

The Properties of Neutron Halo Structure for ^{17}B and ^{22}N nuclei

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Abstract

The radial wave functions of the Harmonic Oscillator and Woods-Saxon potentials within the three-body [two-body] model of $\text{Core} + 2n$ [$\text{Core} + n$] have been used to study some of the basic structural features such as the ground state proton, neutron and matter densities, the associated root mean square (rms) radii and elastic form factors of neutron-rich ^{17}B [^{22}N] halo nucleus. The long tail manner is clearly shown in the results obtained of neutron and matter densities of these nuclei. According to the calculated results it is found that this model provides a good description on the nuclear structure of above exotic nuclei. The reaction cross sections for these nuclei have been studied using the Glauber model with an optical limit approximation at high energy region.

[DOI: [10.22401/ANJS.22.3.09](https://doi.org/10.22401/ANJS.22.3.09)]

Keywords: Glauber model, Halo nuclei, Neutron-rich nuclei.

Introduction

The study of the halo phenomena has become a hot topic in nowadays nuclear physics [1-3]. The cause of halo phenomena lies in both the small separation energy of the last few nucleons and their occupation on the orbits with low angular momentum ($l = 0, 1$) [4], which allow the wave function of the valence nucleons to extend to large radii [5]. Both on theories and experiments, studying the halo structure is very useful to well understand the structure of nucleus. Halo nuclei are so short lived that they cannot be used as targets at rest. Therefore, experiments have been done in inverse kinematics (i.e., the role of target and projectile are exchanged) with a beam of exotic nuclei incident on a stable target at radioactive ion beam (RIB) facilities [6].

Halo systems are well described by the few body models, which assumes that halo nuclei consist of a core and a few outside nucleons [7]. There are two types of halo systems: the two-body system where one nucleon is surrounding the core nucleus, such as the one-neutron halo ^{11}Be and ^{19}C and the one-proton halo ^8B ; and the three-body system where two valence nucleons are around the core nucleus, such as ^6He , ^{11}Li and ^{14}Be . The three body system is called Borromean because where the binary subsystems (core plus one nucleon or the di-nucleon) are unbound, while the three body system is bound [8].

Hamoudi *et al.* [9] have been used the two-frequency shell model (TFSM) and the binary

cluster model (BCM) to study the ground state densities of exotic ^8He and ^{17}B nuclei. The long tail property is shown in the calculated neutron and matter densities of these nuclei. The calculated results of matter densities were in good accordance with the experimental results. Xia *et al.* [12] have been studied the properties of neutron-rich boron isotopes in the relativistic continuum Hartree-Bogoliubov theory in coordinate space with NL-SH, PK1 and TM2 effective interactions. Pairing corrections were taken into account by a density dependent force of zero range. The binding energies calculated for these nuclei agree with the experimental data quite well. The neutron-rich nucleus ^{17}B has been predicted to have a two-neutron halo structure in its ground state. The halo structure of ^{17}B was reproduced in a self-consistent way, and this halo was shown to be formed by the valence neutron level $2s_{1/2}$. Suzuki *et al.* [11] studied the interaction cross section for ^{17}B on a carbon target at 880 A MeV. The result for σ_I was 1118 ± 22 mb. They also studied the root mean square (rms) matter radii of ^{17}B using the Glauber-type calculation based on the optical limit approximation and a few-body reaction model. Their results for these two different methods were 2.90 ± 0.06 and 2.99 ± 0.09 fm, respectively.

Abdullah [12] have been used a three-body model of $(\text{Core} + 2n)$ to study the ground state densities of halo ^6He , ^{11}Li , ^{12}Be and ^{14}Be nuclei. The long tail property is shown in the

calculated neutron and matter densities of these nuclei. The calculated results of matter densities were in good accordance with the experimental results. Abdullah [13] has been investigate the ground state properties such as, the binding energy per nucleon, the ground state densities and the corresponding rms radii of two-neutron ${}^6\text{He}$, ${}^{11}\text{Li}$, ${}^{12}\text{Be}$ and ${}^{14}\text{Be}$ halo nuclei by means of the Skyrme-Hartree-Fock (SHF) method with MSK7 Skyrme parameter. In the present work, we will use the radial wave functions of the Harmonic Oscillator (HO) and Woods-Saxon (WS) potentials within the three-body model of *Core + 2n* for ${}^{17}\text{B}$ and two-body model of *Core + n* for ${}^{22}\text{N}$ to study some of the basic structural features for these halo nuclei such as the ground state proton, neutron and matter densities, the associated root mean square (rms) radii and elastic form factors. The reaction cross sections for these nuclei will be study using the Glauber model with an optical limit approximation at high energy region.

