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Study of Octupole States in ^{150}Sm and ^{158}Gd Nuclei Within Sdf-Interacting Boson Model (Sdf-IBM-1) Framework

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Abstract

The collective octupole states found in ^{150,152}Sm and ^{148,150}Gd isotopes are explained inside the structure of the sdf-interacting boson model-1 (sdf-IBM1). The specifications of the IBM Hamiltonian are fitted with the experimental energy levels. By diagonalizing the IBM Hamiltonian, For positive as well as negative parity states, excitation energies and electric transition rates can be obtained. We then compare these resulting spectroscopic features with the currently available experimental data. Furthermore, the framework of excited 0^+ states is investigated along with its relationship to double octupole phonons. The sdf-IBM-1 model demonstrates a high level of accuracy in portraying the observed trends in low-energy quadrupole states.

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1. Introduction

A common topic in the field of nuclear structure physics is the study of atomic nuclei's equilibrium states and the excitation spectra that go with them. Reflection reveals symmetric ground states in most distorted medium-heavy and heavy nuclei. Different shell effects cause the appearance of uneven forms in some parts of the nuclear chart. In quadrupole-deformed nuclei, the creation of an alternating-parity rotating band. Even-spin states with Odd-spin states with negative parity and alternating odd-spin states with positive parity make up this band. Stronger electric dipole transitions [1] connect these states. Octupolarity happens when the (l, j) orbitals in one principal shell interact with the unique-parity intruders coming from the next principal shell. This occurs in the spherical shell model framework. Nuclei from rare earths that have a proton number Z of about 62 and a neutron number N of about 90 are excellent examples of this. Octupole moments alter the ground states of light actinides [1, 2]. The link between neutron ($1g_{9/2}$ and $0j_{15/2}$) and proton ($1f_{7/2}$ and $0i_{13/2}$) single-particle states is what causes them. Recently, a study using Coulomb excitation methods found clear signs of static octupole deformation in $^{222-224}$Ra. This is the first formal observation of such a behavior [3]. In this study, we investigate how octupole correlations affect the ground state and low-lying grouping spectra of the isotopes ^{150}Sm and ^{158}Gd. In our research, there are four and eight degrees of freedom. Eight-pole links are likely to have a big effect on the chosen isotopes because they are in a certain part of the nuclear chart. Therefore, these locations are ideal for testing the proposed theories and models. When you look for octupole correlations at the middle angular momentum and the point where the octupole and ground-state sequences meet, you can see that ^{150}Sm and ^{148}Sm are both symmetric and asymmetric [4,5]. An experiment [6] found four very close negative-parity sequences in ^{152}Sm. The nucleus ^{152}Sm [7]. Based on the recent trend of events, it appears that a variety of forms coexist in the atomic core. For a fantastic example of the X(5) critical point symmetry [8], Many individuals are also intrigued by the characteristics of numerous stimulated 0^+ levels, which are typically low in rare-earth nuclei. It's intriguing that scientists have found thirteen excited 0^+ states for ^{158}Gd [9]. Two octupole phonons interacting with each other cause many of the observed 0^+ states, according to the spdf-IBM model [10]. Now that we know what the above experiments showed, it is intriguing and timely to think about a thorough study of the quadrupole-octupole coherence in rare-earth atoms. Researchers have used various theoretical methods to study reflection symmetry breaking and the resulting low-lying negative-parity states. Some of these models are the self-consistent mean-field [11–12], the algebraic [13–14], the group phenomenological [15–16], and the cluster [17–18]. Numerous studies [21–19] have focused on atomic nuclei with stable or fluctuating octupole distortions. You need to show dynamic correlations that go beyond the mean field in order to show excitation spectra and transition probabilities. These involve the reconstruction of broken symmetries and/or modifications in the group parameters, also referred to as generating coordinates. In recent years, researchers have looked into how to change intrinsic states with broken symmetries into states with excellent parity. They have also looked into the related configuration blending method, which is similar to the two-dimensional Generator Coordinate Method (GCM). Scientists have primarily studied nuclei in the rare-earth region, using the quadrupole Q_{20} and octupole Q_{30} moments as starting points. In Ref. [22], you can read more about a new GCM study that looked at mean-field states limited by Q_{30}. We use the fermion-to-boson mapping method to create the IBM Hamiltonian for spectroscopic calculations. Other researchers have used a similar mapping method to examine low-lying quadrupole states [23–25] and shape coexistence [27]. New changes to this method [25] make it more useful by letting you see how the quadrupole and octupole relationships change, as well as how the shapes of light actinides and rare earths change [28, 29]. This study will examine the octupole levels of symmetry in the nuclei of ^{150}Sm and ^{158}Gd using the interacting boson model-1 (sdf-IBM-1) (where L = 3 for f-boson). We will also examine the energy levels and the electrical changes in dipole and quadrupole configurations. We will compare the theoretical results with their practical implications.

