Weak Soft (1,2)*- $\widetilde{D}_{\omega}$-Sets and Weak Soft (1,2)*- $\widetilde{D}_{\omega}$-Separation Axioms in Soft Bitopological Spaces

Sabiha I. Mahmood and Asaad Adel Abdul-Hady
Department of Mathematics, College of Science, Mustansiriyah University, Baghdad-Iraq.
Corresponding Author: ssabihaa@uomustansiriyah.edu.iq.

Abstract

In this article we introduce and characterize new types of soft sets in soft bitopological spaces, namely, soft (1,2)*-omega difference sets (briefly soft (1,2)*- $\widetilde{D}_{\omega}$-sets) and weak forms of soft (1,2)*-omega difference sets. Moreover we use these soft sets to study new types of soft separation axioms, namely, soft (1,2)*-ω- $\widetilde{D}_j$-spaces, soft (1,2)*-α-ω- $\widetilde{D}_j$-spaces, soft (1,2)*-pre-ω- $\widetilde{D}_j$-spaces, soft (1,2)*-b-ω- $\widetilde{D}_j$-spaces and soft (1,2)*-β-ω- $\widetilde{D}_j$-spaces, for $j = 0,1,2$. Furthermore we investigate the characterizations and the relations between these types of soft separation axioms and other soft separation axioms. [DOI: 10.22401/ANJS.22.2.07]

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Introduction

Shabir and Naz [1] introduced and studied the concept of soft topological spaces by using the notion of soft sets which is introduced by Molodtsov [2]. Senel and Çagman [3] investigated the concept of soft bitopological spaces over an initial universe set with a fixed set of parameters. Mahmood and Abdul-Hady [4,5] introduced and studied new types of soft sets in soft bitopological spaces called soft (1,2)*-omega open sets and weak forms of soft (1,2)*-omega open sets such as soft (1,2)*-α-ω-open sets, soft (1,2)*-pre-ω-open sets, soft (1,2)*-b-ω-open sets and soft (1,2)*-β-ω-open sets and we use them to define and study new classes of soft separation axioms called soft (1,2)*-omega separation axioms and weak soft (1,2)*-omega separation axioms in soft bitopological spaces. The purpose of this paper is to define and study new types of soft separation axioms called weak soft (1,2)*- $\widetilde{D}_{\omega}$ -separation axioms in soft bitopological spaces by using weak soft (1,2)*- $\widetilde{D}_{\omega}$-sets such as soft (1,2)*-ω- $\widetilde{D}_j$-spaces, soft (1,2)*-α-ω- $\widetilde{D}_j$-spaces, soft (1,2)*-pre-ω- $\widetilde{D}_j$-spaces, soft (1,2)*-b-ω- $\widetilde{D}_j$-spaces, and soft (1,2)*-β-ω- $\widetilde{D}_j$ -spaces, for $j = 0,1,2$. Moreover we study the basic properties and the relationships between these types of soft separation axioms and other soft separation axioms.

1. Preliminaries:

Throughout this paper X is an initial universe set, $P(X)$ is the power set of X, P is the set of parameters and $A \subseteq P$.

Definition (1.1)[2]: A soft set over X is a pair (U, A), where U is a function defined by $U: A \to P(X)$ and A is a non-empty subset of P.

Definition (1.2)[6]: A soft set (U, A) over X is called a soft point if there is exactly $a \in A$ such that $U(a) = \{x\}$ for some $x \in X$ and $U(a') = \varphi, \forall a' \in A \setminus \{a\}$ and is denoted by $\tilde{x} = (a, \{x\})$.

Definition (1.3)[6]: A soft point $\tilde{x} = (a, \{x\})$ is called soft belongs to a soft set (U, A) if $a \in A$ and $x \in U(a)$, and is denoted by $\tilde{x} \tilde{\in} (U, A)$.

Definition (1.4)[6]: A soft set (U, A) over X is called countable (finite) if the set $U(a)$ is countable (finite) $\forall a \in A$.

Definition (1.5)[1]: A soft topology on X is a family $\tilde{\tau}$ of soft subsets of $\tilde{X}$ having the following properties:

(i) $\tilde{\varphi} \tilde{\in} \tilde{\tau}$ and $\tilde{X} \tilde{\in} \tilde{\tau}$.

(ii) If $(U_1,P),(U_2,P) \tilde{\in} \tilde{\tau}$, then $(U_1,P)\tilde{\bigcap}(U_2,P) \tilde{\in} \tilde{\tau}$

(iii) If $(U_j,P) \tilde{\in} \tilde{\tau}, \forall j \in \Lambda$, then $\bigcup_{j\in\Lambda}(U_j,P) \tilde{\in} \tilde{\tau}$.

The triple $(X,\tilde{\tau},P)$ is called a soft topological space over X. The elements of $\tilde{\tau}$ are called soft open sets in $\tilde{X}$. The complement of a soft open set is called soft closed.

Definition (1.6)[3]: Let X be a non-empty set and let $\tilde{\tau}_1$ and $\tilde{\tau}_2$ be soft topologies over X. Then $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft bitopological space over X.

