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On Coc-Preopen Sets in Topological Spaces

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Abstract

In this research we introduced a new concept, which is coc-preopen set, with some properties, theorems, and functions related to this concept. We also benefited from the properties of some spaces in order to find new results related to the concept of coc-preopen.

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1. Introduction

Mashhour, A. S. established the idea of preopen sets in topological spaces in 1982 [1]. Al-Ghour and Samarah in [2] defined coc-open set. We recall the definition of connected [3], compact (resp., compact mapping) [4] and coc-connected spaces [5]. we presented a new definition similar the definition of coc-connected using preopen sets which is called coc-pre-connected space and we give several properties of this definition. In this research, we used Kc-space [6] in some theorems to obtain some important results.

2. Preliminaries

Definition 2.1, [1]. The set $B \subseteq \mathrm{N}$ can be thought of as preopen set (resp, preclosed) if $B \subseteq \mathrm{int}(\mathrm{cl}(B))$ (resp, $\mathrm{cl}(\mathrm{int}(B)) \subseteq B$).

Definition 2.2, [2]. When for every $n \in$ Ç, there exists an open set $\mathrm{U_n} \subseteq \mathrm{N}$ and a compact subset $K_n \in \mathrm{C}(\mathrm{N},\tau)$ where $\mathrm{n} \in \mathrm{U_n} \setminus K_n \subseteq$ Ç, then a subset Ç of a space (N,τ) is called a co-compact open set (notation: coc-open set). Coc-closed is the complement of a coc-open subset of N.

Definition 2.3, [6]. A space N can be considered KC-space if all compact sets in it are closed.

Definition 2.4.
1. If every cover of a topological space N made up of preopen sets admits finite subcover, then this space can be defined as a strongly compact [7].
2. A subset S is deemed strongly compact with respect to N if each cover of the subset S consisting of preopen sets in space N admits a finite subcover [7].

Definition 2.5, [8]. A topological space N is said to be submaximal if and only if every dense subset of N is open.

Corollary 2.1, [8]. Any finite intersection of sets that are preopen is preopen if (N,τ) is submaximal.

Definition 2.6. In space N, a subset L is considered:
1. α -open if $L \subseteq \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(\mathrm{L})))$ [9].
2. semi-open if $L \subseteq \mathrm{Cl}(\mathrm{Int}(\mathrm{L}))$ [10].

Lemma 2.1, [11]. The result of the intersection between preopen set and α-open set is preopen set.

Definition 2.7, [12]. The topological space (N,τ) is said to be preconnected iff N is not the union of two non-empty disjoint preopen sets, equivalently, if $\mathrm{N} = \mathrm{P}_1 \cup \mathrm{P}_2$, $\mathrm{P}_1 \in \mathrm{PO}(\mathrm{N})$, $\mathrm{P}_2 \in \mathrm{PO}(\mathrm{N})$, $\mathrm{P}_1 \neq \emptyset$, $\mathrm{P}_2 \neq \emptyset$ implies $\mathrm{P}_1 \cap \mathrm{P}_2 \neq \emptyset$.

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Definition 2.8. It is argued that a function $f: N \to M$ is:

1. Continuous mapping [13], if $f^{-1}(A)$ is an open set in N for every open set A in M.
2. M-precontinuous [14], if each preopen subset of M has an inverse image that is a preopen subset in N, or, equivalently, if each preclosed subset of M has an inverse image that is preclosed subset in N.
3. M-preopen (resp. M-preclosed) [14] If every preopen (resp. preclosed) subset of N has an image that is a preopen (resp. preclosed) subset in M.

3. Coc-Preopen Set

Definition 3.1. A is referred to as a cocompact preopen set (abbreviated coc-preopen set or c-po) in topological space N if, for each $n \in A$, there is a compact set K_n and a preopen set U_n such that $n \in U_n \setminus K_n \subseteq A$.

Remark 3.1. While the opposite is not usually true, any open set is a c-po.

Examples 3.1.