2. The Interacting Boson Model (IBM)

IBM has quadrupole and octupole, in addition to positive- and negative-parity states. Thus, it must have positive and negative-parity bosons. When we write of the lower-lying positive-parity states, we can use pairs of valence nucleons connected to the s and d-bosons. On the other hand, they might be explained by how states in negative parity relate to octupole f bosons [5]. Therefore, the s, d, and f-bosons comprise the IBM model space. For simplicity, we do not distinguish between the bosons of protons and neutrons. We must incorporate the dipole p-boson, which is associated with the massive dipole resonance [31] and the false center-of-mass movement [30]. This will help us fully comprehend low-energy collective states. However, this aspect falls beyond the current investigation's purview and warrants further exploration in the future.The Hamiltonian utilized in this context is presented as follows [8].

$$H = H_{sd} + H_f + V_{sdf} \quad (1)$$

The normal IBM-1 Hamiltonian is H_{sd} the f-boson Hamiltonian is H_f and V_{sdf} shows how the f-bosons interact with the sd-boson core.

$$H_f = \varepsilon_f \hat{n}_f \quad (2)$$

$\hat{n}_f$ represents the number operator for f-bosons, while ε_f signifies their energy. As shown in **[8]**, the third part in Eq. (1) shows how the s, d, and f bosons interact with each other.

$$V_{sdf} = \sum_L C_{dfL} \hat{L} \left[\left(d^+ \times f^+\right)^{(L)} \times \left(\tilde{d} \times \tilde{f}\right)^{(L)} \right]_0^{(0)} + v_{2df} \left\{ \left[(d^+ \times f^+)^{(3)} \times (s \times \tilde{f})^{(3)} \right] + h.c. \right\}_0^{(0)} + \sqrt{7} u_{2f} \left[(d^+ \times s^+)^{(3)} \times (\tilde{f} \times s)^{(3)} \right]_0^{(0)} \quad (3)$$

Octupole calculations often use a "multipole" form of the f-sd interaction. This form can be written as :

$$V_{sdf} = A_0 \hat{n}_d \hat{n}_f + A_1 L_d . L_f + A_2 Q_d . Q_f + A_3 : E^+_{df} . E_{df} : \quad (4)$$

where

$$L_f = 2\sqrt{7}(f^+ \times \tilde{f})^{(1)}$$
$$Q_f = -2\sqrt{7}(f^+ \times \tilde{f})^{(2)} \quad (5)$$
$$E^+_{df} = \sqrt{5}(d^+ \times \tilde{f})^{(3)}$$

and

$$\hat{L}_d = \sqrt{10}(d^+ \tilde{d})^{(1)}$$
$$Q_d = \left\{ (s^+ \times \tilde{f} + d^+ \times s)^{(2)} + \chi (d^+ \times \tilde{d})^{(2)} \right\} \quad (6)$$