Theory

In the case of halo nuclei, it is reasonable to parameterize the core and halo densities separately. Therefore, the ground state matter density distribution for halo nuclei can be written as [14]:

$$\rho_m(r) = \rho_c(r) + \rho_v(r), \dots\dots\dots (1)$$

where $\rho_c(r)$ and $\rho_v(r)$ are the core density and valence (halo) density, respectively.

The ground state densities of exotic (halo) nuclei have been calculated via two methods, these are HO+HO (Harmonic Oscillator) and WS+WS (Woods-Saxon).

In the HO+HO method, the core and halo densities are parameterized with harmonic oscillator wave functions [15, 16]:

$$\rho_c(r) = \frac{1}{4\pi} \sum_{n\ell} X_c^{n\ell} |R_{n\ell}(r)|^2 \dots\dots\dots (2)$$

$$\rho_v(r) = \frac{1}{4\pi} \sum_v X_v^{n\ell} |R_{n\ell}(r)|^2 \dots\dots\dots (3)$$

where $X_{c(v)}^{n\ell}$ represents the number of neutrons or protons, in the sub-shell $n\ell j$.

In the WS+WS method, the core and halo densities are calculated by solving the

eigenvalue problem of Woods-Saxon potential [16]:

$$\frac{\hbar^2}{2m} \frac{d^2 R_{nlj}(r)}{dr^2} + \left[\varepsilon_{nlj} - V(r) - \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] R_{nlj}(r) = 0 \dots\dots\dots (4)$$

Where m is the reduced mass of the core and single nucleon, ε_{nlj} is the single-particle energy, $R_{nlj}(r)$ is the radial eigen function of WS potential and $V(r)$ is the potential of the core and can be written as [16]:

$$V(r) = V_0(r) + V_{so}(r) \vec{l} \cdot \vec{s} + V_c(r), \dots\dots\dots (5)$$

where $V_0(r)$ is the spin-independent central potential:

$$V_0(r) = \frac{-V_0}{1 + [e^{(r-R_0)/a_0}]} \dots\dots\dots (6)$$

$V_{so}(r)$ is the spin-orbit potential:

$$V_{so}(r) = V_{so} \frac{1}{r} \left[\frac{d}{dr} \frac{1}{1 + (e^{(r-R_{so})/a_{so}})} \right] \dots\dots\dots (7)$$

and $V_c(r)$ is the Coulomb potential generated by a homogeneous charged sphere of radius R_c [17]:

$$V_c(r) = \frac{Ze^2}{r} \quad \text{for } r \geq R_c \dots\dots\dots (8)$$

$$V_c(r) = \frac{Ze^2}{R_c} \left[\frac{3}{2} - \frac{r^2}{2R_c^2} \right] \quad \text{for } r < R_c \dots\dots\dots (9)$$

The radii R_0, R_{so} and R_c are usually expressed as [12]:

$$R_i = r_i A^{1/3} \dots\dots\dots (10)$$

where A is the nuclear mass number.

The matter density of Eq. (1) can be written as [6]:

$$\rho_m(r) = \rho^p(r) + \rho^n(r), \dots\dots\dots (11)$$

where $\rho^p(r)$ and $\rho^n(r)$ are the proton and neutron densities of halo nuclei, respectively and written as [12]:

$$\rho^p(r) = \rho_c^p(r) + \rho_v^p(r) \dots\dots\dots (12)$$

$$\rho^n(r) = \rho_c^n(r) + \rho_v^n(r) \dots\dots\dots (13)$$

The rms radii of the neutron and proton distributions can be calculated by [16]:

$$\langle r_g^2 \rangle^{1/2} = \left[\frac{\int r^2 \rho_g(r) dr}{\int \rho_g(r) dr} \right]^{1/2} \quad g = n, p \dots\dots\dots (14)$$

We study the elastic form factors for considered nuclei using the plane wave Born approximation (PWBA) within the proton density distribution $\rho_p(r)$. In the PWBA, the elastic form factors are written as [16]:

$$F(q) = \frac{4\pi}{Z} \int_0^\infty \rho_p(r) j_0(qr) r^2 dr \dots\dots\dots (15)$$

where $j_0(qr)$ is the zero-order spherical Bessel function and q is the momentum transfer from the incident electron to the target nucleus. Inclusion the corrections of the finite nucleon size $F_{fs}(q) = \exp(-0.43q^2/4)$ and the center of mass $F_{cm}(q) = \exp(b^2q^2/4A)$ [19] in the calculations (when the harmonic oscillator wave functions is used) needs multiplying the form factor of Eq. (15) by these corrections.