1. Let N = {1, 2, 3} and τ any topology. Obviously any subset is c-po. To clarify further, if we take the set {1 2}, then for every $a \in \{1\ 2\}$, there is a preopen set N and a compact set N\{a} such that $a \in N\setminus(N\setminus\{a\}) \subseteq \{1\ 2\}$.
2. Let N = R and τ = I. Obviously any subset is c-po. To clarify further, if we take the set D, then for every $a \in D$, there is a preopen set R, and a compact set R\{a} such that $a \in R\setminus(R\setminus\{a\}) \subseteq D$.
3. Let N = R and τ = co-finite topology. Obviously any subset is c-po and the example in paragraph (2) makes this clear.
4. Let N= R and $\tau = \tau_u$. Obvious $R \setminus (0,1)$ isn't c-po because there is no preopen set A and compact set K such that $a \in A\setminus K \subseteq R \setminus (0,1)$ for all $a \in R \setminus (0,1)$. Although each singleton belongs to (0,1), then $R \setminus \{singleton\}$ is a c-po and this is evidence that the infinite intersection of all c-po does not give a c-po.

Remark 3.2. Although the opposite is not usually true, every preopen set is a c-po.

Example 3.2. If we say N = {ς, ρ, υ} with τ = {φ, N, {ς}, {ρ}, {ς, ρ}}. Obviously {υ} is c-po but not preopen, because the set N can be considered preopen set and N\{υ} is compact.

Definition 3.2. If there is a c-po C in which $n \in C \subseteq G$ exists, then $n \in N$ is said to be a coc-pre-interior point to G.

Definition 3.3. If for any c-po C containing n, $C \cap (G \setminus \{n\}) \neq \emptyset$, then a point $n \in N$ is considered the coc-pre-limit point of G.

Definition 3.4. If for any c-po C containing n, $C \cap G \neq \emptyset$, then a point $n \in N$ is considered the coc-pre-adherent point of G.

Theorem 3.1. Consider the submaximal space (N, τ) . (N, τ^{pk}) is then a topological space.

Proof.

1. As of right now, $\varphi, N \in \tau^{pk}$.
2. To demonstrate that $\mathcal{H} \cap \mathcal{D}$ is a c-po for any $\mathcal{H}, \mathcal{D}$ belongs to τ^{pk}. Assume that if $n \in \mathcal{H} \cap \mathcal{D}$, then $n \in \mathcal{H}$ and $n \in \mathcal{D}$. It is natural that there are two preopen sets $U_n, V_n \subseteq N$ and two compact subsets K_n , L_n where $n \in U_n \setminus K_n \subseteq \mathcal{H}$, $n \in V_n \setminus L_n \subseteq \mathcal{D}$. This implies that $n \in U_n \cap (N\setminus K_n) \cap V_n \cap (N\setminus L_n) \subseteq \mathcal{H} \cap \mathcal{D}$, and so we obtain $n \in (U_n \cap V_n) \cap (N\setminus K_n \cap N \setminus L_n) \subseteq \mathcal{H} \cap \mathcal{D}$, and so we obtain $n \in (U_n \cap V_n) \setminus (K_n \cup L_n) \subseteq \mathcal{H} \cap \mathcal{D}$. Since $K_n \cup L_n$ is a compact set in N and $U_n \cap V_n$ is preopen (Corollary (2.1)), $\mathcal{H} \cap \mathcal{D}$ is c-po.
3. To demonstrate that is $\cup_{\alpha \in \lambda} B_\alpha$ is c-po, let $B_\alpha, \alpha \in \lambda$ be c-po. If $n \in \cup_{\alpha \in \lambda} B_\alpha$, then $n \in B_\alpha$ for some $\alpha \in \lambda$. Since B_α is c-po, there exist a preopen set U_α and a compact subset K_α where $n \in U_\alpha \setminus K_\alpha \subseteq B_\alpha$ for some $\alpha \in \lambda$, since $B_\alpha \subseteq \cup_{\alpha \in \lambda} B_\alpha$ then $\cup_{\alpha \in \lambda} B_\alpha$ is c-po. Obviously The intersection of any family of c-pcs is a c-pc. ■

Example 3.3. Suppose that $N = \{n_1, n_2, n_3\}$ with $\tau = \{\varphi, N, \{n_1\}, \{n_1, n_2\}\}$. As a result, (N, τ^{pk}) is topological space even though space (N, τ) not submaximal space.