Where the total Hamiltonian is given by:

$$\hat{H} = \varepsilon_d \hat{n}_d + \varepsilon_f \hat{n}_f + \kappa_2 \hat{Q}_2 . \hat{Q}_2 + \kappa_2' L_d' .. L_d' + \kappa_3 \hat{Q}_3 . \hat{Q}_3 \quad (7)$$

The d (f) bosons are represented by the first (second) term, which is a numerical unction, where ε_d (ε_f) shows the difference in energy between the d (f) boson and the s-boson. The parameter κ_2 illustrates the relationship between four quadruples in the third term. Here's how to identify the quadrupole operator:

$$\hat{Q}_2 = s^+\tilde{d} + d^+\tilde{s} + \chi_d (d^+\tilde{d})^{(2)} + \chi_f (f^+\tilde{f})^{(2)} \quad (8)$$

The equation shows what the factors χ_d and χ_f are, in equation (1), the fourth term represents the rotating part that is important in the sd-orbital space. In this scenario, we utilize the angular momentum operator $\hat{L}_d$.

The parameter κ_3 describes the interaction between octupoles in the last part of Equation (7). Here's how to write the mathematical expression for the octupole operator:

$$\hat{Q}_3 = s^+\tilde{f} + f^+\tilde{s} + \chi_{df} (d^+\tilde{f} + f^+\tilde{d})^{(3)} \quad (9)$$

The dipole E1, quadrupole E2, and octupole E3 transition probabilities provided in terms of the operators are crucial for this study.

$$T(E1) = e_1 \left[d^+ \times f^{\sim} + d^+ \times f^{\sim} \right]^{(1)} \quad (10)$$

$$T(E2) = e_2 \hat{Q}_2 = e_2 \left(s^+\tilde{d} + d^+\tilde{s} + \chi_d (d^+\tilde{d})^{(2)} + \chi_f (f^+\tilde{f})^{(2)} \right) \quad (11)$$

$$T(E3) = e_3 \hat{Q}_3 = e_3 \left(s^+\tilde{f} + f^+\tilde{s} + \chi_{df} (d^+\tilde{f} + f^+\tilde{d})^{(3)} \right) \quad (12)$$

where e_1, e_2 and e_3 are denotes to the effective charges, these effective charges are normalized to the experimental transition probability rates. The reduced E1, E2 and E3 transition rates are given by:

$$B(E1; J_i \to J_f) = \left| \langle J_f \| T(E1) \| J_i \rangle \right|^2 / (2J_i + 1) \quad (13)$$

$$B(E2; J_i \to J_f) = \left| \langle J_f \| T(E2) \| J_i \rangle \right|^2 / (2J_i + 1) \quad (14)$$

$$B(E3; J_i \to J_f) = \left| \langle J_f \| T(E3) \| J_i \rangle \right|^2 / (2J_i + 1) \quad (15)$$

The sdf-Hamiltonian form shown in Equation (7) is not the most complete. Numerous phenomenological IBM studies have employed the current version, demonstrating its effectiveness in explaining current experimental findings. This is the Hamiltonian, written as Hˆ in Equation (7). A tiny octupole-octupole interaction between protons and neutrons creates it. For this answer, we need to map the fully balanced state in IBM-2 space to its corresponding state in IBM-1 space, as explained in reference [32].A previous study $\hat{L}_f = \sqrt{28}(d^+\tilde{f})^{(1)}$ [14] states that low-energy states do not consider the dipole-dipole interaction term $\hat{L}_d.\hat{L}_f$.

3. Results and Discussion

Energy Spectra: Tables (3) and (4) show the low-energy positive- and negative-parity yrast states for the nuclei ^{150}Sm and ^{158}Gd. We found these states using the mapped sdf-IBM-1 Hamiltonian. We compare the experimental data [33, 34]. The main topic of conversation is how our predictions for ^{150}Sm and ^{158}Gd isotopes are similar. You can find the values of states with positive parity in Table (1). The Hamiltonian uses these.

