



## The Non-Adjacency Incompatible Vertices Topology on Digraphs

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### Abstract

In this work, we introduce a new type of topology associated to directed graph on vertices set, which is named as non-adjacency incompatible vertices topology (*NIV*-topology) of directed graph. A sub-basis family of this topology is generated on the set of vertices, and formed via taking the non-adjacency vertices which arises (path of length two) in different direction to each vertex. We investigate some properties and discuss it on an important and certain types of directed graphs. Our motivation is to giving fundamental steps toward investigation of various properties of directed graphs via their corresponding non-adjacency incompatible vertices topology. Some results are obtained, the presence of the isolated vertex is not necessary in our definition, observations about the specification of this new topology with certain types of digraphs (which type of digraphs achieves the discrete non-adjacency incompatible vertices topology and which does not, and when it achieves  $T_1$  and  $T_2$ -space). Furthermore, application (applied example in biomathematics) to the Pulmonary circulation in humans is introduced in this paper.

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### 1. Introduction

Numerous applications have utilized the relationship between topology and graph to generate many new types of topologies generated by graph, because of the importance of topological graph theory as it is part of graph theory that has a great role and illustrious history in mathematics [1–4]. On the basis of vertices or edges, some topological models are developed or based. In the directed and undirected graphs, for locally finite which does not have isolated vertex was started using a graphic topology by [5] in 2013. In 2018, Kilicman and Abdulkalek [6] defined the compatible edges topology, such that a sub-base family on the edge set of directed graphs as a collection of sets of adjacent edges with each edge and construct a path of length two. In 2022, Asmhan and Zainab, present the independent compatible edges topology [7], the outlined it as the topology linked to the set  $\mathbb{E}$  of edges via a not adjacent edges which construct a path of length three. In 2022, Asmhan and Iman related an independent incompatible Edges topology based on di-graphs with some

applications [8]. Section 2 in this work involves main definitions of topologies and graphs. In the section 3, the definition of a non-adjacency incompatible vertices topology (*NIV* – topology) associated with digraph and some examples on a basic digraph are presented. In section 4, a biomathematical application of the non-adjacency incompatible vertices topological space (*NIV* – topological space), in human greater circulation is viewed. In the last section the conclusion of our new topology is presented with some future work.

### 2. Preliminaries

Basic definitions and introductions to graph theory and topological space are covered in this section. These ideas are all commonly used and can be found in [1]. Usually the di-graph is as  $G_\theta = (V, \mathbb{E})$ , where  $V$  represents the set of vertices and  $\mathbb{E}$  represents the set of directed edges, an edge of the form  $\sigma = (v, v)$  is loop. Parallel edges are those with the identical end vertices. If a graph contains no parallel edges or loop, it is considered simple. If the vertex  $v$  and  $u$  are

connected by edge then they are adjacent. All of these ideas are well-known and are available in books mentioned above. We use the symbols  $K_n$  for the complete graph with  $n$  vertices and  $C_n$  is cycle graph on  $n$  vertices and  $P_n$  is path on  $n$  vertices and  $K_{n_1, n_2}$  is a whole bipartite graph of size partite  $n_1$  and  $n_2$ . A topology  $T$  is a family of subsets that are open to the non-empty set  $X$  if the following conditions are hold:  $X, \emptyset \in T$  for every  $H, M \in T, H \cap M \in T$  and  $\cup_{i \in \Delta} H_i \in T$  for every sub-sets  $H_i$  of  $T$ , then  $(X, T)$  is called a topological space, an open set is sub-set of  $T$ . Indiscrete topology is defined as  $T = \{\emptyset, X\}$  on  $X$ , while discrete topology is defined as  $T = P(X)$  on  $X$ .

### 3. The Non-Adjacency Incompatible Vertices Topological Space

This section includes the definition of a non-adjacency incompatible vertices topological space (NIV – topological space) associated with di-graph and some examples on a basic di-graph.