Definition 3.5. If a space N has a subset J, then:

1. $coc-pre-int(J) = \cup \{Ç: Ç \subseteq J, Ç \text{ is a } c-po\}$.
2. $coc-pre-cl(J) = \cap \{F: J \subseteq F, F \text{ is a } c-pc\}$.

Remarks 3.3. If we say G, Q are two subsets of a space N and $G \subseteq Q$, then:

1. $coc-pre-int(G) \subseteq G$.
2. $coc-pre\ int(G)$ is c-po in N.
3. $coc-pre-int(coc-pre-int(G)) = coc-pre-int(G)$.

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4. $coc-pre-int(N) = N$ and $coc-pre-int(\emptyset) = \emptyset$.
5. $coc-pre-int(G) \subseteq coc-pre-int(Q)$.
6. $G^{oP} \subseteq G^{oP-coc}$.

Proof.
1. By definition of coc-pre-int(G).
2. Through the union of all c-po sets is a c-po set.
3. Since coc-pre-int(G) is the largest c-po set contained in G and by (ii), then $coc-pre-int(coc-pre-int(G)) = coc-pre-int(G)$.
4. Since N and Ø are c-po sets, then by definition 3.5, $coc-pre-int(N) = \cup\{Ç: Ç$ is a c-po, $Ç \subseteq N\} = N \cup$ all c-po sets = N. On the other hand, since Ø is the only c-po set contained in Ø, then, $coc-pre-int(\emptyset) = \emptyset$.
5. Let $x \in coc-pre-int(G)$, then there can be found c-po set Ç such that $x \in Ç \subseteq G$. For the reason that $G \subseteq Q$, then $x \in Ç \subseteq Q$. Consequently, $x \in coc-pre-int(Q)$.
6. Let $x \in G^{oP}$, then there can be found preopen set U such that $x \in U \subseteq G$. For the reason that every preopen set is c-po set, therefore $x \in G^{oP-coc}$.

Remarks 3.4. Assume that U and V are two subsets of a space N, then:
1. $U \subseteq coc-pre-cl(U)$.
2. $coc-pre-cl(U)$ is a c-pc.
3. $coc-pre-cl(coc-pre-cl(U)) = coc-pre-cl(U)$.
4. $coc-pre-cl(\emptyset) = \emptyset$ and $coc-pre-cl(N) = N$.
5. If $U \subseteq V$, then $coc-pre-cl(U) \subseteq coc-pre-cl(V)$.
6. $U^{-P-coc} \subseteq U^{-P}$.

Proof.
1. By definition of $coc-pre-cl(U)$.
2. Since the union of all c-po sets is a c-po set, then the intersection of all c-pc sets is a c-pc set, and thus $coc-pre-cl(U)$ is a c-pc.
3. Thus from (ii) and definition of $coc-pre-cl(U)$.
4. By definition $coc-pre-cl(U)$, then $coc-pre-cl(X) = \cap\{F: X \subseteq F, F$ is c-pc $\}$. But X is the only c-pc comprising X. In this way $coc-pre-cl(X) = X$. Also by the definition of $coc-pre-cl(\emptyset)$, $coc-pre-cl(\emptyset) = \cap\{F: \emptyset \subseteq F, F$ is a c-pc $\} = \emptyset \cap$ any c-pc sets comprising $\emptyset = \emptyset$. In this way $coc-pre-cl(\emptyset) = \emptyset$.
5. Let $x \in coc-pre-cl(U)$, then each c-po C comprise x intersect U, since $U \subseteq V$, then the set C intersect V. Consequently, $x \in coc-pre-cl(V)$.
6. Let $x \notin U^{-P}$, then there can be found M as a preopen set, such that $x \in M$ and $M \cap U = \emptyset$. For the reason that every preopen set is a c-po, then $x \notin U^{-P-coc}$ and consequently $U^{-P-coc} \subseteq U^{-P}$. ∎

Proposition 3.1. In the event that Ç is a subset of space N, then:
1. A set Ç is considered c-po if and only if $coc-pre-int(Ç) = Ç$.
2. The set $\mathcal{F}$ is considered c-pc iff $coc-pre-cl(\mathcal{F}) = \mathcal{F}$.

