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Fuzzy Almost Nearly Prime and Fuzzy Almost Nearly 2 Absorbing Submodules

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Abstract

We present in this work the concepts of fuzzy AlmostNearlyprime submodules and fuzzy AlmostNearly2Absorbiing submodules as popularizations of fuzzy Nearly prime submodules and fuzzy 2-Absorbing submodules. Numerous additional fundamental characteristics, attributes, and connections between these ideas and other ideas are provided.

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1. Introduction

In this work, R mean a reciprocal ring with unitary and M is an R-module. In this work, two definitions were studied fuzzy AlmostNearlyprime submodules and fuzzy AlmostNearly2Absorbiing submodules These concepts are strong form of (prime and 2Absorbiing) submodules. Lotfi A. Zadeh, in 1965, proposed the idea of a fuzzy set to tackle such problems by giving each member of a given set a certain grade of membership [1]. This allowed vagueness to be mathematically described in all its abstractness. Algebra was one of the first fields of pure mathematics to put the idea of a fuzzy set among other branches. The notions of fuzzy sub groupoid and fuzzy subgroups were initially presented in A. Rosenfeld's 1971 publication, which was the first on fuzzy groups [2]. J.M. Anthony and H. Sherwood (1979) examined few Rosenfeld results under the new concept known as the triangular norm function by redefined fuzzy subgroup [3]. After that, many concepts in fuzzy algebra were clarified and studied, especially modules and sub modules. We denoted fuzzy set by Fu-set.

2. Results and Discussion (Basics of Research)

Definition (2.1)[1]:
If $\Upsilon_a: B \to [0,1]$ be a Fu-set in B, where $\Upsilon \in B$, $a \in [0,1]$ defined by: $\Upsilon_a(x) = \begin{cases} a & if\ \Upsilon = x \\ 0 & if\ \ \Upsilon \neq x \end{cases}$ $\forall\ x \in B$, Υ_a is named to be a Fu-singleton or Fu-point in B.

Lemma (2.2) [4]:
If B be an essential Fu-submodule of M, if $\Upsilon_a \subseteq$ M, then $\exists$ a Fu-singleton Υ_d of R, such that $0_1 \neq \Upsilon_d \Upsilon_a \subseteq B$.

Definition (2.3)[5]:
If B be a Fu-set in K, the set $B_a = \{y \in K; B(y) \geq a\}$ is named to be a level subset of B, such that, $B_a \subset K$ in the ordinary.

Definition (2.4)[6]:
If $\Upsilon = 0$ & $a = 1$, then $0_1(x) = \begin{cases} 1 & if\ x = 0 \\ 0 & if\ \ x \neq 0 \end{cases}$

Definition (2.5)[4]:
If M be a Fu-module, M is named to be faithful if Fu-annM $= 0_1$. Where Fu-annM $=\{\Upsilon_v : \Upsilon_v \Upsilon_a = 0_1; \forall\ \Upsilon_a \subseteq$M & Υ_v be a Fu-singleton of R.

Proposition (2.6)[7]:
If M is a faithful multiplication $\mathcal{R}$-module, then J(M)= J(R)M.

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Proposition (2.7)[8]
A Fu-module M of R is multiplication iff every non-empty Fu- submodule B of M such that $B = (B:_R \mathrm{M})$ M.

Definition (2.8) [9]:
If M be a Fu-module of R, M is named to be cancellative if
$\Upsilon_v \Upsilon_a = \Upsilon_v \Upsilon_b$, $\forall$ $\Upsilon_a, \Upsilon_b \subseteq$ M and Υ_v be a Fuzzysingleton of R, then $\Upsilon_a = \Upsilon_b$.

Definition (2.9)[10]:
A non-zero submodule B of an $\mathcal{R}$-module M is called an essential in M if $B \cap C \neq (0)$ for each non-zero submodule C of M.

3. (FANP and FAN2A) Submodules and Related Concepts.