#### Definition 3.1.

Let  $G_\theta = (V, E)$  be any directed graph. The non-adjacency incompatible vertices topology (NIV - topology) is a topology that relates to the set  $V$  of vertices for  $G_\theta$ , and brought on by sub-basis  $S_{NIV}$  whose components are the sets  $\omega \subseteq V, |\omega| \leq 2$ ; if  $v \in \omega$  and the vertex  $w$  is non-adjacent with vertex  $v$ , such that  $v$  and  $w$  connected by path of length two in the different direction then  $w \in \omega$ . The set of vertices with  $T_{NIV}$  initiate the topological space  $(V, T_{NIV})$  which called non-adjacency incompatible vertices topological space (NIV – topological space).

#### Example 3.2.

Consider the digraph  $G_\theta = (V, E)$  see Figure 1, such that:

$$V(G_\theta) = \{v_1, v_2, v_3, v_4, v_5\},$$

$$E(G_\theta) = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}.$$

$T_{NIV}$  has a sub-basis:

$$S_{NIV} = \{\{v_2, v_4\}, \{v_3\}, \{v_5, v_1\}\}$$

Then by using finite intersection, the following base  $\beta_{NIV}$  is produced:

$$\{\{v_2, v_4\}, \{v_3\}, \{v_5, v_1\}, \emptyset\}.$$

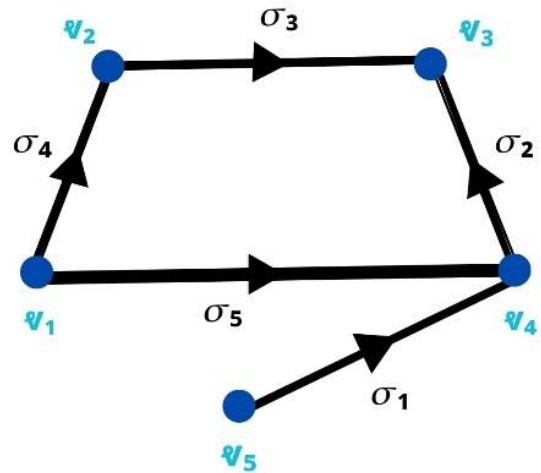


Figure 1. Di- graph with different direction.

Then, utilizing each union, generate a non-adjacency incompatible vertices topology  $T_{NIV}$  as the following:

$$T_{NIV} = \{\emptyset, V, \{v_2, v_4\}, \{v_3\}, \{v_5, v_1\}, \{v_3, v_2, v_4\}, \{v_3, v_5, v_1\}, \{v_2, v_4, v_5, v_1\}\}$$

Clearly, it is not discrete topology.

#### Remark 3.3

1. The topology  $T_{NIV}$  on di-graph star  $S_n$  represents a discrete topology.
2. The topology  $T_{NIV}$  on every directed tree represents a discrete topology.
3. The topology  $T_{NIV}$  on complete di-graph  $K_n$  is discrete topology.

#### Remark 3.4

1. The topology  $T_{NIV}$  on any di-graph  $C_n$  does not represent a discrete topology.
2. The topology  $T_{NIV}$  on any directed path  $P_n$  does not represent a discrete topology.

The following examples illustrate the remarks above.

**Example 3.5.** Consider the digraph  $G_\theta = (V, E)$  in Figure 2 such that:

$$V(G_\theta) = \{v_1, v_2, v_3, v_4, v_5, v_6\}, E(G_\theta) = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}.$$

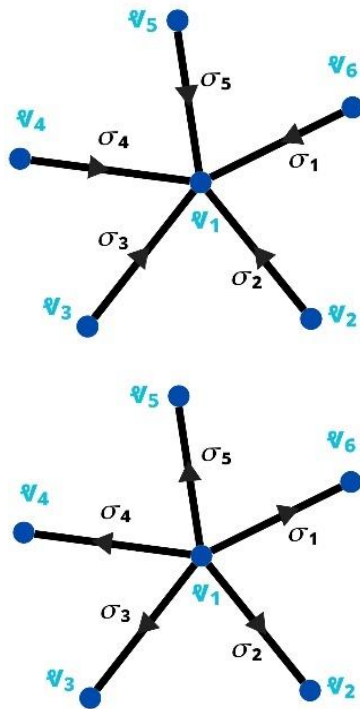


Figure 2. A Star di-graph  $S_5$  with different directed edges.

