Enhanced Nuclear State Density Equation in a Two-Component Pre-Equilibrium Exciton Model for Heavy Nuclei

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Abstract

The properties of nuclear state density which increases with increasing excitation energy are very important in many nuclear reactions. Previously, several theoretical calculations were performed and added to correct the state density equation. In this work, various corrections, i.e. the pairing effect, Pauli exclusion principle, parity, spin distribution, the finite depth and the bound state effect were added to enhance the state density results using the energy-dependent single-particle level densities (non-ESM) in ^{114}Sn and ^{116}Sn nuclei. The results of this work were compared to the available experimental (OSLO technique), and theoretical (E&B and HF-BCS) results. The calculations were performed for energies up to 80 MeV and show a reasonable match with experimental and theoretical results at low energies. At high energies the deformation effect affects the results and leads to mismatch between the results of this work and other results. The inclusion of the spin distribution function in case of $j = \frac{1}{2}$, gives results far from the experimental data and previous theoretical results, which shows the importance of the introduction of a new formula that deals with a non-ESM system.

Keywords: Exciton model, state density, pre-equilibrium, two components.

Introduction

The exciton model [1] of nuclear reactions states that, when a particle incident on a target nucleus creates complex states of excited particles and holes in a compound system (projectile+target). The nuclear excited levels display a discrete spectrum for low excitation energies. When the excitation energy increases, the spacing of these levels reduces, and above 1 or 2 MeV, depending upon the mass of the nucleus, the level spacing became so weak, that it is experimentally impossible to distinguish all of them. Therefore, the description of these levels is replaced by state density. One of the most known models that evaluate the state density is the Fermi Gas Model (FGM). Using the FGM, Griffin [1] proposed the exciton model. This model describes the system as a number of excitons (the exciton refers to a particle-hole pair created, where a particle is excited to a higher-energy level leaving a hole) composed of p particles and h holes so that the exciton number is given by $n = p + h$. This may be denoted in two equivalent forms:

$n_0 \rightarrow n_0 + 2 \rightarrow n_0 + 4 \rightarrow n_0 + 6 \ldots.$,

$p_0 , h_0 \rightarrow (p_0 + 1 , h_0 + 1) \rightarrow (p_0 + 2 , h_0 + 2) \rightarrow (p_0 + 3 , h_0 + 3) \ldots..$

Each stage has one more particle-hole pair (or two more excitons) than the preceding stage. In this case, all particles and holes degrees of freedom were grouped together as excitons, and treated as undistinguishable from one another, so it is a one-dimensional problem (one-component system). When proton and neutron are distinguished from one another, each class of the states in above is divided into $h = p - A_a$ subclasses, and now it is a two-dimensional problem (two-component system). The excited states in the compound nuclei described by the state density $\omega(p, h, E)$, which defined as the number of states per unit energy, as a function of the numbers of particles and holes and excitation energy E, the models that used to describe the level spacing are the ESM (Equi spacing model) and the non-ESM. In the ESM model the single-particle level density, g, is constant and does not depend on the excitation energy of particles and holes, it gives a constant value for all values of excitation energies, and it just changed according to the mass number of the nuclei. By introducing the non-ESM model, g is no longer constant, and becomes energy dependent. In this work the non-ESM model is used to describe the state

density levels. The state density is very important because it provides a useful description of the compound nucleus, and also it gives a basic description for the pre-equilibrium emission [2]. Several corrections have been added to the state density equation to get a good agreement between theoretical calculations and experimental data. Next, the importance of these corrections is explained briefly.

Theory

Pauli principle requires that no two excitons of the same type are allowed to be in the same state, which implies that they cannot have the same energy [3]. The two component state density equation corrected by the inclusion of Pauli Exclusion Principle [3] is given by [4].

$$\omega_2(n,E) = \frac{g_\pi^{p_\pi+h_\pi} g_\nu^{p_\nu+h_\nu} \left(E-A_{p_\pi,h_\pi,p_\nu,h_\nu}\right)^{n-1}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \Theta\left(E - \alpha_{p_\pi,h_\pi,p_\nu,h_\nu}\right) \quad \text{..... (1)}$$

where, ω_2 refers to the two component state density model, and Θ is the step function which equals one for positive arguments and equals zero for negative ones.