Proof.
1. As the union of each c-po is c-po, then $coc-pre-int(Ç)$ is the largest c-po contained in Ç. Since Ç is c-po, then $coc-pre-int(Ç) = Ç$. Conversely, whenever $coc-pre-int(Ç) = Ç$, then Ç is c-po, since $coc-pre-int(Ç)$ is a c-po.
2. As the intersection of each c-pc is c-pc, then $coc-pre-cl(\mathcal{F})$ is the smallest c-pc comprises $\mathcal{F}$. Since $\mathcal{F}$ is a c-pc, then $coc-pre-cl(\mathcal{F}) = \mathcal{F}$. Conversely, whenever $coc-pre-cl(\mathcal{F}) = \mathcal{F}$, then $\mathcal{F}$ is a c-pc, (since $coc-pre-cl(\mathcal{F})$ is a c-pc). ∎

Lemma 3.1. C-po is the crisscrossing of α-open set and a c-po.

Proof. If we say $n \in Q \cap Ç$ where Q is an α-open and Ç a c-po, we obtain a preopen set $P_n \subseteq N$ and compact subsets K_n such that $n \in P_n \setminus K_n \subseteq Ç$, and also by Lemma (2,1), $Q \cap P_n$ is preopen. There is now a preopen set $Q \cap P_n$ in which $n \in (Q \cap P_n) \setminus K_n \subseteq Q \cap Ç$. As a result, $Q \cap Ç$ is c-po. ∎

Proposition 3.2. Let N be a Kc-space. Any nonempty c-po, contains a preopen set.

Proof. If we say $n \in Ç$ and Ç is a nonempty c-po, then P_n is a preopen set and K_n is a compact subset of N such that $n \in P_n \setminus K_n \subseteq Ç$. Since N is a Kc-space and by lemma (2,1), $P_n \setminus K_n$ is preopen set. ∎
The example below demonstrates that in the event that N is not a Kc-space, a nonempty c-po will exist, devoid of a nonempty preopen set.

Example 3.4. Assume N = {r, f, g} and τ = {φ, N, {r}, {f}, {r, f}}. Hence, {g} is a c-po that is devoid of any nonempty preopen sets.

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Theorem 3.2. $\mathcal{F} \subseteq$ N, with N being a space. Given some compact subset K_n and a preclosed subset $\mathcal{C}$, if $\mathcal{F}$ is c-pc, then $\mathcal{F} \subseteq \mathcal{C} \cup K_n$.

Proof. A preopen set $Ᵽ_n$ and a compact set K_n exist for any $n \in N \setminus \mathcal{F}$ in which $n \in Ᵽ_n \setminus K_n \subseteq N \setminus \mathcal{F}$. The result is that $\mathcal{F} \subseteq N \setminus (Ᵽ_n \setminus K_n) = N \setminus (Ᵽ_n \cap (N \setminus K_n)) = (N \setminus Ᵽ_n) \cup K_n$. Let $\mathcal{C}$ equal $(N \setminus Ᵽ_n)$. As a result, $\mathcal{F} \subseteq \mathcal{C} \cup K_n))$. ■

Definition 3.6. An expression for a function f: N→ M is as follows:

1. If $f^{-1}(Q)$ is a c-po in N for any open set Q in M, then f is a coc-pre-continuous function.
2. If $f^{-1}(Ç)$ is an open set in N for any c-po Ç in M, then f is a coc-pre-* continuous.
3. If $f^{-1}(Ç)$ is a c-po in N for every c-po Ç in M, then f is a coc-pre-** continuous.
4. If $f(Ᵽ)$ is c-po (resp., c-pc) in M for any preopen (resp., preclosed) subset Ᵽ of N, then f is a coc-preopen function (resp., coc-preclosed function).
5. coc-preopen* function (resp., coc-preclosed* function) if $f(Ç)$ is c-po (resp., c-pc) in M for any c-po (resp., c-pc) subset Ç of N.

Proposition 3.3. If f: N → M has characteristics including continuous, M-preopen and injective function, then the image of c-po of N will be c-po in M.