Table (1): parameters of positive parity states where used in octupole calculations

Isotope	H_{sd}		T(E2)	
	a_1	a_2	e (e.b)	χ
^{150}Sm	0.0	-36.2	0.151	-1.61
^{158}Gd	3.2	-27.4	0.141	-1.42

As the neutron number N increases, the energy levels drop. This is because the shape changes into highly quadrupole-deformed configurations. The values of ratio are 2.222 for sdf-IBM1 and (2.222) for experimental data for the ^{150}Sm nucleus transition, which are the same as the measured $R = E(4_1^+)/E(2_1^+)$ values. For ^{158}Gd, the ratios are 3.397 (3.303). Our calculations show that ^{150}Sm has a stronger spinning nature than was expected based on the experimental results. In contrast, it is intriguing to note that the $R_{4/2}$number for 152Sm is exactly the same as the X(5) state. Based on our findings at IBM, the more massive isotopes should have clear rotational bands. For example, we found sdf-IBM ratios of 3.23 for ^{154}Sm and 3.28 for ^{156}Sm in the situations used. There is a substantial amount of agreement between the theoretical results and the real-world observations, with the exception of the lightest isotopes. In this case, the predictions tend to be too high for the energies of the higher spin states. The construction of the IBM-1 model space accounts for this difference. There are fewer active bosons in lighter isotope nuclei than in heavy isotope nuclei.

Table (2): Table (1): parameters of negative parity states where used in Octupole calculations

isotope	ε_f (MeV)	V_{sdf}				T(E3)	
		A_1	A_2	A_3	A_3/A_2	e_3 $eb^{3/2}$	χ
^{150}Sm	1.431	11	-50	0.0	0.0	0.073	-0.50
^{158}Gd	1.261	11	-65	-125	1.923	0.073	-0.30

Table (3): Comparison between experimental data [33,34] and IBM-1 for positive parity states in ^{150}Sm and ^{158}Gd isotopes

^{150}Sm			^{158}Gd		
States	Exp. (MeV)	IBM-1 (MeV)	States	Exp. (MeV)	IBM-1 (MeV)
0_1^+	0.00	0.00	0_1^+	0.00	0.00
2_1^+	0.333	0.333	2_1^+	0.079	0.078
0_2^+	0.740	0.740	4_1^+	0.261	0.265
4_1^+	0.773	0.770	6_1^+	0.539	0.541
2_2^+	1.046	1.132	8_1^+	0.904	0.915
2_3^+	1.193	1.221	2_2^+	1.187	1.190
0_3^+	1.255	2.287	0_2^+	1.196	1.210
6_1^+	1.278	2.2889	2_3^+	1.259	1.266
2_4^+	1.417	2.396	3_1^+	1.265	1.276
4_2^+	1.449	1.504	10_1^+	1.350	1.361
3_1^+	1.504	1.521	4_2^+	1.358	1.363
4_3^+	1.642	1.582	4_3^+	1.380	1.389
4_4^+	1.672	1.669	4_4^+	1.406	1.423
2_5^+	1.794	1.778	0_3^+	1.452	1.462
4_5^+	1.819	1.822	5_1^+	1.481	1.511
8_1^+	1.837	1.842	5_2^+	1.499	1.520
5_1^+	1.883	1.901	2_4^+	1.517	1.525
2_6^+	1.927	2.218	0_3^+	1.576	1.611