Pauli effect correction has the form [4],

$$A_{p_\pi,h_\pi,p_\nu,h_\nu} = \frac{1}{4}\left[\frac{p_\pi(p_\pi+1)+h_\pi(h_\pi-1)}{g_\pi} + \frac{p_\nu(p_\nu+1)+h_\nu(h_\nu-1)}{g_\nu}\right] - \frac{1}{2}\left(\frac{h_\pi}{g_\pi}+\frac{h_\nu}{g_\nu}\right) \quad \text{..... (2)}$$

with

$$\alpha_{p_\pi,h_\pi,p_\nu,h_\nu} = \frac{p_\pi(p_\pi+1)+h_\pi(h_\pi-1)}{2g_\pi} + \frac{p_\nu(p_\nu+1)+h_\nu(h_\nu-1)}{2g_\nu} \quad \text{..... (3)}$$

$g = 3A/2F$ is the single particle level density [4]. F is Fermi energy,p, h refers to the number of particles and holes respectively. π refers to proton and ν refers to neutron. A is the nuclear mass number.

A is the mass number, p and h are the number of particles and holes respectively. π and ν refer to a proton and neutron respectively.

$g_\pi = Z/13$ and $g_\nu = N/13$ where Z and N are the protons and neutrons number [5].

The bound state and finite depth corrections [6] are important corrections that correct the state density equation. These corrections imply that the single particle excitation should not exceed the nucleon binding energy B and the single hole excitation energy should not exceed the Fermi energy F. The state density formula corrected by Pauli, bound state and finite depth corrections can be written as [6,7]:

$$\omega_2(p_\pi,h_\pi,,p_\nu,h_\nu,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \sum_{i_\pi=0}^{p_\pi}\sum_{i_\nu=0}^{p_\nu}\sum_{j_\pi=0}^{h_\pi}\sum_{j_\nu=0}^{h_\nu} (-1)^{i_\pi+i_\nu+j_\pi+j_\nu} \times C_{p_\pi}^{i_\pi} C_{p_\nu}^{i_\nu} C_{h_\pi}^{j_\pi} C_{h_\nu}^{j_\nu} \left(E - A_{p_\pi h_\pi p_\nu h_\nu} - i_\pi B_\pi - i_\nu B_\nu - j_\pi F_\pi - j_\nu F_\nu\right)^{n-1} \times \Theta\left(E - \alpha_{p_\pi p_\nu h_\pi h_\nu} - i_\pi B_\pi - i_\nu B_\nu - j_\pi F_\pi - j_\nu F_\nu\right) \quad \text{..... (4)}$$

F is Fermi energy and assumed to be 38 MeV, B is the binding energy, and in all calculations was taken to be 10 MeV.

The nucleons have to couple by pairs inside the nucleus. Before exciting the nucleons, pairs have to be broken which requires an additional energy. The state density of even-even nuclei is lower than that for odd-even nuclei which in turn is lower than the state density of an odd-odd nucleus. Adding the pairing correction, the state density equation becomes [8, 9],

$$\omega_2(p_\pi,h_\pi,p_\nu,h_\nu,E,A_k) = \left(\frac{g}{2}\right)^n \frac{(E-A_k)^{n-1}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \quad \text{..... (5)}$$

where A_k has the form [9],

$$A_k(p_\pi,h_\pi,p_\nu,h_\nu) = A_k(p_\pi,h_\pi) + A_k(p_\nu,h_\nu) \quad \text{..... (6)}$$

The function $F(p,h)$ in case of two component system is given by [9],

$$F(p_i,h_i) = 12n_i/n \quad \text{..... (7)}$$

where i can be either π or ν and E_i can be evaluated using [10]

$$E_\pi = \frac{En_\pi}{n}, \quad E_\nu = \frac{En_\nu}{n} \quad \text{..... (8)}$$

n is the total exciton number [5]

$$n = n_\pi + n_\nu \quad \text{..... (9)}$$
$$n_\pi = p_\pi + h_\pi \quad \text{..... (10)}$$
$$n_\nu = p_\nu + h_\nu \quad \text{..... (11)}$$