$\mathcal{T}_{NIV}$  has a sub-basis:

$$\mathcal{S}_{NIV} = \{ \{v_1\}, \{v_2, v_6\}, \{v_3, v_2\}, \{v_4, v_3\}, \{v_4, v_5\}, \{v_5, v_6\} \}.$$

Then by using finite intersection, the following base  $\beta_{NIV}$  is produced:

$$\{ \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_2, v_6\}, \{v_3, v_2\}, \{v_4, v_3\}, \{v_4, v_5\}, \{v_5, v_6\}, \emptyset \}$$

Then, utilizing each union, generate a topology  $\mathcal{T}_{NIV}$  as the following:

$$\begin{aligned} \mathcal{T}_{NIV} &= \{ \emptyset, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}, \{v_1, v_2\}, \{v_3, v_1\}, \\ &\{v_6, v_4\}, \{v_5, v_3\}, \{v_6, v_3\}, \{v_4, v_2\}, \{v_5, v_1\}, \{v_1, v_4\}, \\ &\{v_2, v_3\}, \{v_2, v_5\}, \{v_2, v_6\}, \{v_3, v_4\}, \{v_1, v_6\}, \{v_4, v_5\}, \{v_5, v_6\}, \\ &, \{v_1, v_2, v_6\}, \{v_6, v_4, v_1\}, \{v_5, v_3, v_1\}, \{v_6, v_3, v_1\}, \{v_2, v_4, v_1\}, \\ &\{v_3, v_2, v_1\}, \{v_6, v_4, v_2\}, \{v_5, v_3, v_2\}, \{v_6, v_3, v_2\}, \{v_5, v_2, v_1\}, \\ &\{v_6, v_4, v_3\}, \{v_3, v_4, v_2\}, \{v_3, v_4, v_1\}, \{v_5, v_4, v_3\}, \{v_5, v_4, v_1\}, \\ &\{v_6, v_4, v_5\}, \{v_5, v_4, v_2\}, \{v_6, v_5, v_3\}, \{v_6, v_5, v_1\}, \\ &\{v_3, v_1, v_6, v_4\}, \{v_1, v_3, v_4, v_2\}, \{v_6, v_4, v_5, v_3\}, \\ &\{v_6, v_4, v_5, v_1\}, \{v_5, v_1, v_3, v_2\}, \{v_1, v_2, v_4, v_5\} \\ &, \{v_1, v_2, v_6, v_3\}, \{v_1, v_2, v_5, v_6\}, \{v_3, v_1, v_4, v_5\}, \\ &\{v_3, v_1, v_5, v_6\}, \{v_2, v_3, v_4, v_6\}, \{v_2, v_4, v_5, v_6\} \\ &, \{v_3, v_5, v_2, v_4\}, \{v_2, v_1, v_4, v_6\}, \{v_1, v_2, v_3, v_4, v_5\}, \\ &\{v_2, v_3, v_4, v_5, v_6\}, \{v_3, v_4, v_5, v_6, v_1\}, \{v_4, v_5, v_6, v_1, v_2\} \\ &, \{v_5, v_6, v_1, v_2, v_3\}, \{v_6, v_1, v_2, v_3, v_4\} \}. \end{aligned}$$

Clearly,  $\mathcal{T}_{NIV}$  is a discrete topology.

**Example 3.6.**

Consider the digraph  $G_\theta = (V, E)$  be a tree, show in Figure 3 s. t.:  $V(G_\theta) = (v_1, v_2, v_3, v_4, v_5, v_6, v_7)$   $E(G_\theta) = \{ \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6 \}$ .

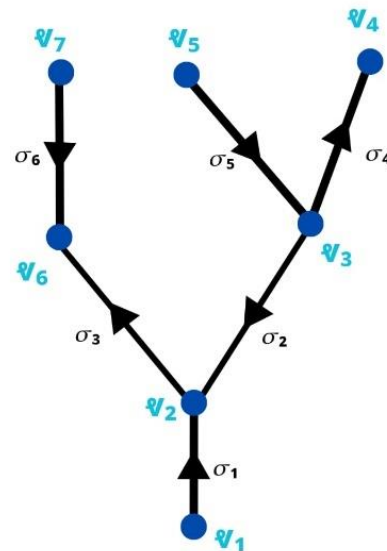


Figure 3. A tree directed graph.