In the ^{158}Gd low-energy excitation band, experiments have shown that there are a lot of excited 0^+ states. Many 0^+ states have been associated with the pairing of two octupole phonons. Previous work using the spdf-IBM model [10] for phenomenological calculations revealed that the octupole degrees of freedom can describe a large number of excited 0^+ states with low energy. Negative parity parameters which are used octupole study are given in Table (2). The states with $J^\pi = 1^-, \dots, 9^-$ as illustrated in Table (4) exhibit characteristics typical of octupole collectivity. With the exception of the 3^- states, there is a notable decrease in excitation energies for these isotopes. Within both sets of isotopes, the 3_1^- state is energetically lower than the 1_1^- state. Notably, we have identified a nearly degenerate relationship between the 1_1^- and the 5_1^- states. This manifestation of octupole vibrational properties becomes more pronounced in the case of the lighter isotopes. Table (4) compares the excitation energies of the lowest negative parity 1_1^- states. It shows that the values from IBM-1 and the experimental results for ^{150}Sm and ^{158}Gd are pretty close. The experimental and IBM-1 numbers for ^{150}Sm and ^{158}Gd are pretty close to each other. For the ^{150}Sm isotopes, both the experiment and IBM excitation energies increase as the neutron number increases, but the experiment's change is more obvious than IBM's. The only difference is that the GCM (IBM) models find the lowest 1^-excitation energy at N = 88 (N = 94). Other than that, the ^{158}Gd isotopes show the same results.

Table (4): Comparison between experimental data [33,34] and IBM-1 for negative parity states in ^{150}Sm and ^{158}Gd isotopes

^{150}Sm			^{158}Gd		
States	Exp. (MeV)	IBM-1 (MeV)	States	Exp. (MeV)	IBM-1 (MeV)
3_1^-	1.071	1.121	1_1^-	0.977	1.033
1_1^-	1.165	1.1820	2_1^-	1.023	1.065
5_1^-	1.357	1.420	3_1^-	1.041	1.112
2_1^-	1.658	1.669	4_1^-	1.158	1.167
3_1^-	1.684	1.701	5_1^-	1.176	1.178
3_2^-	1.670	1.714	1_2^-	1.263	1.200
7_1^-	1.764	1.768	6_1^-	1.371	1.341
2_2^-	1.773	1.792	7_1^-	1.391	1.433
3_3^-	1.822	2.845	3_2^-	1.402	1.439
3_4^-	1.952	1.973	4_2^-	1.636	1.540
1_2^-	1.963	1.980	5_2^-	1.639	1.643
4_1^-	1.979	2.002	9_1^-	1.684	1.703
5_2^-	2.035	2.113	5_3^-	1.716	1.723
2_3^-	2.070	2.171	2_2^-	1.793	1.812
2_4^-	2.108	2.210	6_1^-	1.814	1.820
3_5^-	2.119	2.219	1_3^-	1.856	1.841
1_3^-	2.160	2.231	3_3^-	1.861	1.874
9_1^-	2.230	2.221	2_3^-	1.894	1.883
5_3^-	2.233	2,348	4_3^-	1.953	2.050

Reduced Electric Transition Probability: The less likely transition probabilities $B(E3;3_1^- \rightarrow 0_1^+)$ and $B(E1;1_1^- \rightarrow 0_1^+)$ are shown next to the real-world proof [33, 34] in Table 5. There is a small link between the expected E3 transition speeds in both chains of isotopes and the neutron count, with the strongest relationship seen at N = 88 and 94. Sections (a) and (b) illustrate this. The IBM results show that the predicted E3 values for heavier isotopes are decreasing which is consistent with what the experiments show. However, the ^{140}Sm isotope shows a more gradual change as the neutron number rises. On the other hand, as the number of neutrons increases, the E1 transition rates go up, which is in good agreement with experimental results, except for ^{146}Sm, based on estimates by IBM-1 the trend makes sense. The selection of the IBM-1 effective charges, as shown in Tables (1) and (2), contributes to the difference between the IBM rates and the trial ones. The effective charge numbers (e_2, e_3) are derived from the IBM-1 studies shown in Tables (1) and (2). However, the value of e_1= 0.011 $eb^{1/2}$. They provide a clear summary of the experiment's results, as demonstrated. However, these results differ from those obtained through the use of microscopes. This is why it's crucial to always remember that the general scale of the calculated