In the excited nucleus, the single particle state density should increase with increasing energy, so Kalbach [11] included the energy dependent single particle state density to the state density equation. The state density formula including the energy dependent single particle state density is given by [4],

$$\omega_2(p_\pi, h_\pi, p_\nu, h_\nu, E) = \frac{[g_\pi(p)]^{p_\pi}[g_\pi(h)]^{h_\pi}[g_\nu(p)]^{p_\nu}[g_\nu(h)]^{h_\nu}}{p_\pi! h_\pi!} \times \frac{[E-A(p_\pi,h_\pi,p_\nu,h_\nu)]^{n-1}}{p_\nu! h_\nu! (n-1)!} \times \sum_{i=0}^{h}(-1)^i \binom{h}{i}\left(\frac{E-iV}{E}\right)^{n-1}\Theta(E-iV) \quad \text{..... (12)}$$

where $g_p(p,h)$ and $g_h(p,h)$ are the Energy Dependent Single Particle and hole Level Density (EDSPLD) respectively, where $g_p(p,h)$ and $g_h(p,h)$ can be written as [10],

$$g_p(p,h) = g\left(F + \overline{u_p}\right) \quad \text{..... (13)}$$
$$g_h(p,h) = g(F - \overline{u_h}) \quad \text{..... (14)}$$

$\overline{u_p}$ and $\overline{u_h}$ are the single particle and hole energies respectively where [10],

$$\overline{u_p} = \frac{E}{n}\frac{f_k^+(p,h,E,F)}{f_k(p,h,E,F)} \quad \text{..... (15)}$$
$$\overline{u_h} = \frac{E - p\overline{u_p}}{h} \quad \text{..... (16)}$$

The spin distribution correction R_J [12] was added to the state density equation to include the dependence of the nuclear state density on the angular momentum J [13]. The particle-hole state density including all the above corrections is given by,

$$\omega_2(p_\pi, h_\pi, , p_\nu, h_\nu, E) = \frac{1}{2} \times \frac{[g_\pi(p)]^{p_\pi}[g_\pi(h)]^{h_\pi}[g_\nu(p)]^{p_\nu}[g_\nu(h)]^{h_\nu}}{p_\pi! h_\pi! p_\nu! h_\nu! (n-1)!} \times \sum_{i_\pi=0}^{p_\pi}\sum_{i_\nu=0}^{p_\nu}\sum_{j_\pi=0}^{h_\pi}\sum_{j_\nu=0}^{h_\nu}(-1)^{i_\pi+i_\nu+j_\pi+j_\nu} \times C_{p_\pi}^{i_\pi} C_{p_\nu}^{i_\nu} C_{h_\pi}^{j_\pi} C_{h_\nu}^{j_\nu} (E - A_k(p_\pi, h_\pi, p_\nu, h_\nu) - i_\pi B_\pi - i_\nu B_\nu - j_\pi F_\pi - j_\nu F_\nu)^{n-1} \times \Theta(E - E_{thresh} - i_\pi B_\pi - i_\nu B_\nu - j_\pi F_\pi - j_\nu F_\nu) \frac{2J+1}{2\sqrt{2\pi}\sigma^3} \; exp\left(-\frac{(J+1/2)^2}{2\sigma^2}\right) \quad \text{..... (17)}$$

where [13],

$$R_J = \frac{2J+1}{2\sqrt{2\pi}\sigma^3} \; exp\left(-\frac{(J+1/2)^2}{2\sigma^2}\right) \quad \text{..... (18)}$$

J was assumed to be ½ and the spin cutoff parameter σ can be written as,

$$\sigma^2 = c_n n A^{2/3} \quad \text{..... (19)}$$

Where,

$$c_n = 0.24 + 3.8 \times 10^{-3} E \quad \text{..... (20)}$$

The state density in equation (17) was multiplied by ½ [14, 15] under the assumption that the number of states with positive parities is identical to that with negative ones.

All the corrections that are described above are collected together to get the composite formula which includes all the recommended corrections that should be added to the state density equation.

Results and Discussion

To improve the state density equation, several corrections were collected together and added to it, i.e. pairing effect, Pauli exclusion principle, parity, spin distribution, the shell and bound state effect using the non ESM exciton model. Figs. (1, 2, 3), show the state density for two component systems (1, 1, 1, 1) in ^{116}Sn and ^{114}Sn adding several corrections, and compared with the theoretical (HF-BCS [14] and EB [16, 17] and OSLO experimental data [18, 19, 20] respectively. In these figures, the step function neglects the negative arguments. Because of this, curves start from 2 MeV excitation energy.