Proof. Let f: N → M be injective M-preopen and Ç a c-po of N. For any m ∈ f(Ç), there exists n ∈ Ç in which f(n) = m. Since Ç is c-po, there be found a preopen set $Ᵽ_n$ and compact subset K_n of N such that $n \in Ᵽ_n \setminus K_n \subseteq Ç$. Since f is M-preopen, $f(Ᵽ_n)$ is a preopen in M in which $m = f(n) \in f(Ᵽ_n \setminus K_n) \subseteq f(Ç)$ and so $m = f(n) \in f(Ᵽ_n) \setminus f(K_n) \subseteq f(Ç)$. As the continuous image of compact is compact, then $f(Ç)$ is c-po in M. ■

Proposition 3.4. If f: N → M is an M-precontinuous bijective, compact mapping and Ç is c-po in M, then $f^{-1}(Ç)$ is c-po in N.

Proof. Let $n \in f^{-1}(Ç)$. Then f(n) ∈ Ç and there be found a preopen set $Ᵽ_n$ and compact subset K_n of M such that $f(n) \in Ᵽ_n \setminus K_n \subseteq Ç$. Then $n \in f^{-1}(Ᵽ_n) \setminus f^{-1}(K_n) \subseteq f^{-1}(Ç)$. Since f is an M-precontinuous, $f^{-1}(Ᵽ_n)$ is a preopen set and by compact mapping, $f^{-1}(K_n)$ is compact in N and so $f^{-1}(Ç)$ is c-po in N. ■

Theorem 3.3. Let N be a Kc-space. The following statements are equivalent:

1. N is S-C.
2. any proper c-pc is S-C.

Proof. (1). ⇒ (2) If we say F is a proper c-pc and $\{F_Ɛ : Ɛ \in ɣ\}$ be a cover of F by preopen sets of N. Now for each n ∈ N \ F, there is a preopen set p_n and compact subset K_n of N in which $n \in p_n \setminus K_n \subseteq N \setminus F$. Through the data of the theorem and by lemma (2,1), $\{F_Ɛ : Ɛ \in ɣ\} \cup \{p_n \setminus K_n : n \in N \setminus F\}$ is a preopen cover of N. If we say N is S-C, we get a finite subset $ɣ_0$ of ɣ and a finite subset G such that j ∈ G. So $N = (\cup\{F_Ɛ : Ɛ \in ɣ_0\}) \cup (\cup\{p_{nj} \setminus K_{nj}, j \in G\})$, hence $F \subseteq (\cup\{F_Ɛ : Ɛ \in ɣ_0\}) \cup (\cup\{(p_{nj} \setminus K_{nj}), j \in G\})$. Therefore, we obtain $F \subseteq \cup\{F_Ɛ : Ɛ \in ɣ_0\}$. This shows that F is S-C.

(2)⇒(1) If we say $\{W_\propto : \propto \in ɣ\}$ is a preopen cover of N. The fix point $\propto_0 \in ɣ$ is our choice. Then $\{W_\propto : \propto \in ɣ \setminus \{\propto_0\}\}$ is a preopen cover of a c-pc $N \setminus W_{\propto_0}$. There be found a finite subset $ɣ_0$ of $ɣ \setminus \{\propto_0\}$ in which $N \setminus W_{\alpha_0} \subseteq \cup\{W_\propto : \propto \in ɣ_0\}$. Therefore, $N = \cup\{W_\propto : \propto \in ɣ_0 \cup \{\propto_0\}\}$. As a result, N is S-C. ■

Theorem 3.4. Let (N, τ) be a Kc-space, $(N, τ^{pk})$ is compact iff (N, τ) is S-C.

Proof. If we say $\{W_Ɛ : Ɛ \in ɣ\}$ is a cover of N by c-po then all n ∈ N, we get that $n \in W_{Ɛ(n)}$, $Ɛ(n) \in ɣ$ and $W_{Ɛ(n)}$ is a c-po. So there exists a preopen set $P_{Ɛ(n)}$ and a compact subset $K_{Ɛ(n)}$ of N where $n \in P_{Ɛ(n)} \setminus K_{Ɛ(n)} = G_{Ɛn} \subseteq W_{Ɛ(n)}$. Since N is a Kc –space and by lemma (2,1) The family $\{G_{Ɛn}\}$ is a preopen cover of (N, τ). If we say (N, τ) is S-C, then we get $ɣ_0$ of ɣ such that $N = \cup\{G_{Ɛn}\ Ɛn \in ɣ_0\}$. Since for every $G_{Ɛn} \subseteq W_{Ɛn}$, then $N = \cup\{W_Ɛ, Ɛ \in ɣ_0\}$. Hence $(N, τ^{pk})$ is compact. On the other hand, let P be a preopen cover of (N, τ). As $P \subseteq τ^{pk}$ and $(N, τ^{pk})$ is compact, a finite sub cover of P exists for N. As a result, (N, τ) is S-C. ■