The reaction cross section of considered nuclei is studied by the Glauber model within optical limit (OL) approximation which can be expressed as [20]:

$$\sigma_R = 2\pi \int [1 - T(b)] b db \left(1 - \frac{B_c}{E_{cm}} \right) \dots\dots (16)$$

where E_{cm} is the kinetic energy in the center of mass system, B_c is Coulomb barrier and $T(b)$ is the transparency function at impact parameter b .

In optical limit (OL) approximation the $T(b)$ is written as [21]:

$$T(b) = |S_{el}^{OL}(b)|^2 \dots\dots\dots (17)$$

Where $S_{el}^{OL}(b)$ is the elastic S – matrix for the target-projectile system given as [21]:

$$S_{el}^{OL}(b) = \exp[iO_{PT}(b)] \dots\dots\dots (18)$$

$$O_{PT}(b) = \int_{-\infty}^{\infty} dR_3 \int dr_1 \int dr_2 \rho_P(r_1) \rho_T(r_2) f_{NN}(|R+r_1-r_2|) \dots\dots\dots (19)$$

where $O_{PT}(b)$ is the overlap of the ground state densities of projectile and target (ρ_P and ρ_T , respectively). P denotes the projectile and T denotes the target and f_{NN} the effective nucleon-nucleon (NN) amplitude.

Results and discussion

The radial wave functions of the Harmonic Oscillator (HO) and Woods-Saxon (WS)

potentials within the three-body [two-body] model of *Core + 2n* [*Core + n*] have been used to study some of the basic structural features such as the ground state proton, neutron and matter densities, the associated rms radii and elastic form factors of neutron-rich ^{17}B [^{22}N] halo nucleus. In HO+HO method the core and halo parts are parameterized with HO functions, while in WS+WS methods the core and halo parts are parameterized with WS functions. The reaction cross sections for these nuclei have been studied using the Glauber model with an optical limit approximation at high energy region. A core ^{15}B (^{21}N) plus two halo neutrons (one halo neutron) structure is assumed for ^{17}B (^{22}N). For core ^{15}B and ^{21}N nuclei the configurations, respectively, $\{(1s_{1/2})^4, (1p_{3/2})^7, (1p_{1/2})^2, (1d_{5/2})^2\}$ and $\{(1s_{1/2})^4, (1p_{3/2})^8, (1p_{1/2})^3, (1d_{5/2})^6\}$ are assumed. We assumed that the two halo neutrons of ^{17}B and one halo neutron of ^{22}N occupies in $2s_{1/2}$ orbit.

Table (1) exhibits some features [22,23] of halo ^{17}B and ^{22}N nuclei. The WS parameters for stable nuclei (^{10}B and ^{14}N) and the depth of WS potential (V_0) for core nucleons of halo nuclei ^{17}B and ^{22}N are taken from Ref. [24] whereas the V_0 for valence neutron and other parameters (*i.e.* V_{so} , r_0 , r_{so} , a_0 , a_{so} and r_c) are adjusted to reproduce the experimental single particle energy of a valence neutron and the experimental matter rms radii. In ^{17}B case, the experimental single particle energy of a valence neutron is assumed as a free parameter [25] in a range from ~ 0 MeV to the two-neutron separation energy ($S_{2n} = 1.38 \pm 0.21$ MeV) to get a good agreement between the calculated and experimental matter densities. The best description of the experimental densities is obtained when the V_0 for valence neutron and other parameters are adjusted to reproduce the single particle energy of a valence neutron is equal to 1.38 MeV as well as the experimental matter rms radius. The HO size parameters b_c and b_v are adjusted to reproduce the experimental rms matter radii for core (^{15}B , ^{21}N) and halo nuclei (^{17}B , ^{22}N), respectively. The HO size parameters (b_c , b_v) and WS parameters utilized in our calculations are tabulated in Table (2).

Table (1)
Some properties for nuclei under study.

Halo Nucleus	Type of halo nucleus	Half life time ($\tau_{1/2}$) [22]	Separation Energy (MeV) [23]
^{17}B	Two- neutron halo	5.08 ms	$S_{2n} = 1.38 \pm 0.21$
^{22}N	One neutron halo	24 ms	$S_n = 1.28 \pm 0.21$

Table (2)
The HO size parameter and WS parameters utilized in our calculations.