The composite formula of the state density which includes the pairing effect, Pauli exclusion principle, parity, the shell and bound state effect with EDSPLD, gives results close to the experimental and other theoretical results at energies lower than 7 MeV. The inclusion of the spin distribution function gives results far from other results. This function previously performed to correct the state densities using Equal-Spacing Model (ESM). That model suggested that the spacing between energy states is constant and independent on the excitation energy. In fact, the level spacing changed according to excitation energy, as the excitation energy increases the level spacing decrease. At high excitation energies the energy levels overlap and become as a single level. The spin distribution function did not deal with a non-ESM system, and for this the state density

is corrected with the spin distribution function gives unreasonable results, since this work has been done using a non ESM system.

At energies higher than 7 MeV, there is no correspondence between our results and other theoretical and experimental data. This inaccuracy is due to the deformation in heavy nuclei. In ^{56}Fe the deformation effect is much less than that in ^{116}Sn and ^{114}Snas seen in Fig.(4) [21]. In this figure a good agreement between the theoretical results for ^{56}Fe and experimental data (OSLO and evaporation technique) up to 10 MeV, and also a good match with other theoretical results (HF-BCS and E&B) up to 14 MeV is observed. For this reason the effect of deformation in ^{116}Sn and ^{114}Sn is clearly evident at excitation energies less than^{56}Fe and, hence, the deformation effect correction need to be considered in the state density equation when dealing with heavy nuclei to get good results and match with other theoretical and experimental data.

Conclusions

A modified formula (composite formula), for the state density has been developed in the present work, using the non-ESM exciton model including pairing effect, Pauli exclusion principle, parity, spin distribution, the finite depth and bound state effect. These corrections acted largely on the state density behavior, and they will reduce the state density value. A good match between the results obtained in this work, and the empirical and previous theoretical data is seen at energies lower than 5MeV, when including pairing effect, Pauli's exclusion principle, parity, the finite depth and bound state effect. The inclusion of the spin distribution function gives results far from the experimental and previous theoretical data; this is due to the inclusion of the non-ESM effects in the state density calculations.

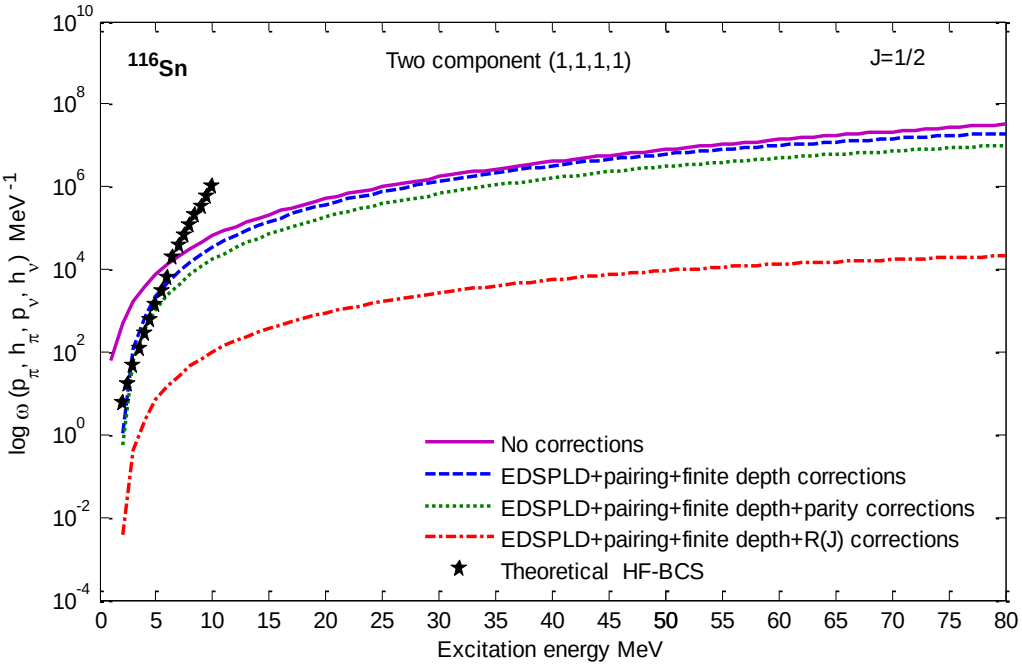

Fig. (1) The state density in ^{116}Sn as a function of excitation energy.