Theorem 3.5. Let f: N → M be a c-pco and surjective function. If N is Kc-space and S-C, then M is compact.

Proof. If we say $\{W_Ɛ : Ɛ \in ɣ\}$ be a cover of M by open set, we get that $\{f^{-1}(W_Ɛ :) : Ɛ \in ɣ\}$ is a c-po cover of N. According to the data and Theorem (3.4), then we get a finite subset ɣ0 of ɣ in which $N = \cup\{f^{-1}(W_Ɛ :) : Ɛ \in ɣ0\}$; so $M = \cup\{W_Ɛ : : Ɛ \in ɣ0\}$. As a result, M is compact. ■

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Theorem 3.6. Let $f: N \to M$ be a c-p**co and surjective function. If N is Kc-space and S-C, then M is S-C.

Proof. Let $\{P_{\varepsilon} : \varepsilon \in \gamma\}$ be a cover of M by preopen set. Since every preopen set is a c-po, then $\{f^{-1}(P_{\varepsilon}) : \varepsilon \in \gamma\}$ is a c-po cover of N. According to the data of the theorem and by theorem (3.4), there is a finite subset $\gamma 0$ of γ in which $N = \cup\{f^{-1}(P_{\varepsilon}) : \varepsilon \in \gamma 0\}$; so $M = \cup\{P_{\varepsilon} : \varepsilon \in \gamma 0\}$. As a result M is S-C.

4. Coc-Pre-Connected Space

Definition 4.1. In a space N, two subsets $Ƥ_1$ and $Ƥ_2$ are referred to as coc-pre-separated if $Ƥ_1^{-P-COC} \cap Ƥ_2 = Ƥ_2^{-P-COC} \cap Ƥ_1 = \emptyset$. In the case of finite space, any two sets are coc-pre-separated

Definition 4.2. If Ç is not the combine of any two nonempty coc-pre-separated sets, it is denoted as a coc-pre-connected set. Space (R, τ_u) is an example of this

Definition 4.3. If a set is both c-pc and c-po, it is referred to as coc-pre-clopen. Any subset of any finite space is coc-pre-clopen.

Proposition 4.1. Let N be a space. The following statements are equivalent:
1. N is a C-pcon.
2. The only sets that are coc-pre-clopen in space N are N and Ø.
3. There are no two non-empty disjoint c-po sets that combine to form the space N.

Proof. (1) ⇒ (2) Let Z is a coc-pre-clopen set such that $Z \neq \emptyset$ and $Z \neq N$ and let $Z^C = W$. Consequently, $N = Z \cup W$. As W is a c-pc, then $W^{-P-COC} = W$ and $Z \cap W^{-P-COC} = Z \cap W = \emptyset$, and $W \cap Z^{-P-COC} = W \cap Z = \emptyset$. So, N is not a C-pcon spacewhich is contradictory. As a result, only c-po in the space N is N and Ø.
(2)⇒(3) Let the only sets that are coc-pre-clopen in space N are N and Ø and let $N = Z \cup W$ where Z and W are non-empty disjoint c-po sets, then $Z = W^C$ and in this way Z is a c-pc. This contradicts the truth of paragraph (ii). So the statement of (iii) is fulfilled.
(3) ⇒ (1) Suppose that N is a coc-pre-disconnected space. Then there exists non-empty subset Z, W of N such that $Z \cap W^{-P-COC} = \emptyset$ and $W \cap Z^{-P-COC} = \emptyset$ and $Z \cup W = N$. Since $W \subseteq W^{-P-COC}$, then $Z \cap W = \emptyset$. Since $W \cap Z^{-P-COC} = \emptyset$, then $Z^{-P-COC} \subseteq W^C = Z$ and therefore $Z^{-P-COC} = Z$. So Z is a c-pc. As $W = Z^C$, then W is a c-po. Likewise, shown that Z is a c-po, which is contradictory. As a result, N is a C-pcon. ∎

Remark 4.1 Every C-pcon space is a connected (resp., preconnected space) in that every open, respectively preopen set is a c-po. Likewise, the empty set can be considered a compact set. However, the result is not true for the opposite.