$\mathcal{T}_{NIV}$  has a sub-basis:

$$\mathcal{S}_{NIV} = \{ \{v_1, v_3\}, \{v_2, v_4\}, \{v_5\}, \{v_2, v_7\}, \{v_6\} \}.$$

Then, via finite intersection, the following base  $\beta_{NIV}$  is produced:

$$\{ \{v_1, v_3\}, \{v_2, v_4\}, \{v_5\}, \{v_2, v_7\}, \{v_6\}, \{v_2\}, \emptyset \}$$

Then, utilizing each union, generate a topology  $\mathcal{T}_{NIV}$  as the following:

$$\begin{aligned} \mathcal{T}_{NIV} &= \{ \emptyset, V, \{v_2\}, \{v_5\}, \{v_6\}, \{v_1, v_3\}, \{v_2, v_4\}, \{v_2, v_7\}, \{v_5, v_6\}, \\ &\{v_5, v_2\}, \{v_6, v_2\}, \{v_1, v_3, v_5\}, \{v_1, v_3, v_2\}, \{v_1, v_3, v_6\}, \\ &\{v_2, v_4, v_5\}, \{v_2, v_4, v_6\}, \{v_2, v_5, v_7\}, \{v_2, v_6, v_7\}, \\ &, \{v_1, v_3, v_5, v_2\}, \{v_1, v_3, v_2, v_6\}, \{v_6, v_4, v_5, v_2\}, \\ &\{v_5, v_2, v_7, v_6\}, \{v_1, v_2, v_4, v_3, v_5\}, \{v_1, v_3, v_4, v_2, v_6\}, \\ &\{v_1, v_2, v_4, v_3, v_6\}, \{v_1, v_3, v_2, v_7, v_5\}, \{v_1, v_3, v_7, v_2, v_6\}, \\ &\{v_1, v_2, v_3, v_4, v_5, v_6\}, \{v_7, v_2, v_3, v_4, v_5, v_6\}, \\ &\{v_7, v_1, v_3, v_4, v_5, v_6\}, \{v_7, v_1, v_2, v_4, v_5, v_6\}, \\ &\{v_7, v_1, v_2, v_3, v_5, v_6\}, \{v_7, v_1, v_2, v_3, v_4, v_6\}, \\ &\{v_1, v_2, v_3, v_4, v_5, v_7\} \}. \end{aligned}$$

Clearly, it is not discrete topology.

**Example 3.7.** Consider  $G_\theta = (V, E)$  be a complete di-graph  $K_4$ , show in Figure 4, such that:

$$V(G_\theta) = \{v_1, v_2, v_3, v_4\}.$$

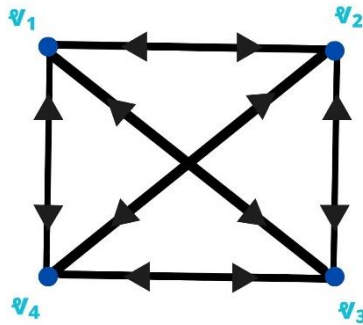


Figure 4. A complete di-graph  $K_4$ .

$\mathcal{T}_{NIV}$  has a sub-basis

$$\mathcal{S}_{NIV} = \{\{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}.$$

Then via using finite intersection, the following base  $\beta_{NIV}$  is produced:

$$\{\emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}\}.$$

Then, utilizing each unions generate a topology  $\mathcal{T}_{NIV}$  as the following:

$$\begin{aligned} \mathcal{T}_{NIV} &= \{\emptyset, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \\ &\{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_3\}, \\ &\{v_1, v_2, v_4\}, \{v_2, v_3, v_4\}\} \end{aligned}$$

Clearly, a discrete topology.

**Example 3.8.** Consider the di-graph  $G_\theta = (V, E)$  in Figure 5 such that:

$$V(G_\theta) = \{v_1, v_2, v_3, v_4\}, E(G_\theta) = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}.$$

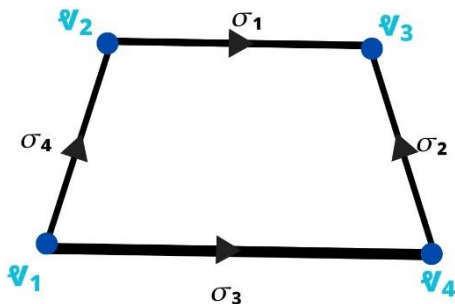


Figure 5. A Di-graph  $C_4$ .