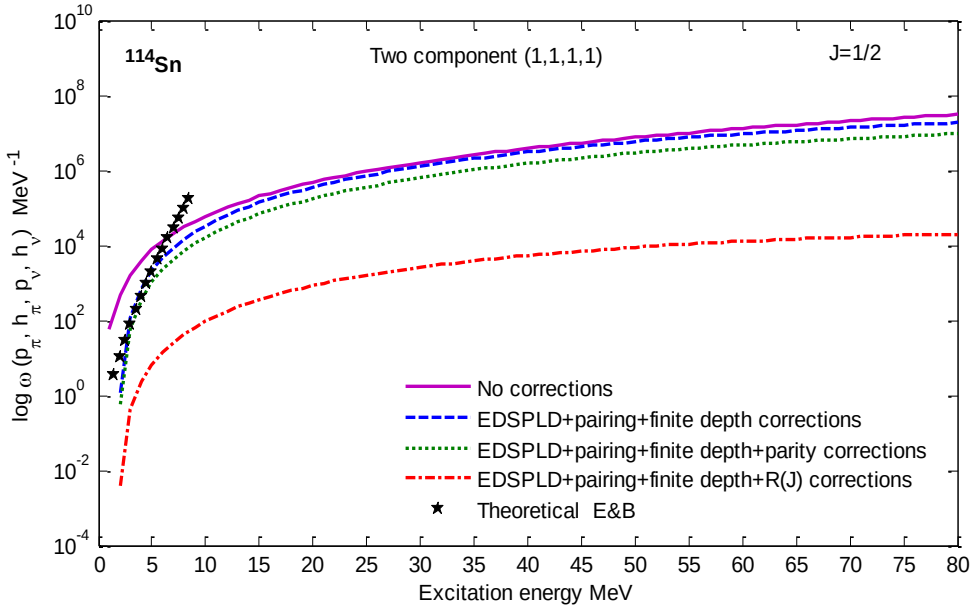

Fig. (2) The state density in ^{114}Sn as a function of excitation energy.

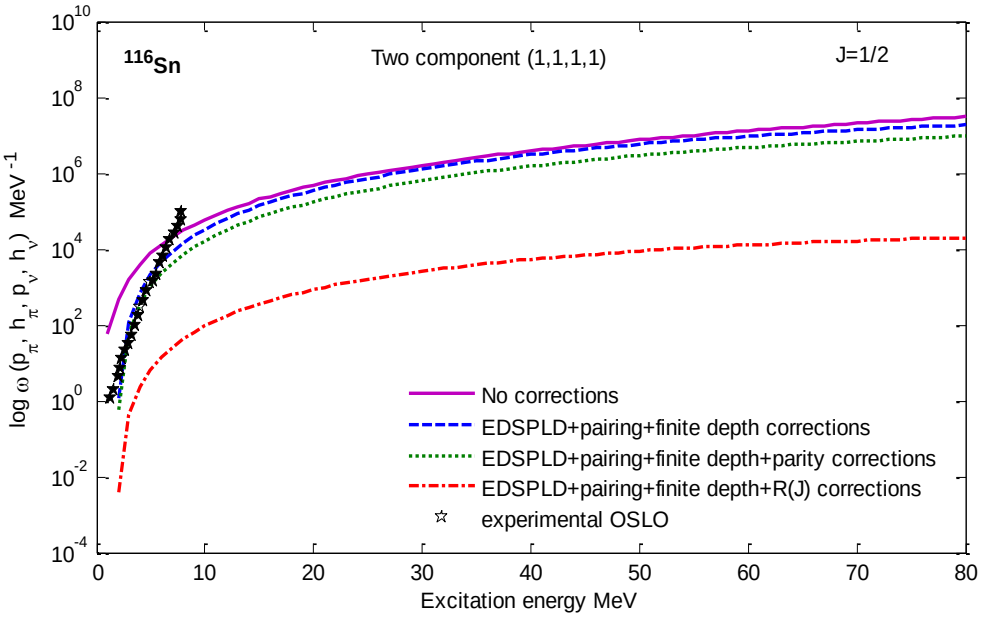

Fig.(3) The state density in ^{116}Sn as a function of excitation energy.