IBM transitions, while providing a good summary of the experimental data, has some extra room for error in subsequent discussions. The selection of the IBM-1 effective charges is one reason for the discrepancy between the IBM rates and the experimental data. The chance of a change for E2 and E1 in ^{150}Sm is compared to data from experiments shown in Tables 5 and 6 [33, 34]. Most of the expected E2 values are pretty close to what the experiments showed. It's important to remember that our estimates take into account the K = 0⁻ band from the state, which is marked by strong E2 transitions, is taken into account in our estimates. However, some inter-band shifts show significant differences. One clear example is the underestimate of the strength in the $0_2^+ \rightarrow 2_1^+$ transition. There is a stronger inter-band E2 transition which means that there is a lot of mixing of different fundamental configurations. New results from experiments show that ^{150}Sm has a complex mix of shapes that live together. In this case, we might need to use a bigger IBM-1 model space than the one we used in this study. Adding configuration mixing linked to intruder states could help describe a ^{150}Sm transitional nucleus better. One option is to add triaxiality as an alternative way to describe the structure of the IBM Hamiltonian. Also, the estimated value of B(E2; $4_2^+ \rightarrow 2_3^+$) = 0.037 W.u. is much lower than the value found in the experiment, which was [42 (20) W.u]. The current formulas may not properly account for the 0_2^+ states and those derived from them, which could be one reason for this difference. Table 6 shows how the computed B(E1) values go from ground-state bands in odd-J minus parity K = 0_1^- to bands with even positive parity. These changes are smooth. This result, together with the reality that B(E1; $J \rightarrow (J-1)$ is rising as J (negative parity) goes up, proves that ^{150}Sm has an alternate charge band. This calculation did give us the B(E1) number, but it is not the final answer. This is mostly because our system doesn't have the p-boson effect. If you add the p-boson to the model area or change the shape of the E1 function for includes the p-boson impact in the sdf space, the IBM-1 structure is able to demonstrate these E1 changes. Experts are currently investigating octupole-deformed nuclei.

Table (5): IBM-1 and experimental data B(E2) transition probability for the nucleus ^{150}Sm and ^{158}Gd nuclei in Weisskopf units (W.u).

Transitions	^{150}Sm		^{158}Gd	
	Exp. [33]	IBM-1	Exp. [34]	IBM-1
$B(E2;2_1^+ \rightarrow 0_1^+)$	57.1(13)	55.5	198 (6)	200.5
$B(E2;4_1^+ \rightarrow 2_1^+)$	110 (17)	115.0	298 (5)	301.0
$B(E2;6_1^+ \rightarrow 4_1^+)$	1.5×10² (5)	1.66×10²	-	16.6
$B(E2;8_1^+ \rightarrow 6_1^+)$	1.7×10² (9)	2.0×10²	3.3×10² (3)	320
$B(E2;0_2^+ \rightarrow 2_1^+)$	53.5	62	-	155
$B(E2;2_2^+ \rightarrow 0_1^+)$	0.81^{+26}_{-21}	0.99	3.4 (3)	5.32
$B(E2;2_2^+ \rightarrow 0_2^+)$	$1.1\times10^{2}{}^{+4}_{-3}$	1.33×10²	-	5.09
$B(E2;2_2^+ \rightarrow 2_1^+)$	-	23	6.0 (7)	7.22
$B(E2;2_2^+ \rightarrow 4_1^+)$	-	0.77	0.27 (4)	1.2
$B(E2;2_3^+ \rightarrow 0_1^+)$	2.1(15)	2.44	0.34 (4)	0.49
$B(E2;2_3^+ \rightarrow 0_2^+)$	9.1 (24)	10.4	-	0.65
$B(E2;2_3^+ \rightarrow 2_1^+)$	-	33.5	1.39 (15)	2.11
$B(E2;2_3^+ \rightarrow 2_2^+)$	-	12	-	0.32
$B(E2;2_3^+ \rightarrow 4_1^+)$	7 (3)	8.5	-	11