Nuclei	V_0 (MeV)		V_{so} (MeV)	$a_0 = a_{so}$ (fm)	$r_0 = r_{so}$ (fm)	r_c (fm)	Ref.	b (fm)	
	Core [24]	Valence						b_c	b_v
^{17}B	60.827	41.510	6.0	0.472	1.374	1.464	---	1.685	3.134
^{22}N	60.335	34.011	6.0	0.751	1.325	1.376	---	1.708	3.80
^{10}B	57.684		6.0	0.542	1.236	1.362	[24]	1.54	
^{14}N	70.862		6.0	0.715	1.319	1.431	[24]	1.64	

In Tables (3) and (4) a comparison is made between the obtained and experimental results [26,27] of the proton, neutron and matter rms radii for ^{17}B and ^{22}N halo nuclei. Within errors

the obtained results are consistent with experimental ones. Table (5) displays the calculated results of the single-particle energies (ϵ) for the investigated nuclei.

Table (3)
Calculated and experimental results of the proton and neutron rms radii.

Nuclei	$\langle r_p^2 \rangle^{1/2}$		$\langle r_p^2 \rangle_{exp}^{1/2}$ [26]	$\langle r_n^2 \rangle^{1/2}$		$\langle r_n^2 \rangle_{exp}^{1/2}$ [26]
	HO	WS		HO	WS	
^{17}B	2.44	2.46	2.8 ± 0.5	3.41	3.20	3.0 ± 0.6
^{22}N	2.54	2.64	----	3.30	3.26	----

Table (4)
Calculated and experimental results of the core and matter rms radii.

Nuclei	$\langle r_{core}^2 \rangle^{1/2}$		$\langle r_{core}^2 \rangle_{exp}^{1/2}$ [27]	$\langle r_m^2 \rangle^{1/2}$		$\langle r_m^2 \rangle_{exp}^{1/2}$ [27]
	HO	WS		HO	WS	
^{17}B	2.59	2.59	2.59 ± 0.03	3.15	3.0	3.0 ± 0.6
^{22}N	2.75	2.86	2.75 ± 0.03	3.08	3.08	3.08 ± 0.13

Table (5)
The calculated single-particle energies.

Nucleus	nl_j	proton	neutron
		ϵ_{cal} (MeV)	ϵ_{cal} (MeV)
^{17}B	1s _{1/2}	42.213	44.276
	1p _{3/2}	28.796	30.741
	1p _{1/2}	----	28.007
	1d _{5/2}	----	16.146
	2s _{1/2}	----	1.38
^{22}N	1s _{1/2}	38.856	41.868
	1p _{3/2}	26.312	29.114
	1p _{1/2}	23.777	26.586

	$1d_{5/2}$	----	16.031
	$2s_{1/2}$	----	1.07

Fig.(1) exhibits the core (black curves), valence (blue curves) and matter (dashed-red curves) densities for ^{17}B and ^{22}N calculated by HO+ HO (left panel) and WS+ WS (right panel) parameterizations. The experimental matter densities (grey region) of ^{17}B [28] and ^{22}N [29] are also plotted in this figure. The densities of ^{17}B are displayed in figures 1(a)

and 1(b) whereas those of ^{22}N are presented in figures 1(c) and 1(d). As shown from these figures there is a very good accordance between the dashed-red curves and grey region for selected nuclei. Moreover, the dashed-red curves support extended matter distributions in these nuclei.