$\mathcal{T}_{NIV}$  has a sub-basis;

$$\mathcal{S}_{NIV} = \{\{v_1\}, \{v_3\}, \{v_2, v_4\}\}.$$

Then by using finite intersection, the following base  $\beta_{NIV}$  is produced

$$\{\{v_1\}, \{v_3\}, \{v_2, v_4\}, \emptyset\}.$$

Then, utilizing each union generate a topology  $\mathcal{T}_{NIV}$  as the following:

$$\mathcal{T}_{NIV} = \{\emptyset, V, \{v_1\}, \{v_3\}, \{v_2, v_4\}, \{v_1, v_3\}, \{v_1, v_2, v_4\}, \{v_3, v_4, v_2\}\}$$

It is clear that, the topology  $\mathcal{T}_{NIV}$  does not represent a discrete topology.

**Example 3.9.** Consider the digraph  $G_\theta = (V, E)$  be directed path  $P_6$  as shown in Figure 6; such that:

$$V(G_\theta) = \{v_1, v_2, v_3, v_4, v_5, v_6\}, E(G_\theta) = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}.$$

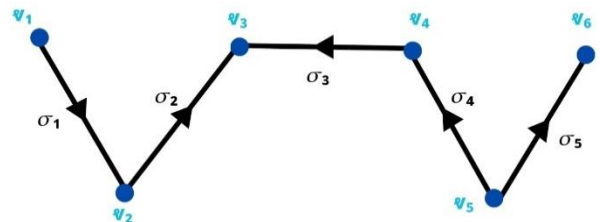


Figure 6. A directed path  $P_6$ .

$\mathcal{T}_{NIV}$  has a sub-basis;

$$\mathcal{S}_{NIV} = \{\{v_1\}, \{v_2, v_4\}, \{v_3\}, \{v_4, v_6\}, \{v_5\}\}.$$

Then by using finite intersection, the following base  $\beta_{NIV}$  is produced:

$$\{\{v_1\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_2, v_4\}, \{v_4, v_6\}, \emptyset\}.$$

Then, utilizing each union, generate a topology  $\mathcal{T}_{NIV}$  as the following:

$$\begin{aligned} \mathcal{T}_{NIV} &= \{V, \emptyset, \{v_1\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_2, v_4\}, \{v_4, v_6\}, \{v_1, v_3\}, \\ &\{v_1, v_4\}, \{v_1, v_5\}, \{v_4, v_3\}, \{v_4, v_5\}, \{v_3, v_5\}, \{v_1, v_2, v_4\}, \\ &\{v_1, v_4, v_6\}, \{v_1, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_1, v_3, v_5\}, \\ &\{v_3, v_2, v_4\}, \{v_3, v_4, v_6\}, \{v_5, v_4, v_2\}, \{v_5, v_4, v_6\}, \\ &\{v_2, v_4, v_6\}, \{v_2, v_4, v_1, v_3\}, \{v_2, v_4, v_1, v_5\}, \{v_4, v_6, v_1, v_3\}, \end{aligned}$$

$\{v_4, v_6, v_1, v_5\}, \{v_4, v_6, v_3, v_5\}, \{v_1, v_3, v_4, v_5\},$   
 $\{v_1, v_5, v_4, v_3\}, \{v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4, v_5\}$   
 $, \{v_2, v_3, v_4, v_5, v_6\}, \{v_3, v_4, v_5, v_6, v_1\}, \{v_4, v_5, v_6, v_1, v_2\},$   
 $\{v_5, v_6, v_1, v_2, v_3\}, \{v_6, v_1, v_2, v_3, v_4\}$ .

It is clear that not discrete.

**Proposition 3.10.** The topological space  $(V, \tau_{NIV})$  is  $T_1$  – space if and only if  $\tau_{NIV}$  is discrete topology.

**Proof.** Directly via the proposition (Any topological space  $(X, \tau)$  is represent  $T_1$  – space if and only if  $\tau = P(X)$ ). ■

**Proposition 3.11.** Let  $G_\theta = (V, E)$  be any directed graph,  $\tau_{NIV}$  be a discrete on  $G_\theta$ , afterwards  $(V, \tau_{NIV})$  is  $T_2$  – space  $\Leftrightarrow (V, \tau_{NIV})$  is  $T_1$  – space.