Definition (3.1):
A proper Fu-submodule B of an Fu-R-module M is named Fuzzy Almost Nearlyprime (simply FANP) submodule, if whenever $\Upsilon_a \Upsilon_b \subseteq B$, $\forall$ Υ_a Fu-singleton of R, $\Upsilon_b \subseteq$ M, implies that:

$\Upsilon_b \subseteq B \cap F - J(\mathrm{M})$ or $\Upsilon_a \subseteq [B \cap F - J(\mathrm{M}):_R \mathrm{M}]$.

And an *fuzzy* ideal J of a fuzzy ring R is named (FANP) ideal of R if J is an FANP submodule of R.

Proposition (3.2):
If B be a proper Fu-submodule of a Fu-module M, therefore the following claims are equivalent:

- B is a FANP submodule of M.
- $(B:\boldsymbol{J})$ is a FANP submodule of M, $\forall$ Fu-ideal $\boldsymbol{J}$ of R.
- $(B:\Upsilon_a)$ is a FANP submodule of M, $\forall$ Fu-singleton Υ_1 of R.

Proof 1→ 2
Assume that
$\Upsilon_a \Upsilon_b \subseteq (B:\boldsymbol{J})$,
$\forall$ Υ_1 Fu-singleton of R,
$\Upsilon_b \subseteq$ M
& $\boldsymbol{J}$ Fu-ideal of R from the above $\Upsilon_c \Upsilon_a \Upsilon_b \subseteq B$, $\forall$ Υ_c Fu-singleton of $\boldsymbol{J}$ (but B is a FANP),
then
$\Upsilon_c \Upsilon_b \subseteq B \cap J(\mathrm{M}) \implies \Upsilon_b \subseteq (B \cap J(\mathrm{M}) : \boldsymbol{J})$
or
$\Upsilon_c \Upsilon_a \subseteq B \cap J(\mathrm{M}) \implies \Upsilon_a \subseteq (B \cap J(\mathrm{M}) : \boldsymbol{J})$.
Since by theorem (3.3)[8] we have
$B \subseteq (B:\boldsymbol{J})$, it follows
$\Upsilon_b \subseteq (B \cap J(\mathrm{M}) : \boldsymbol{J}) \subseteq ((B:\boldsymbol{J}) \cap J(\mathrm{M})) : \boldsymbol{J})$
or
$\Upsilon_a \subseteq (B \cap J(\mathrm{M}) : \boldsymbol{J}) \subseteq ((B:\boldsymbol{J}) \cap J(\mathrm{M})) : \boldsymbol{J})$,
thus
$\Upsilon_b \subseteq (((B:\boldsymbol{J}) \cap J(\mathrm{M})) : \boldsymbol{J})$
or
$\Upsilon_b \subseteq (((B:\boldsymbol{J}) \cap J(\mathrm{M})) : \boldsymbol{J})$
hence $(B:\boldsymbol{J})$ is FANP submodule of M.

Proof 2→3 Evident.

Proof 3→1
It is simple to follow by taking $\Upsilon = 1$, $a = 1$.

Proposition (3.3):
If B be a Fu-submodule of a Fu-module M, then the level B_v is an FANP submodule of M_v iff B is a FANP submodule of M.

Proof:
$\Longrightarrow$ Assume that $\Upsilon \mathrm{K} \in B_v$, $\forall \Upsilon \in R$, $\mathrm{K} \in M_v$
then

$B(\Upsilon \mathrm{K}) \geq v$, so $(\Upsilon \mathrm{K})_v \subseteq B$ implias that $\Upsilon_a \mathrm{K}_b \subseteq B$

where $v = min\{a, b\}$, since B is a FANP
then
$\mathrm{K}_b \subseteq B \cap F - J(\mathrm{M})$
or
$\Upsilon_a \subseteq [B \cap F - J(\mathrm{M}):_R \mathrm{M}]$,
we have
$v = min\{a, b\}$,
thus
$\mathrm{K}_v \subseteq B \cap F - J(\mathrm{M})$ or $\Upsilon_v \subseteq [B \cap F - J(\mathrm{M}):_R \mathrm{M}]$,
so that
$\mathrm{K} \in B_v \cap J(M_v)$
or
$\Upsilon \in [B_v \cap J(M_v):_R M_v]$
where
$[B \cap F - J(\mathrm{M}):_R \mathrm{M}]_v = [B_v \cap J(M_v):_R M_v]$.
Therefore B_v is an FANP submodule of M_v.