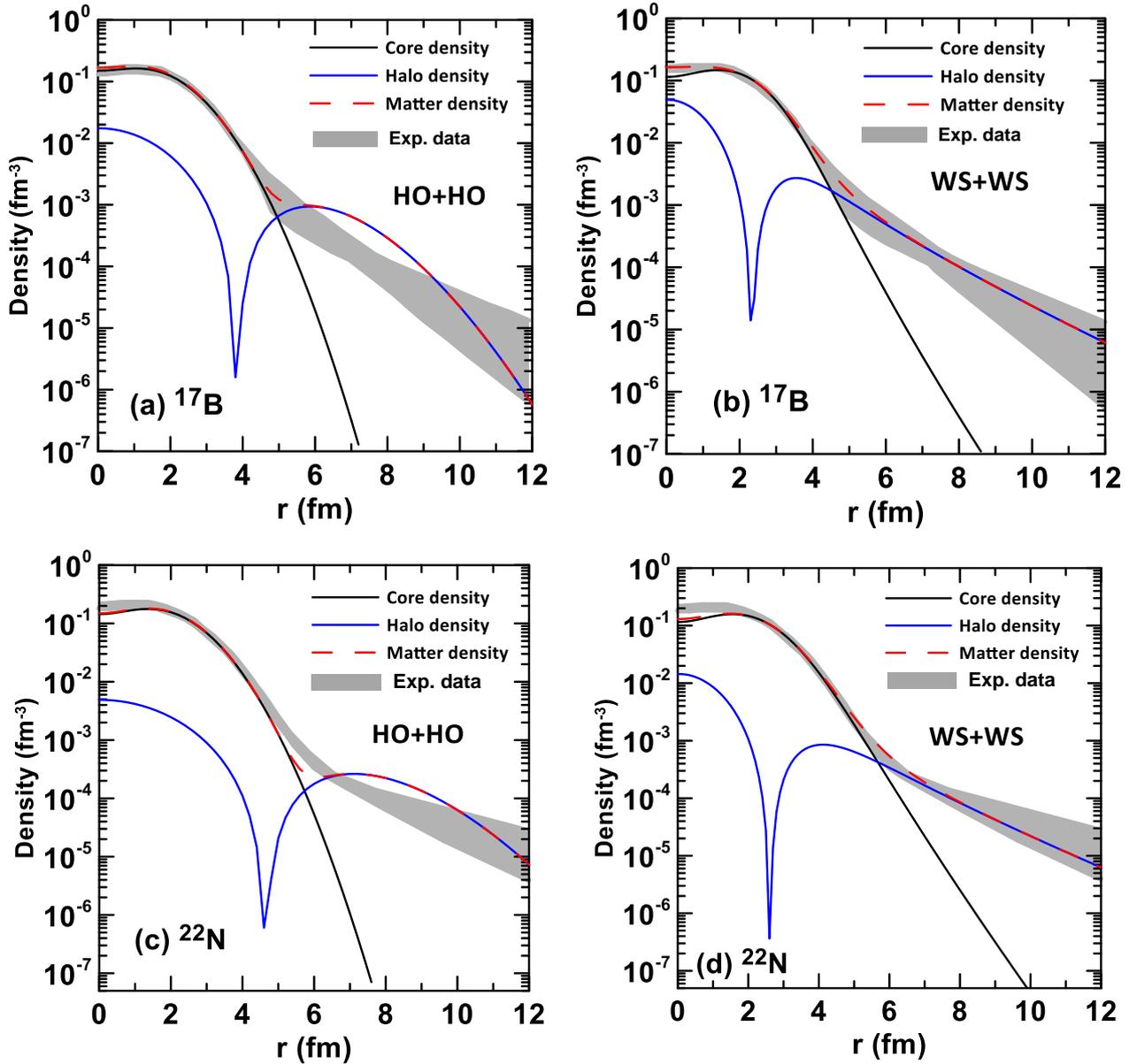


Fig.(1): The core, valence and matter densities for halo ^{17}B and ^{22}N nuclei.

Fig.(2) demonstrates the proton (black curves), neutron (blue curves) and matter (dashed-red curves) densities for ^{17}B (top panel) and ^{22}N (bottom panel). These densities are obtained by the HO+ HO parameterization [Figures 2 (a) and 2(c)] and WS+ WS

parameterization [Figures 2 (b) and 2(d)]. One can see from this figure that the typical performance of a halo nucleus (*i.e.* long tail) is clearly shown in the neutron density distributions.