**Proof.**  $\Rightarrow$ ) Intelligible.

$\Leftarrow$ ) Assume  $(V, \tau_{NIV})$  is  $T_1$  – space, then  $(V, \tau_{NIV})$  is discrete (using Proposition 3.7).

This implies  $\forall v \in V, \{v\} \in \tau_{NIV}$ . So  $\forall v, w \in V, s.t. v \neq w, \exists \{v\}, \{w\} \in \tau_{NIV}$  s.t.  $w \in \{w\}$  and  $v \in \{v\}$  and  $\{w\} \cap \{v\} = \emptyset$ . Hence  $(V, \tau_{NIV})$  is  $T_2$  – space. ■

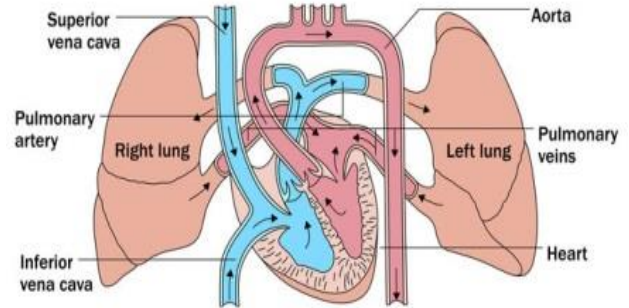
#### 4. Applied Example of the Non-Adjacency Incompatible Vertices Topological Space

This section deals with the biomathematical application of the non-adjacency incompatible vertices topological space (applied example of the  $\tau_{NIV}$  in pulmonary circulation in humans).

##### 4.1. $\tau_{NIV}$ in the human pulmonary circulation

The microcirculatory system is part or section of the circulatory system, it includes the cardiovascular system. The small circulation includes a blood vessels that carry on deoxygenated blood from the heart arrives to the lungs, and then return oxygenated blood again to the heart through the ventricle. This is contrary to what occurs in the major circulation. Deoxygenated blood departs the right part (right ventricle) of the heart during the pulmonary arteries, which takes blood to the lungs, where red blood cells free carbon dioxide and combine with oxygen through breathing. The oxygenated blood departs the lungs during the pulmonary veins, which drain into the left part, or what is called the left atrium, of the heart, thus completing the small (pulmonary) blood circulation [9].

### Pulmonary circulation



**Figure 7.** The pulmonary circulation in humans.

Blood that has lost oxygen enters the heart, passes through the lungs, and then returns to the heart. Here, it passes from the right atrium to the right ventricle via tricuspid valve, The right ventricle then will pump the fluid to the main pulmonary artery via the pulmonary valve, which is the right atrioventricular valve.

##### 4.2. Lungs $\{v_3, v_5\}$

Deoxygenated blood is transported by the pulmonary arteries to the lungs, where breathing causes the release of carbon dioxide and the inhalation of oxygen. The capillaries that are formed from the arteries have extremely thin walls. The heart's left atrium takes the oxygenated blood back from pulmonary vein.

##### 4.3. Veins $\{v_4\}$

After passing via the pulmonary veins and into left atrium of heart, where it is pumped via the mitral valve and into the left ventricle, oxygenated blood departs the lungs and enters the heart left side, completing the pulmonary circulation. After pumping blood through the aortic valve to reach the aorta, left ventricle distributes blood throughout the body via the greater circulation before return it to the lesser circulation.

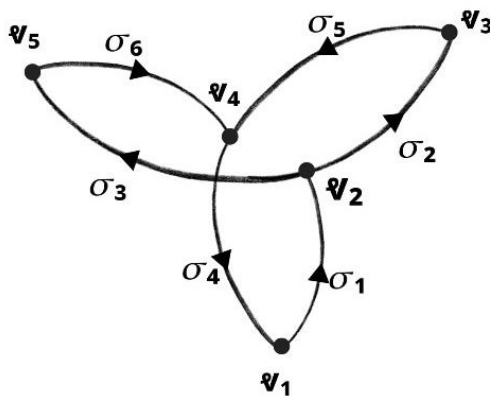
##### 4.4. Arteries $\{v_2\}$

The semilunar pulmonary valve lets blood to pass from right ventricle to left and right main pulmonary arteries, which then split off to form smaller pulmonary arteries that distribute throughout the lungs.