Definition (1.7)[3]: A soft subset (U,P) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-open if $(U,P)=(U_1,P)\tilde{\bigcup}(U_2,P)$ such that $(U_1,P)\tilde{\in}\tilde{\tau}_1$ and $(U_2,P)\tilde{\in}\tilde{\tau}_2$. The complement of a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$ is called soft $\tilde{\tau}_1\tilde{\tau}_2$-closed.

Definition (1.8)[4]: A soft subset (A,P) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called soft (1,2)*-omega open (briefly soft (1,2)*-ω-open) if for each $\tilde{x}\tilde{\in}(A,P)$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set (U,P) in $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,P)$ and $(U,P)-(A,P)$ is countable. The complement of a soft (1,2)*-ω-open set is called soft (1,2)*-omega closed (briefly soft (1,2)*-ω-closed).

Definitions (1.9)[4]: A soft subset (A,P) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called:

(i) A soft (1,2)*-α-ω-open set if $(A,P)\tilde{\subseteq}$ (1,2)*-$\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl($(1,2)*-$\omega int(A,P)))$.

(ii) A soft (1,2)*-pre-ω-open set if $(A,P)\tilde{\subseteq}$ (1,2)*-$\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(A,P))$.

(iii) A soft (1,2)*-b-ω-open set if $(A,P)\tilde{\subseteq}$ (1,2)*-$\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(A,P))$ $\tilde{\bigcup}$ $\tilde{\tau}_1\tilde{\tau}_2 cl($(1,2)*-$\omega int(A,P))$.

(iv) A soft (1,2)*-β-ω-open set if $(A,P)\tilde{\subseteq}$ $\tilde{\tau}_1\tilde{\tau}_2 cl($(1,2)*-$\omega int(\tilde{\tau}_1\tilde{\tau}_2 cl(A,P)))$.

Proposition (1.10)[4]: If $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space. Then:

(i) Every soft $\tilde{\tau}_1\tilde{\tau}_2$-open set is soft (1,2)*-ω-open.

(ii) Every soft (1,2)*-ω-open set is soft (1,2)*-α-ω-open.

(iii) Every soft (1,2)*-α-ω-open set is soft (1,2)*-pre-ω-open.

(iv) Every soft (1,2)*-pre-ω-open set is soft (1,2)*-b-ω-open.

(v) Every soft (1,2)*-b-ω-open set is soft (1,2)*-β-ω-open.

Definitions (1.11)[5],[7]: A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_0$-space (resp. soft (1,2)*-ω-$\tilde{T}_0$-space, soft (1,2)*-α-ω-$\tilde{T}_0$-space, soft (1,2)*-pre-ω-$\tilde{T}_0$-space, soft (1,2)*-b-ω-$\tilde{T}_0$-space, soft (1,2)*-β-ω-$\tilde{T}_0$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-ω-open, soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set in $\tilde{X}$ containing one of the soft points but not the other.

Definition (1.12)[5],[7]: A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_1$-space (resp. soft (1,2)*-ω-$\tilde{T}_1$-space, soft (1,2)*-α-ω-$\tilde{T}_1$-space, soft (1,2)*-pre-ω-$\tilde{T}_1$-space, soft (1,2)*-b-ω-$\tilde{T}_1$-space, soft (1,2)*-β-ω-$\tilde{T}_1$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-ω-open, soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets (U,P) and (V,P) in $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,P), \tilde{y}\tilde{\notin}(U,P)$ and $\tilde{y}\tilde{\in}(V,P), \tilde{x}\tilde{\notin}(V,P)$.

Definition (1.13)[5],[7]: A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{T}_2$-space (resp. soft (1,2)*-ω-$\tilde{T}_2$-space, soft (1,2)*-α-ω-$\tilde{T}_2$-space, soft (1,2)*-pre-ω-$\tilde{T}_2$-space, soft (1,2)*-b-ω-$\tilde{T}_2$-space, soft (1,2)*-β-ω-$\tilde{T}_2$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft $\tilde{\tau}_1\tilde{\tau}_2$-open (resp. soft (1,2)*-ω-open, soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets (U,P) and (V,P) in $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,P),\tilde{y}\tilde{\in}(V,P)$ and $(U,P)\tilde{\cap}(V,P)=\tilde{\varphi}$.

Proposition (1.14)[5]: Every soft bitopological space is a soft (1,2)*-ω-$\tilde{T}_j$-space (resp. soft (1,2)*-α-ω-$\tilde{T}_j$-space, soft (1,2)*-pre-ω-$\tilde{T}_j$-space, soft (1,2)*-b-ω-$\tilde{T}_j$-space, soft (1,2)*-β-ω-$\tilde{T}_j$-space), for $j=0,1$.

Definition (1.15)[5]: A soft function $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is called strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) if $f^{-1}((U,P))$ is a soft $\tilde{\tau}_1\tilde{\tau}_2$-open set in $\tilde{X}$ for each soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set (U,P) in $\tilde{Y}$.