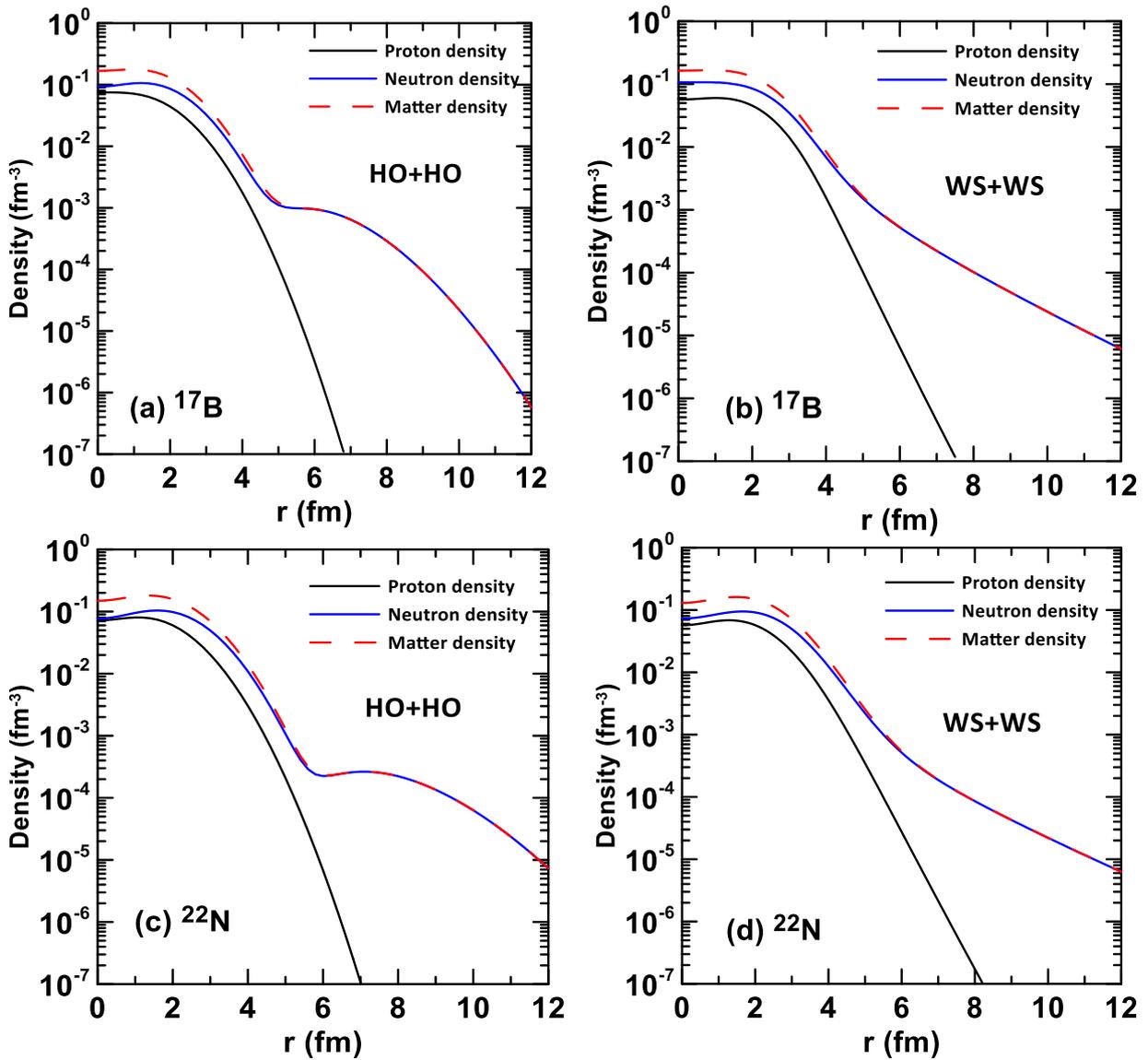


Fig.(2): The proton, neutron and matter densities for halo ^{17}B and ^{22}N nuclei.

Fig.(3) exemplifies the matter densities of $^{10,17}\text{B}$ [Figures 3 (a) and 3(b)] and $^{14,22}\text{N}$ [Figures 3 (c) and 3(d)] obtained by HO+ HO (two left figures) and WS+ WS (two right figures) parameterizations. Therein, the matter densities of unstable ($^{17}\text{B}, ^{22}\text{N}$) and stable ($^{10}\text{B}, ^{14}\text{N}$) nuclei are plotted by the dashed-red and blue distributions, respectively. It is apparent from these figures that the dashed-red and blue distributions are quite different. As the last two neutrons [one neutron] in ^{17}B [^{22}N] are [is] weakly bounded, the dashed-red distributions extend much farther than the blue distributions.