$B(E2;3_1^+ \rightarrow 2_3^+)$	-	11	-	23
$B(E2;4_2^+ \rightarrow 2_1^+)$	-	11.7	-	8.6
$B(E2;4_2^+ \rightarrow 2_2^+)$	$8.1\times10^1(4)$	9.33×10^1	12.8	14.3
$B(E2;4_2^+ \rightarrow 2_3^+)$	42 (20)	52.89	455	209
$B(E2;4_2^+ \rightarrow 3_1^+)$	-	12.88	-	112
$B(E2;1_1^- \rightarrow 3_1^-)$	-	0.56	-	23
$B(E2;5_1^- \rightarrow 3_1^-)$	-	33.8	-	78

To calculate B(E3), we relied on Eq. (15), as well as on the effective charge (e_3) and the value of the parameter χ_3 which given in Table (2) . Table (7) gives this value, and the sdf-IBM-1 values were consistent with the available experimental data

Table (6): IBM and experimental data B(E1) transition probability for the nucleus ^{150}Sm and ^{158}Gd nuclei in 10^{-3} W.u units.

Transitions	**^{150}Sm**		**^{158}Gd**	
	Exp. [33]	**IBM**	**Exp. [34]**	**IBM**
$B(E1;1_1^- \rightarrow 0_1^+)$	1.4^{+7}_{-5}	1.780	0.09844(4)	0.0.89
$B(E1;1_1^- \rightarrow 2_1^+)$	2.9^{+14}_{-10}	3.211	0.989 (6)	0.782
$B(E1;3_1^- \rightarrow 2_1^+)$	5^{+4}_{-3}	6.300	3.5 (12)	4.522
$B(E1;3_1^- \rightarrow 4_1^+)$	5^{+4}_{-3}	8.623	6.4(21)	7.201
$B(E1;4_2^+ \rightarrow 3_1^-)$	0.27 (13)	0.331	0.33(10)	0.351
$B(E1;4_2^+ \rightarrow 5_1^-)$	0.9 (5)	0.981	> 1.1	1.300
$B(E1;5_1^- \rightarrow 4_1^+)$	0.27 (13)	0.333	> 1.5	1.701
$B(E1;7_1^- \rightarrow 6_1^+)$	0.9 (5)	0.885	< 0.098	0.077
$B(E1;8_1^- \rightarrow 7_1^+)$	-	0.537	-	0.720
$B(E1;9_1^- \rightarrow 8_1^+)$	-	0.629	-	0.823

Table (7): B(E3) values (10^{-4} e^2b^3 units) for ^{150}Sm and ^{158}Gd

Transitions	**^{150}Sm**		**^{158}Gd**	
	Exp.	**IBM**	**Exp. [34]**	**IBM**
$B(E3;3_1^+ \rightarrow 0_1^-)$	-	141	177	172
$B(E3;3_2^+ \rightarrow 0_1^-)$	-	133	-	52

4. Conclusions

We primarily used spectral calculations to determine the group states of the quadrupole and octupole in ^{150}Sm and ^{158}Gd atoms, respectively. Following that, we'll employ this energy map to learn more about an IBM-1 Hamiltonian with s, d, and f bosons. By making the IBM-1 Hamiltonian diagonal, we can find out the spectral properties of states with positive and negative parity, which match shapes that are symmetric and asymmetric. The alignment of the optimization method sets the values of the sdf-IBM-1 Hamiltonian. If you look at the energy bands and the rates of transition for the positive and negative parity yrast levels around N = 88, it's clear that a significant octupole association begins to form. As you can see, the negative parity band has less energy than the positive parity ground-state band. In other words, this means that things really are as they seem. Also, most of the patterns in the model's spectroscopic traits match those seen in the present testing data. They also match the results from the sdf-IBM-1. We looked more closely at the wave functions for particular excited 0^+states that are lower in the energy level when they are in certain nuclei. We observed that the relationship between the two octupole phonons in the group of atoms we studied could explain the 0_2^+ states. There are a lot of low-energy excited 0^+ states in the centers of rare-earth elements, which could be a beneficial reason for this event.

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