**4.5. Heart  $\{v_1\}$**

The right side of heart pumps the blood to the lungs, where oxygen is added to blood and carbon dioxide is removed. The left side of heart pumps the blood to the rest of the body, where the tissues are supplied with oxygen and nutrients, and waste products (like carbon dioxide) are transported into blood for removal via other organs (like the lungs and kidneys), as shows in Figure 8.



**Figure 8.** Di-graph of the pulmonary circulation in humans, as in Figure 7.

$\mathcal{T}_{NIV}$  has a sub-basis:  $\mathcal{S}_{NIV} = \{\{v_1\}, \{v_2\}, \{v_3, v_5\}, \{v_4\}\}$ . Then via finite intersection, the following base  $\beta_{NIV}$  is produced  $\{\emptyset, \{v_1\}, \{v_2\}, \{v_3, v_5\}, \{v_4\}\}$ . Then, utilizing each union will generate a topology  $\mathcal{T}_{NIV}$  as the following:

$$\begin{aligned} \mathcal{T}_{NIV} &= \{ \emptyset, V, \{v_1\}, \{v_2\}, \{v_3, v_5\}, \{v_4\}, \{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_4\}, \\ &\{v_1, v_3, v_5\}, \{v_2, v_3, v_5\}, \{v_4, v_3, v_5\}, \{v_1, v_2, v_4\}, \\ &\{v_2, v_4, v_3, v_5\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_4, v_3, v_5\} \} \end{aligned}$$

Then,  $\mathcal{T}_{NIV}$  Is not discrete topology in the di-graph of the pulmonary circulation in humans.

**5. Conclusions**

We present a definition of new topology on directed graph, which is said to be the non-adjacency incompatible vertices topological space (i.e.  $NIV$  topological space) on the set of vertices, we also show a few of the properties of this topology in some of remarks and examples of basic di-graph types. Some

results are obtained, such as existing of the isolated vertex is not necessary in our definition, also observations about the specification of this new type (which types of di-graphs achieves the discrete non-adjacency incompatible vertices topology and which does not and consequently which achieves  $T_1$  and  $T_2$ -space). We also refer to a useful applied example of non-adjacency incompatible vertices topological space which is not a discrete topology in the di-graph of the pulmonary circulation in humans. And we think in the future we can possibility studying another properties, preliminary results and separation axioms of this new topology.

**Conflict of Interest:** All authors declare that there are no conflicts of interest

**References**

- [1] Gross, J.L.; Yellen J.; “Handbook of Graph Theory.” 1st ed.; CRC press: Boca Raton, USA, 2003.
- [2] Stallings, J.K.; “Topology of finite graphs.” Invent. Math., 71: 551-565, 1983.
- [3] Stadler, B.M.; Stadler P.F.; “Generalized topological spaces in evolutionary theory and combinatorial chemistry”. J. Chem. Inf. Comput. Sci., 42 (3): 577-585, 2002.
- [4] Ray, S.S.; “Graph Theory with Algorithms and its Applications: In Applied Science and Technology.” 1st ed.; Springer: India, 2013.
- [5] Amiri, S.M.; Jafarzadeh, A.; Khatibzadeh, H.; “An Alexandroff topology on graphs.” Bull. Iran. Math. Soc., 39 (4): 647-662, 2013.
- [6] Kilicman, A.; Abdulkalek, K.; “Topological spaces associated with simple graphs.” J. Math. Anal., 9 (4): 44-52, 2018.
- [7] Hassan, A.F.; Zainy, Z.R.; “The independent compatible edges topology of directed graphs.” J. Discrete Math. Sci. Cryptogr., 25 (8): 2683-2695, 2022.
- [8] Ali, I.A.; Hassan, A.F.; “The Independent Incompatible Edges Topology on Di-graphs”. J. Phys.: Conf. Ser., Baghdad, 23-24 March; IOP Publishing Ltd, 2022.
- [9] McKinley, M.; O’Loughlin, V.D.; “Human Anatomy.” 3rd ed.; McGraw-Hill, 2011.