$\Longleftarrow$ If $\Upsilon_a \mathrm{K}_b \subseteq B$, $\forall$ Fu-singleton Υ_a of R, $\mathrm{K}_b \subseteq$ M, hence $(\Upsilon \mathrm{K})_v \subseteq B$
where
$v = min\{a, b\}$, so that $B(\Upsilon \mathrm{K}) \geq v$ implies that
$\Upsilon \mathrm{K} \in B_v$ (B_v is an FANP),
thus
$\mathrm{K} \in B_v \cap J(M_v)$ or $\Upsilon \in [B_v \cap J(M_v):_R M_v]$ but we have $[B \cap F - J(\mathrm{M}):_R \mathrm{M}]_v = [B_v \cap J(M_v):_R M_v]$, then $\Upsilon \in [B \cap F - J(\mathrm{M}):_R \mathrm{M}]_v$,
therefore
$\mathrm{K}_v \subseteq B \cap F - J(\mathrm{M})$ or $\Upsilon_v \subseteq [B \cap F - J(\mathrm{M}):_R \mathrm{M}]$ but $v = min\{a, b\}$, then

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$Ҡ_b \subseteq B \cap F-J(\mathrm{M})$ or $\Upsilon_a \subseteq [B \cap F-J(\mathrm{M}) :_R \mathrm{M}]$. Therefore B is a FANP submodule of M.

Definition (3.4):
A proper fuzzy submodule B of an fuzzy R-module M is named fuzzy almost nearly 2 absorbing (simply FAN2A) submodule, if $\Upsilon_a\Upsilon_b\Upsilon_v \subseteq B$, $\forall$ Υ_a Υ_b Fu-singletons of R, $\Upsilon_v \subseteq \mathrm{M}$, implies that
$\Upsilon_a\Upsilon_v \subseteq B \cap F-J(\mathrm{M})$
or
$\Upsilon_b\Upsilon_v \subseteq B \cap F-J(\mathrm{M})$
or
$\Upsilon_a\Upsilon_b \subseteq [B \cap F-J(\mathrm{M}):_R\mathrm{M}]$.
And an fuzzy ideal J of a fuzzy ring R is named (FAN2A) ideal of R if J is an FAN2A submodule of R.

Proposition (3.5):
Every FANP submodule of M is a FAN2A submodule of M. But not conversely.

Proof:
If $\Upsilon_a(\Upsilon_b\Upsilon_v) \subseteq B$, $\forall$ Υ_a Υ_b Fu-singletons of R, $\Upsilon_v \subseteq \mathrm{M}$, but we have B is FANP submodule of M , thus $(\Upsilon_b\Upsilon_v) \subseteq B \cap F-J(\mathrm{M})$ or $\Upsilon_a \subseteq [B \cap F-J(\mathrm{M}):_R\mathrm{M}]$. Then B is a FAN2A submodule of M.
Generally, the opposite is true, for example:
If $\mathrm{M} : Z_{16} \to \mathrm{L}$ such that $\mathrm{M}(\Upsilon) = \begin{cases} 1 & if\ \Upsilon \in Z_{16} \\ 0 & o.w \end{cases}$
Clearly, that M is Fu-module of Z-module Z.
And $B : Z_{16} \to \mathrm{L}$ such that
$B(\Upsilon) = \begin{cases} v & if\ \Upsilon \in <\bar{4}> \\ 0 & o.w \end{cases}, \quad \forall\, v \in L$
It is obvious that B is a Fu-submodule. it is clear that $B_v =<\bar{4}>$ of M_v is a FAN2A submodule, as an explanation $2.2.\bar{1} \in B_v =<\bar{4}>$.
But
$2.\bar{1} = 2 \notin <\bar{4}> \cap F-J(\mathrm{M}_v) = <\bar{4}> \cap <\bar{2}> = <\bar{4}>$, but we get what we achieve $2.2 = 4 \in [<\bar{4}> \cap F-J(\mathrm{M}_v):_R Z_{16}] = [<\bar{4}> \cap <\bar{2}>:_R Z_{16}] =$
$[<\bar{4}> :_R Z_{16}] = 4Z$.
Now, It is obvious that B_v is not FANP, as an explanation $2.\bar{2} \in B_v = <\bar{4}>$, but
$\bar{2} \notin <\bar{4}> \cap F-J(\mathrm{M}_v) = <\bar{4}> \cap <\bar{2}> = <\bar{4}>$ and $2 \notin [<\bar{4}> \cap F-J(\mathrm{M}_v):_R Z_{16}] =$
$[<\bar{4}> \cap <\bar{2}>:_R Z_{16}] = [<\bar{4}> :_R Z_{16}] = 4Z$.