Example 4.1. Assume that N = {z, x, c} and τ = {φ, N, {z}}. N is clearly connected (resp., preconnected space), but it is not C-pcon.

Proposition 4.2. Consider the following: $Ƥ_1$, $Ƥ_2$ are coc-pre-separated sets, and Ç is a C-pcon set. Either $Ç \subseteq Ƥ_1$ or $Ç \subseteq Ƥ_2$ occurs if $Ç \subseteq Ƥ_1 \cup Ƥ_2$.

Proof. Suppose Ç be a C-pcon set and $Ƥ_1$, $Ƥ_2$ are coc-pre-separated sets and $Ç \subseteq Ƥ_1 \cup Ƥ_2$. Let $Ç \nsubseteq Ƥ_1$ and $Ç \nsubseteq Ƥ_2$. Suppose, $Ç_1 = Ƥ_1 \cap Ç \neq \emptyset$ and $Ç_2 = Ƥ_2 \cap Ç \neq \emptyset$. Then $Ç = Ç_1 \cup Ç_2$. Since $Ç_1 \subseteq Ƥ_1$, $Ç_1^{-P-COC} \subseteq Ƥ_1^{-P-COC}$. Since $Ƥ_1^{-P-COC} \cap Ƥ_2 = \emptyset$, then $Ç_1^{-P-COC} \cap Ç_2 = \emptyset$. In the same way, when $Ç_2 \subseteq Ƥ_2$, we get that $Ç_2^{-P-COC} \cap Ç_1 = \emptyset$. Therefore, Ç is not C-pcon set which is contradictory. As a result, either $Ç \subseteq Ƥ_1$ or $Ç \subseteq Ƥ_2$. ∎

Proposition 4.3. It can be shown that Q^{-P-COC} is C-pcon set if Q is a C-pcon set.

Proof. Let Q be a C-pcon set while Q^{-P-COC} is not. Then, two nonempty coc-pre-separated sets, $Ƥ_1$ and $Ƥ_2$, exist such that $Q^{-P-COC} = Ƥ_1 \cup Ƥ_2$. Since $Q \subseteq Q^{-P-COC}$, then $Q \subseteq Ƥ_1 \cup Ƥ_2$, and since Q is a C-pcon set, either $Q \subseteq Ƥ_1$ or $Q \subseteq Ƥ_2$.
1. If $Q \subseteq Ƥ_1$, then $Q^{-P-COC} \subseteq Ƥ_1^{-P-COC}$ and so $Ƥ_1 \cup Ƥ_2 \subseteq Ƥ_1^{-P-COC}$. As a result, $(Ƥ_1 \cup Ƥ_2) \cap Ƥ_2 \subseteq Ƥ_1^{-P-COC} \cap Ƥ_2$. Hence $Ƥ_2 = \emptyset$, which is contradictory.
2. In the same way as above, we prove the contradiction. As a result, Q^{-P-COC} is a C-pcon set. ∎

Proposition 4.4. C-pcon space has a c-pco image that is connected.

Proof. Let the data of this proposition be achieved from $(N, \tau_n) \to (M, \tau_m)$. To demonstrate M is connected. Assume M is disconnected space, then $M = Ƥ_1 \cup Ƥ_2$ where $Ƥ_1$, $Ƥ_2$ are non-empty disjoints open sets. So, $N = f^{-1}(M) = f^{-1}(Ƥ_1 \cup Ƥ_2) = f^{-1}(Ƥ_1) \cup f^{-1}(Ƥ_2)$. Because of the availability of

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some facts, including f is c-pco , $Ƥ_1 \neq \emptyset$, $Ƥ_2 \neq \emptyset$, f is onto and $f^{-1}(Ƥ_1) \cap f^{-1}(Ƥ_2) = f^{-1}(Ƥ_1 \cap Ƥ_2) = \emptyset$. This leads to the fact that space N is coc-pre-disconnected, and this is a contradiction. ■

Proposition 4.5. C-pcon space has a c-p**co image that is C-pcon.

