(2^+)$, $\varepsilon_o\beta = E(2^+) - E(0^+)$, and $\varepsilon_{ooct} = E(3^-) - E(1^-)$ for ground state, β and octupole bands, respectively. Ground state excitation energy is given by:

$$E(I^\pi) = \frac{\hbar^2}{2\Im}[I(I+1)] \quad ....(12)$$

where $\Im = \frac{2}{5}MAR^2$ is a rigid body, *m* and *A* are nucleon and atomic masses, *R*-nucleus radius.

The rotational energy levels of the ground state bands in even-even deformed nuclei could be interpreted on the basis of the semi-classical model.The energy contains (in addition to the usual rotational term) a potential energy term which depends on the difference of the moment of inertia. We shall use only for ground state band of even -even and odd-odd nuclei.

Results and Discussion

In the present paper, we have calculated the nuclear level density parameters of deformed target isotopes $^{161-168}$Er and $^{203-214}$Bi by using collective excitation modes of nuclear spectra. It has been seen that the nuclear energy levels of different collective excitation bands is obey the eq.(10). Thus, eq.(11) can be applied for determination of the corresponding level density parameters. The calculated values of the level density parameters due to ground state excitation bands for the deformed target isotopes *Er* and *Bi* and the compiled values of those parameters have been represented in Figures 1 and 2 that compiled by William in ESM [5], Rohr [16], Al-Quraishi 1-2 model [20,21], and Mughabgahab and Dunford [22] for s-wave neutron resonances near the neutron binding energy and compard wit non-ESM. In Figs. (1) and (2) we illustrate the comparison of the single-particle level density parameters *a* and the mass number with our calculated values of a_o corresponding to ground state bands for $^{161-168}$Er and $^{204-210}$Bi deformed isotopes, respectively. From figures, it is clear that the present values of the level density parameters a_o calculated by (11) for these considered isotopes are not well consistent with all the compiled values of parameters *a* of different formula. As seen in Fig.(1), the possibility that *a* not be the same for nuclei off the stability line as it is on the stability line as Al-Qurashi [20]. Form Fig. (1a), the dominant bands in the population of $^{161-168}$Er deformed isotopes generally seem to be the well-known collective bands (ground state), as to $^{161-168}$Er deformed isotopes it seems to be the mixed bands (negative and positive parity bands). As clear from Fig. (1b), we can say that the calculated values a_o for the mixed bands (negative and positive parity bands) of $^{204-210}$Bi are generally good dominant bands. The values of the calculated parameters of these bands are well consistent with those of the compiled data, in particular with the data of Mughabgahab and Dunford [22] for $^{204-210}$Bi.

Conclusions

On the basis of the above presented discussion we can conclude that the nuclear level density parameters of deformed target isotope $^{161-168}$Er and $^{203-214}$Bi can be identified by the use of collective rotational bands taking into consideration the equidistant.

Table (1)
Level density parameter (a- MeV^{-1}) of Er and Bi isotopes calculate by different formula comparing with experimental of [22] .Sub-numbers indicated for figures.

Radio nuclide	*Mughabgahab and Dunford¹*	*William² ESM*	*Rohr³*	*Al-Quraishi⁴ 1*	*Al-Quraishi⁵ 2*	*a_o⁶*	*a non -ESM*
161 Er	**19.555**	**20.371**	**20.081**	**16.188**	**12.501**	**1.526**	**1.842**
162	**19.659**	**20.448**	**20.152**	**16.782**	**12.256**	**21.724**	**21.032**
163	**19.763**	**20.624**	**20.223**	**17.230**	**12.003**	**3.727**	**3.927**
164	**19.867**	**20.751**	**20.294**	**17.506**	**11.742**	**22.719**	**23.472**
165	**19.971**	**20.877**	**20.355**	**17.591**	**11.475**	**2.037**	**2.884**
166	**20.075**	**21.004**	**20.436**	**17.301**	**11.203**	**24.492**	**24.979**
167	**20.179**	**21.130**	**20.507**	**17.233**	**10.925**	**2.795**	**3.518**
168	**20.283**	**21.257**	**20.578**	**16.766**	**10.643**	**24.836**	**26.026**
204 Bi	**23.984**	**25.812**	**24.432**	**21.096**	**10.432**	**27.632**	**28.236**
205	**24.085**	**25.939**	**24.540**	**21.626**	**10.079**	**2.773**	**3.279**
206	**24.187**	**26.066**	**24.648**	**21.961**	**9.628**	**27.227**	**27.946**
207	**24.289**	**26.192**	**24.756**	**22.092**	**9.380**	**2.580**	**3.251**
208	**24.391**	**26.318**	**24.864**	**22.116**	**9.034**	**26.872**	**27.938**
209	**24.492**	**26.445**	**24.972**	**21.735**	**8.693**	**1.946**	**3.001**
210	**24.594**	**26.571**	**25.080**	**21.259**	**8.355**	**29.396**	**31.211**

Character of these bands including higher excitations. The nuclear energy level density at any excitation near the neutron binding energy may clearly have generally the same character such as collective rotational. Actually, as it has clearly been seen from Table (1) and Figs. (1) and (2), no dominant band alone is exactly responsible for identification of level density parameters *a* for the considered isotopes. Namely, the nuclear level density for such isotopes apparently should involve combination of partial level densities corresponding to the different bands.