Proposition (3.6):
If B, C be two FAN2A submodules of M with $F-J(\mathrm{M}) \nsubseteq B$ and $F-J(\mathrm{M}) \nsubseteq C$. Then $B \cap C$ is a FAN2A submodules of M.

Proof:
$\Upsilon_a\Upsilon_b\Upsilon_v \subseteq B \cap C$, $\forall$ Υ_a Υ_b Fu-singletons of R, $\Upsilon_v \subseteq \mathrm{M}$, clear that $\Upsilon_a\Upsilon_b\Upsilon_v \subseteq B$ & $\Upsilon_a\Upsilon_b\Upsilon_v \subseteq C$, but we have B is a FAN2A, then $\Upsilon_a\Upsilon_v \subseteq B \cap F-J(\mathrm{M})$ or $\Upsilon_b\Upsilon_v \subseteq B \cap F-J(\mathrm{M})$ or $\Upsilon_a\Upsilon_b \subseteq [B \cap F-J(\mathrm{M}):_R\mathrm{M}]$ and we know C is a FAN2A,
then
$\Upsilon_a\Upsilon_v \subseteq C \cap F-J(\mathrm{M})$ or $\Upsilon_b\Upsilon_v \subseteq C \cap F-J(\mathrm{M})$
or
$\Upsilon_a\Upsilon_b \subseteq [C \cap F-J(\mathrm{M}):_R\mathrm{M}]$
and we know
$F-J(\mathrm{M}) \nsubseteq B$ and $F-J(\mathrm{M}) \nsubseteq C$,
then
$F-J(\mathrm{M}) \nsubseteq B \cap C$,
from the above we conclude
$\Upsilon_a\Upsilon_v \subseteq (B \cap C) \cap F-J(\mathrm{M})$
or
$\Upsilon_b\Upsilon_v \subseteq (B \cap C) \cap F-J(\mathrm{M})$
or
$\Upsilon_a\Upsilon_b \subseteq [(B \cap C) \cap F-J(\mathrm{M}):_R\mathrm{M}]$.
Therefore
$B \cap C$ is a FAN2A submodules of M.

Proposition (3.7):
If B be an essential Fu-submodule of a Fu-module M such that
Fu-$ann\Upsilon_v =$ Fu-$ann\Upsilon_d\Upsilon_v$, $\forall$ $\Upsilon_v \subseteq \mathrm{M}$ for a Fu-singleton Υ_d of R such that
$\Upsilon_d \not\subset$ F-$ann\Upsilon_v$, & $0_1 \subseteq F-J(\mathrm{M})$.
Then M is a FAN2A module iff B is a FAN2A module.