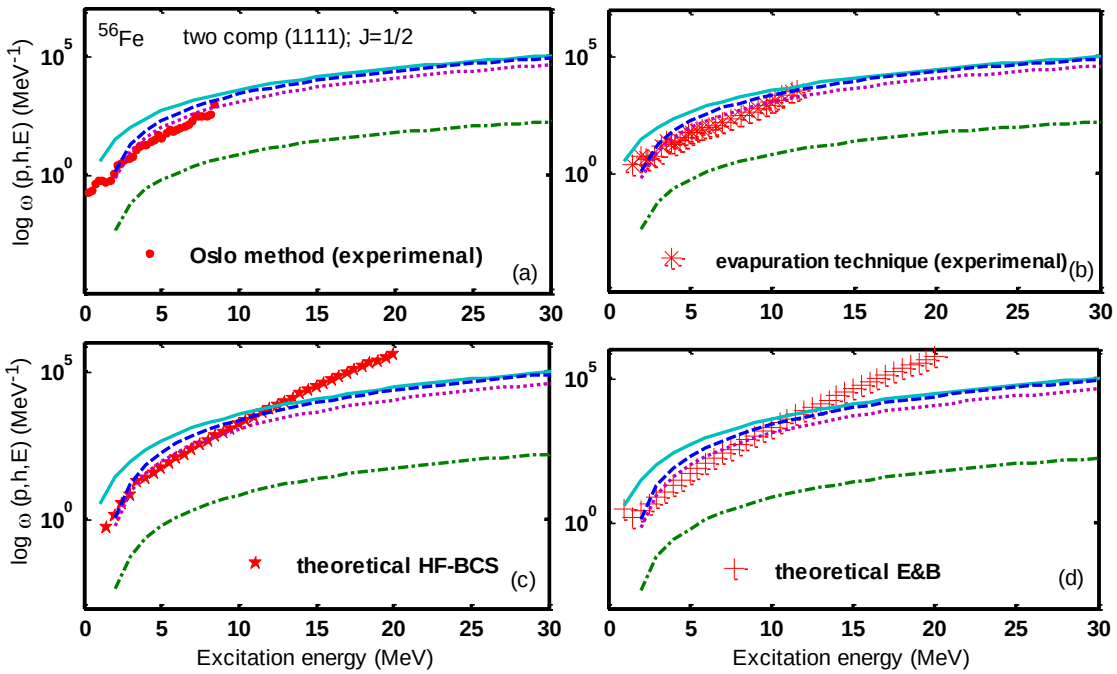

Fig. (4) The state density in ^{56}Fe as a function of excitation energy [21].

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الخلاصة

ان خواص كثافة المستويات النووية والتي تزداد مع ازدياد طاقة التهيج، مهمة جدا في الكثير من التفاعلات النووية. سابقا تم اجراء العديد من الحسابات النظرية واضيفت لتصحيح معادلة كثافة المستويات. في هذا العمل تم اضافة العديد من التصحيات مثل تصحيح الازدواج، مبدأ الاستبعاد لباولي، التناظر، توزيع البرم، القشرة و تأثير حالة الارتباط، لتحسين نتائج كثافة المستويات باستخدام كثافة المستويات للجسيمة المفردة المعتمدة على الطاقة في نواتي ^{114}Sn و ^{116}Sn. تم مقارنة نتائج هذا العمل مع الحسابات العملية المتوفرة (OSLO) والنتائج النظرية (E&B و HF-BCS). اجريت الحسابات لطاقات تصل الى ٨٠ مليون فولط وحصلنا على تطابق معقول عند الطاقات القليلة مقارنة مع الحسابات النظرية والعملية الاخرى. في الطاقات العالية فان تأثير التشوه يؤثر على النتائج ويؤدي الى عدم تظابق بين الحسابات التي اجريت في هذا العمل والحسابات الاخرى. ان تضمين دالة توزيع البرم عندما $j = \frac{1}{2}$ تعطي نتائج بعيدة عن الحسابات العملية والنظرية وهذا يبين اهمية تقديم معادلة جديدة تتعامل مع النظام الغير متساوي المسافات.