Proof. It is clear and in the same context as the proposition (4.4), but by taking advantage of the definition of cp**co. ■

Open problem: It is possible to use coc-preopen in extremally disconnected Spaces, where the following research can be used [15].

Table 1. List of Symbols

Symbol	Description
R	The set of real numbers
(N, τ)	topological space
N\A	The complement to A
τ	The family of all open sets
I	The indiscrete topology
τ_u	The usual topology on R
Cl(L)	The closure of L
Int(L)	The interior of L
$C\,(N, \tau)$	set of each compact subset of (N, τ).
τ^P	The family of all preopen sets
τ^{pk}	the family of all c-po set
S-C	strongly compact
c-pc	coc-preclosed set
c-po	cocompact preopen set
$coc-pre-int(A)$ or A^{oP-coc}	The set of all coc-pre-interior points for A
$coc-pre-cl(A)$ or A^{-P-coc}	The set of all coc-pre-adherent points for A
$coc-pre-d(A)$	The set of all coc-pre-limit points of A
c-pco	coc-pre-continuous function
c-p*co	coc-pre-* continuous
c-p**co	coc-pre-** continuous
c-pof	coc-preopen function
c-pcf	coc-preclosed function
c-po*f	coc-preopen* function
c-pc*f	coc-preclosed* function
C-pcon	coc-pre-connected

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References

[1] Mashhour, A. S.; "On precontinuous and weak precontinuous mappings". Proc. Math. Phys. Soc, 53: 47-53, 1982.
[2] Al Ghour, S.; Samarah, S.; "Cocompact open sets and continuity." Abst. Appl. Anal, 2012 (1): 1-9, 2012.
[3] Hussain, R. A.; Nasser, H. I.; "On Connected Space byUsing N–Open Set". JQCM, 4(2): 1-19, 2012.
[4] Abdullah, H. K.; Fadhila K. R.; "Regular Proper Mappings". JQCM, 3(1): 161-184, 2011.
[5] Jasim, F. H.; "On Compactness Via Cocompact open sets". Msater's Thesis, College of Mathematics and Computer Science, University of Al-Qadissiya, Diwaniyah, Iraq, 2014.
[6] Baldovino, C.; Costantini, C.; "On some questions about KC and related spaces". Topology Appl, 156 (17): 2692-2703, 2009.
[7] Mashhour, A. S.; "S-C spaces." Delta J. Sci, 8: 30-46, 1984.
[8] Reilly, IL.; Vamanamurthy, MK.; "On some questions concerning preopen sets". Kyungpook Math. J, 30 (1): 87-93, 1990.
[9] Njåstad, O.; "On some classes of nearly open sets". Pac J Math, 15 (3): 961-970, 1965.
[10] Levine, N.; "Semi-open sets and semi-continuity in topological spaces". Amer. Math. Monthly, 70 (1): 36-41, 1963.
[11] Popa, V.; Noiri, T.; "Almost weakly continuous functions" Demonstr. mAth. De Gruyter, 25 (1-2): 241-252, 1992.
[12] POPA, V.; "Properties of H-almost continuous functions". Bull. Math. Soc. Sci. Math. Roumanie, 31(2): 163-168, 1987.
[13] Tyagi, B. K.; Singh. S.; Bhardwaj, M. and Chauhan, H.V.; "Some strong forms of connectedness in topological spaces." J. Adv. Stud. Topol , 10 (1): 20-27, 2019.

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[14] Mashhour, A. S.; Abd El-Monsef, M. E. and Hasanein. I. A.; "On pretopological spaces". Bull. Math. Soc. Sci. Math. Roumanie, 28(76): 39-45, 1984.
[15] Salih, A. A.; Ali, H. J.; "A study of mN-extremally Disconnected Spaces With Respect to τ, Maximum mX-N-open Sets" . Iraqi J. Sci., 62 (4): 1265-1270, 2021.