Proof:
$\Rightarrow$Assume that B is a FAN2A module.
Now, If $\Upsilon_a\Upsilon_b\Upsilon_v \subseteq 0_1$, $\forall$ Υ_a, Υ_b Fu-singletons of R, $\Upsilon_v \subseteq \mathrm{M}$. Since B is an essential Fu-submodule, thus by (2.2), $\exists$ Fu-singleton Υ_d of R such that $0_1 \neq \Upsilon_d\Upsilon_v \subseteq B$,that is $\Upsilon_d \not\subset$ Fu-$ann\Upsilon_v$.
hence
$\Upsilon_a\Upsilon_b(\Upsilon_d\Upsilon_v) \subseteq 0_1$.
But B is a FAN2A module, We observed a Fu-module M is a FAN2A module, then any Fu-submodule of M is a FAN2A Fu-module, thus 0_1 is a FAN2A submodule of B, then $\Upsilon_a(\Upsilon_d\Upsilon_v) \subseteq 0_1 \cap F-J(\mathrm{M})$ or $\Upsilon_b(\Upsilon_d\Upsilon_v) \subseteq 0_1 \cap F-J(\mathrm{M})$ or $\Upsilon_a\Upsilon_b \subseteq [0_1 \cap F-J(\mathrm{M}):_R\mathrm{M}]$, we have $0_1 \subseteq F-J(\mathrm{M})$, thus $\Upsilon_a \subseteq$ F-$ann(\Upsilon_d\Upsilon_v)$ or $\Upsilon_b \subseteq$ Fu-$ann(\Upsilon_d\Upsilon_v)$, but Fu-$ann\Upsilon_v=$ Fu-$ann\Upsilon_d\Upsilon_v$, then $\Upsilon_a \subseteq$ Fu-$ann(\Upsilon_v)$ or $\Upsilon_b \subseteq$ Fu-$ann(\Upsilon_v)$ therefore $\Upsilon_a\Upsilon_v \subseteq 0_1 \cap F-J(\mathrm{M})$ or $\Upsilon_b\Upsilon_v \subseteq 0_1 \cap F-J(\mathrm{M})$.
Thus M is a FAN2A module.
$\Leftarrow$ Directly

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Proposition (3.8):
If M be a Fu-finitely generated faithful multiplication module. Let **I** be a FAN2A ideal. Then **I**M is a FAN2A submodule of M.

Proof:
Let $\Upsilon_a\Upsilon_b\Upsilon_v \subseteq \mathbf{I}$M, $\forall\ \Upsilon_a, \Upsilon_b$ Fu-singletons of R, $\Upsilon_v \subseteq$ M and $a, b, v \in [0,1]$. Since $\Upsilon_a\Upsilon_b < k_v > \subseteq \mathbf{I}$M. Now, M is multiplication then, ∃ Fu-ideal J such that $k_v = J$ M, thus $\Upsilon_a\Upsilon_b J$M $\subseteq \mathbf{I}$ M so $\Upsilon_a\Upsilon_b J \subseteq \mathbf{I} + Fu - ann$(M), but we have **I** be a FAN2A ideal of R, then $\Upsilon_a J \subseteq \mathbf{I} \cap F - J$(R) or $\Upsilon_b J \subseteq \mathbf{I} \cap F - J$(R) *or*
$\Upsilon_a\Upsilon_b \subseteq \mathbf{I} \cap F - J$(R), by multiplying both sides with M.
Therefore
$\Upsilon_a J$M $\subseteq (\mathbf{I} \cap F - J$(R))M or $\Upsilon_b J$M $\subseteq (\mathbf{I} \cap F - J$(R))M *or*
$\Upsilon_a\Upsilon_b$M $\subseteq (\mathbf{I} \cap F - J$(R))M , so $\Upsilon_a J$M $\subseteq \mathbf{I}$M $\cap F - J$(R)M
or
$\Upsilon_b J$M $\subseteq \mathbf{I}$M $\cap F - J$(R)M
or
$\Upsilon_a\Upsilon_b$M $\subseteq \mathbf{I}$M $\cap F - J$(R)M.
But
$F - J$(R)M $= F - J$(M),
thus $\Upsilon_a J$M $\subseteq \mathbf{I}$M $\cap F - J$(M)
or
$\Upsilon_b J$M $\subseteq \mathbf{I}$M $\cap F - J$(M)
or
$\Upsilon_a\Upsilon_b$M $\subseteq \mathbf{I}$M $\cap F - J$(M), implies that
$\Upsilon_a\Upsilon_v \subseteq \mathbf{I}$M $\cap F - J$(M)
or
$\Upsilon_b\Upsilon_v \subseteq \mathbf{I}$M $\cap F - J$(M)
or
$\Upsilon_a\Upsilon_b \subseteq \mathbf{I}$M $\cap F - J$(M).
From the above we have proven that **I**M is a FAN2A submodule of M.