Consequently, we remark that the nuclear collective excitation modes are quite meaning full in order to obtain the level density parameters of different isotopes. The calculation of these parameters based on the properties of the measured nuclear low lying level spectra should prove a productive area of study that should over ride the inherent experimental difficulties involve.

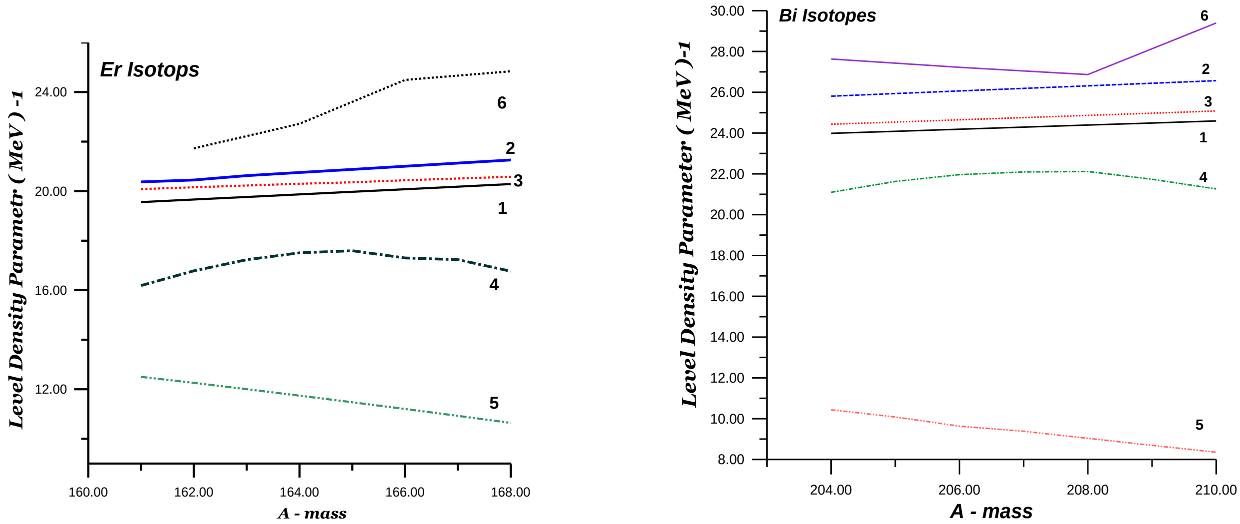

Fig.(1) Deformed isotopes level density parameter(a) as mass dependence compared with Mughabgahab and Dunford[1] Willam[2],Rohr[3],Al-Quraishi1[4] and Al-Quraishi2[5]. (a) for^{161}-^{168}Er (b) $^{204-210}$Bi.

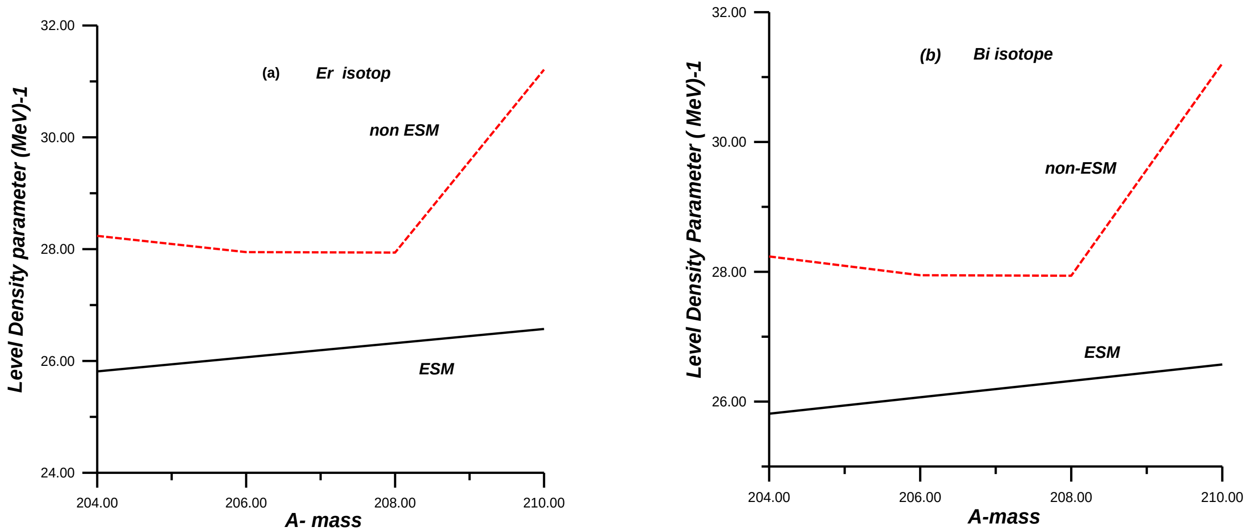

Fig.(2) Deformed isotopes level density parameter (a) comparison between ESM and non-ESM (a) for^{162}-^{168}Er (b) $^{204-210}$Bi.

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الخلاصة

تم في هذه الدراسة حساب معلم كثافة المستوي الجسيم المنفرد باستخدام نمط المستوي الأرضي الدوراني لنظائر الانوية المشوهة لعنصري Er,Bi عند طاقة ربط النيوترون باستخدام انموذج المسافات المتساوية وغيرالمتساوية في انموذج فيرمي الغازي وتمت المقارنة بين القيم المستحصلة لكل نظير من هذه النظائر ثم مقارنة النتائج المستحصلة مع عدد من النماذج النووية النظرية والعملية.