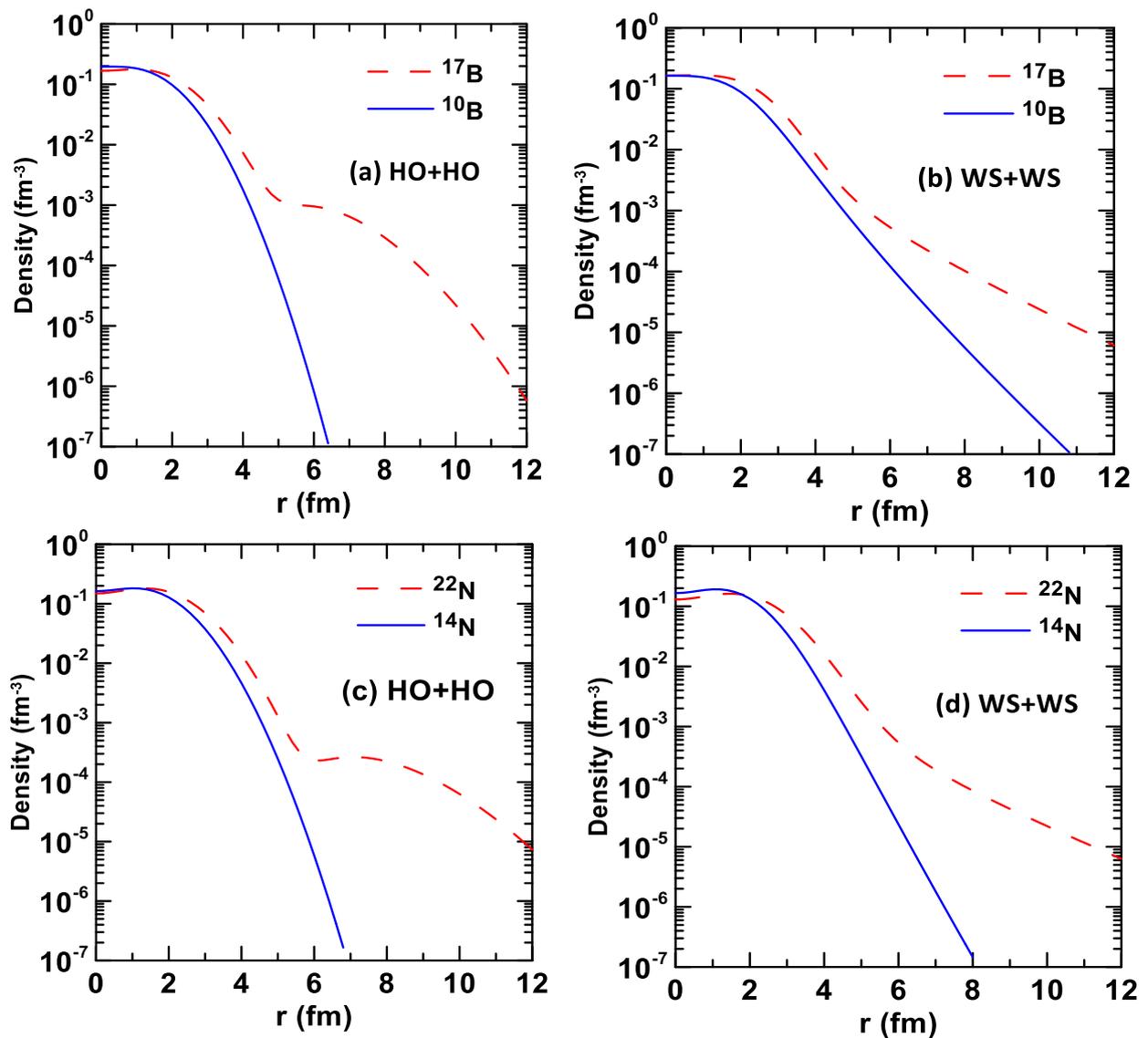


Fig.(3): Distributions of the matter density for unstable (¹⁷B,²²N) nuclei and their stable isotopes (¹⁰B,¹⁴N).

Fig.(4) shows the calculated C0 form factors for unstable nuclei ¹⁷B and ²²N (red curves) compared with those of their stable isotopes ¹⁰B and ¹⁴N (black curves) using the PWBA together with the experimental data (dotted symbols) of stable ¹⁰B [18] and ¹⁴N [30]. The left and right parts of these figures correspond to the results obtained by the HO+HO and WS+ WS parameterizations, respectively. The form factors in two upper and lower figures correspond to nuclei pairs ^{10,17}B and ^{14,22}N, respectively. According to these results that each of the red and black curves has only one diffraction minimum. The

minima location of red curve has inward shift as compared with the minima of black curve.