Corollary (3.9):
If B be a Fu-submodule of a Fu-module M, Then B is a FAN2A submodule of M if Fu-submodule D of M & every Υ_a, Υ_b Fu-singletons of R with $\Upsilon_a\Upsilon_b D \subseteq B$, implies that
$\Upsilon_a D \subseteq B \cap F - J$(M)
or
$\Upsilon_b D \subseteq B \cap F - J$(M)
or
$\Upsilon_a\Upsilon_b \subseteq [B \cap F - J(\mathrm{M}) :_R \mathrm{M}]$.

Proposition (3.10):
If M be a faithful multiplication Fu-module, then a proper Fu-submodule B is FAN2A submodule of iff $[B :_R \mathrm{M}]$ is FAN2A ideal of R.

Proof:
⟹Let $\Upsilon_a\Upsilon_b\Upsilon_v \subseteq [B :_R \mathrm{M}]$, $\forall\ \Upsilon_a, \Upsilon_b$ and Υ_v Fu-singletons of R, and $a, b, v \in [0,1]$.
It follows that
$\Upsilon_a\Upsilon_b(\Upsilon_v\mathrm{M}) \subseteq B$.
Now, B is FAN2A, thus $\Upsilon_a(\Upsilon_v\mathrm{M}) \subseteq B \cap F - J$(M)
or
$\Upsilon_b(\Upsilon_v\mathrm{M}) \subseteq B \cap F - J$(M)
or
$\Upsilon_a\Upsilon_b \subseteq [B \cap F - J(\mathrm{M}) :_R \mathrm{M}]$.
With these basics
$B = [B :_R \mathrm{M}]$ M and $F - J$(M)$= F - J$(R) M,
we get
$\Upsilon_a\Upsilon_v$M $\subseteq [B :_R \mathrm{M}]$M $\cap F - J$(R) M
or
$\Upsilon_b\Upsilon_v$M $\subseteq [B :_R \mathrm{M}]$M $\cap F - J$(R)M
or
$\Upsilon_a\Upsilon_b$M $\subseteq [B :_R \mathrm{M}]$M $\cap F - J$(R)M, implies that
$\Upsilon_a\Upsilon_v \subseteq [B :_R \mathrm{M}] \cap F - J$(R)
or
$\Upsilon_b\Upsilon_v \subseteq [B :_R \mathrm{M}] \cap F - J$(R)
or
$\Upsilon_a\Upsilon_b \subseteq \cap [B :_R \mathrm{M}] \cap F - J$(R).
Therefore
$[B :_R \mathrm{M}]$ is FAN2A ideal of R.

⟸ If $\Upsilon_a\Upsilon_b D \subseteq B$
where Υ_a , Υ_b Fu-singletons of R and D Fu-submodule of M. AFu-module M is multiplication, thus
$D = J$M for some ideal J of R.
From the above we have $\Upsilon_a\Upsilon_b J$M $\subseteq B$, then
$\Upsilon_a\Upsilon_b J \subseteq [B :_R \mathrm{M}]$.
We know $[B :_R \mathrm{M}]$ is FAN2A, get by above corollary $\Upsilon_a J \subseteq [B :_R \mathrm{M}] \cap F - J$(R) or $\Upsilon_b J \subseteq [B :_R \mathrm{M}] \cap F - J$(R) or $\Upsilon_a\Upsilon_b \subseteq [B :_R \mathrm{M}] \cap F - J$(R), by multiplying both sides with M, therefore
$\Upsilon_a J$M $\subseteq ([B :_R \mathrm{M}] \cap F - J$(R)) M
or
$\Upsilon_b J$M $\subseteq ([B :_R \mathrm{M}] \cap F - J$(R)) M
or
$\Upsilon_a\Upsilon_b$M $\subseteq ([B :_R \mathrm{M}] \cap F - J$(R))M,
Subsequently
$\Upsilon_a J$M $\subseteq [B :_R \mathrm{M}]$M $\cap F - J$(R)M
or
$\Upsilon_b J$M $\subseteq [B :_R \mathrm{M}]$M $\cap F - J$(R)M
or
$\Upsilon_a\Upsilon_b$M $\subseteq [B :_R \mathrm{M}]$M $\cap F - J$(R)M, With these basics $B = [B :_R \mathrm{M}]$M & $F - J$(M)$= F - J$(R)M, we get B is FAN2A submodule.

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