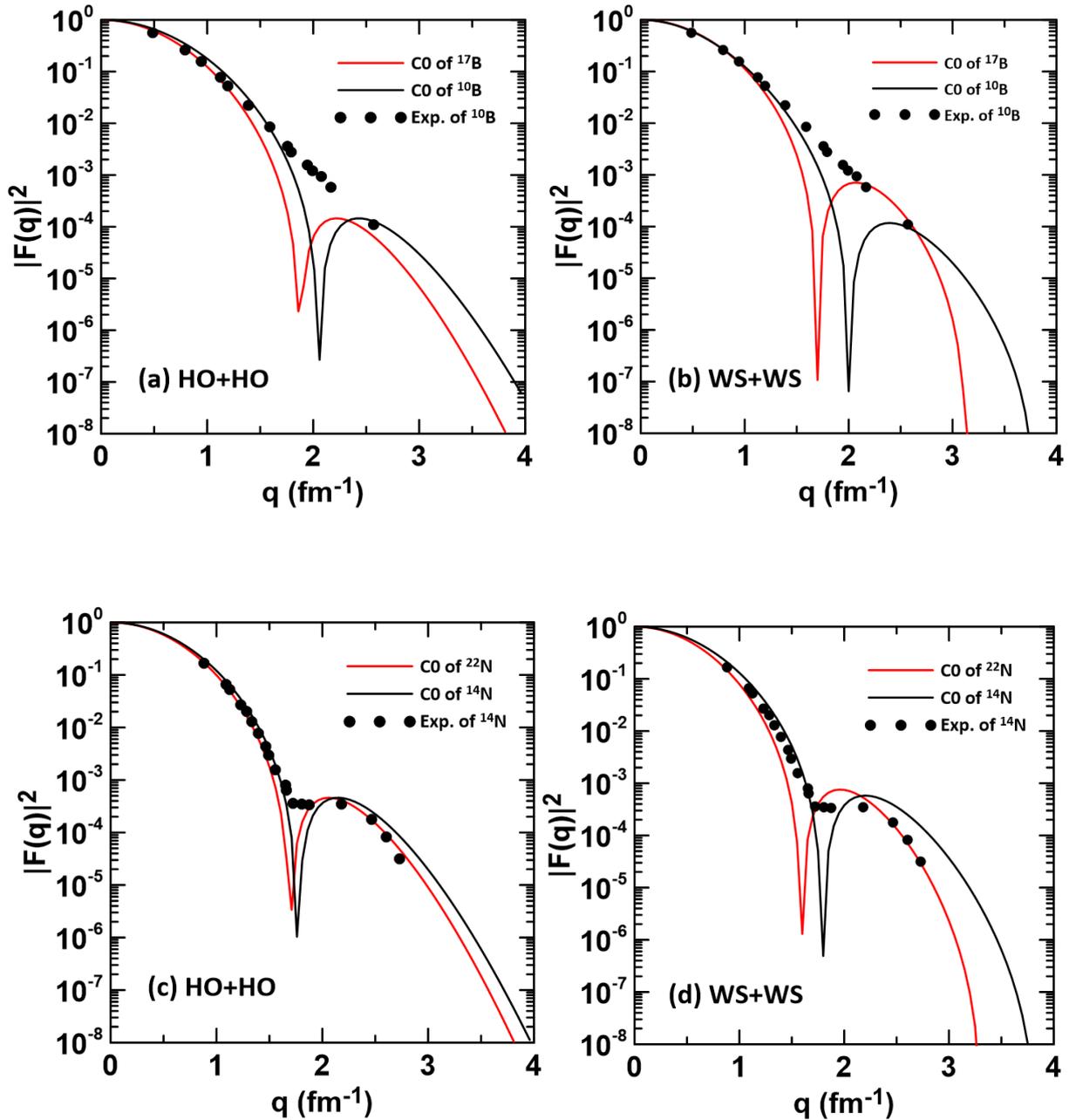


Fig.(4): Elastic form factors of unstable (^{17}B , ^{22}N) nuclei and their stable isotopes (^{10}B , ^{14}N).

The reaction cross sections (σ_R) of ^{17}B and ^{22}N on ^{12}C target have been calculated using the Glauber model with an optical limit approximation at high energy region and tabulated in Table (6) along with experimental data. From this table one can see that the calculated results of σ_R are in a good agreement with experimental data.

Table (6)
Calculated reaction cross sections of ^{17}B and ^{22}N along with experimental data.

Halo nuclei	Calculated σ_R (mb)	Experimental σ_R (mb) [8,27]	Energy (MeV) [8,27]
^{17}B	1125	1118 ± 22	880

	1157	1150 ± 200	790
²² N	1240	1245 ± 49	965

Summary and conclusions

The radial wave functions of the HO and WS potentials within the three-body [two-body] model of *Core + 2n* [*Core + n*] have been used to study some of the basic structural features such as the ground state proton, neutron and matter densities, the associated rms radii and elastic form factors of ¹⁷B [²²N] halo nucleus. The reaction cross sections for these nuclei have been studied using the Glauber model with an optical limit approximation at high energy region. This study confirms the following:

1. According to the calculated results it is found that this model provides a good description on the nuclear structure of above exotic nuclei.
2. The long tail manner is clearly shown in the neutron and matter densities of these nuclei.
3. The structure of the two halo neutrons [one halo neutron] in ¹⁷B [²²N] are [is] in a pure $2s_{1/2}$ leads to good agreement with experimental results.
4. The calculated results of σ_R using the Glauber model with an optical limit approximation at high energy region are in a good agreement with experimental data.

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