Definition (1.16)[5]: A soft function $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is called strongly soft (1,2)*-ω-open (resp. strongly soft (1,2)*-α-ω-open, strongly soft (1,2)*-pre-ω-open, strongly soft (1,2)*-b-ω-open, strongly soft (1,2)*-β-ω-open) if $f((U,P))$ is a soft $\tilde{\sigma}_1\tilde{\sigma}_2$-open set in $\tilde{Y}$ for each soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) set (U,P) in $\tilde{X}$.

2. Weak Soft (1,2)*-$\tilde{D}_\omega$-Sets

In this section, we introduce and study new concepts called soft (1,2)*-$\tilde{D}_\omega$-sets, soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-sets, soft (1,2)*-$\tilde{D}_{pre-\omega}$-sets, soft (1,2)*-$\tilde{D}_{b-\omega}$-sets and soft (1,2)*-$\tilde{D}_{\beta-\omega}$-sets in soft bitopological spaces. Further we investigate the relationships between these types of soft sets and other soft sets.

Definition (2.1): A soft subset (A,P) of a soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-$\tilde{D}_\omega$-set (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set, soft (1,2)*-$\tilde{D}_{pre-\omega}$-set, soft (1,2)*-$\tilde{D}_{b-\omega}$-set, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set) if there exists two soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) sets (A_1,P) and (A_2,P) in $\tilde{X}$ such that $(A_1,P)\neq\tilde{X}$ and $(A,P)=(A_1,P)\setminus(A_2,P)$.

Example (2.2): Let $X=\mathfrak{R}$, $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\varphi},(U,P)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\varphi}\}$ be soft topologies over X, where $(U,P)=\{(p_1,\{4\}),(p_2,\{4\})\}$. The soft sets in $\{\tilde{X},\tilde{\varphi},(U,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Then $(A,P)=\{(p_1,\{3\}),(p_2,\{3\})\}$ is a soft (1,2)*-$\tilde{D}_\omega$-set, since $\exists$ $(A_1,P)=\{(p_1,\mathfrak{R}-\{2\}),(p_2,\mathfrak{R}-\{2\})\}$ and $(A_2,P)=\{(p_1,\mathfrak{R}-\{3\}),(p_2,\mathfrak{R}-\{3\})\}$ are soft (1,2)*-ω-open sets in $\tilde{X}$ such that $(A_1,P)\neq\tilde{\mathfrak{R}}$ and $(A,P)=(A_1,P)\setminus(A_2,P)$.

Remark (2.3): In definition (2.1), if $(A_1,P)\neq\tilde{X}$ and $(A_2,P)=\tilde{\varphi}$, then each proper soft (1,2)*-ω-open (resp. soft (1,2)*-α-ω-open, soft (1,2)*-pre-ω-open, soft (1,2)*-b-ω-open, soft (1,2)*-β-ω-open) subset of $\tilde{X}$ is a soft (1,2)*-$\tilde{D}_\omega$-set (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set, soft (1,2)*-$\tilde{D}_{pre-\omega}$-set, soft (1,2)*-$\tilde{D}_{b-\omega}$-set, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set).

The converse of Remark (2.3) is not true in general as shown in the following examples.

Example (2.4): Let $X=\mathfrak{R}$, $P=\{p_1,p_2,p_3,p_4\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\varphi},(U,P)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\varphi}\}$ be soft topologies over X, where

$(U,P)=\{(p_1,\{1\}),(p_2,\{1\}),(p_3,\{1\}),(p_4,\{1\})\}$. The soft sets in $\{\tilde{X},\tilde{\varphi},(U,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Then $(A,P)=\{(p_1,\{2\}),(p_2,\{2\}),(p_3,\{2\}),(p_4,\{2\})\}$ is a soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set and a soft (1,2)*-$\tilde{D}_{\omega}$-set, but is not soft (1,2)*-α-ω-open set.

Example (2.5): Let $X=\mathfrak{R}$, $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\varphi},(U,P)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\varphi}\}$ be soft topologies over X, where $(U,P)=\{(p_1,\mathfrak{R}-\{1\}),(p_2,\mathfrak{R}-\{1\})\}$. The soft sets in $\{\tilde{X},\tilde{\varphi},(U,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Then $(A,P)=\{(p_1,\{1\}),(p_2,\{1\})\}$ is a soft (1,2)*-$\tilde{D}_{pre-\omega}$-set (resp. soft (1,2)*-$\tilde{D}_{b-\omega}$-set, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set), but is not soft (1,2)*-β-ω-open set.

Proposition (2.6): If $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft bitopological space. Then:

- **(i)** Every soft (1,2)*-$\tilde{D}$-set is a soft (1,2)*-$\tilde{D}_{\omega}$-set.
- **(ii)** Every soft (1,2)*-$\tilde{D}_{\omega}$-set is a soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set.
- **(iii)** Every soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set is a soft (1,2)*-$\tilde{D}_{pre-\omega}$-set.
- **(iv)** Every soft (1,2)*-$\tilde{D}_{pre-\omega}$-set is a soft (1,2)*-$\tilde{D}_{b-\omega}$-set.
- **(v)** Every soft (1,2)*-$\tilde{D}_{b-\omega}$-set is a soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set.

***Proof*:** Follows from proposition (1.10).

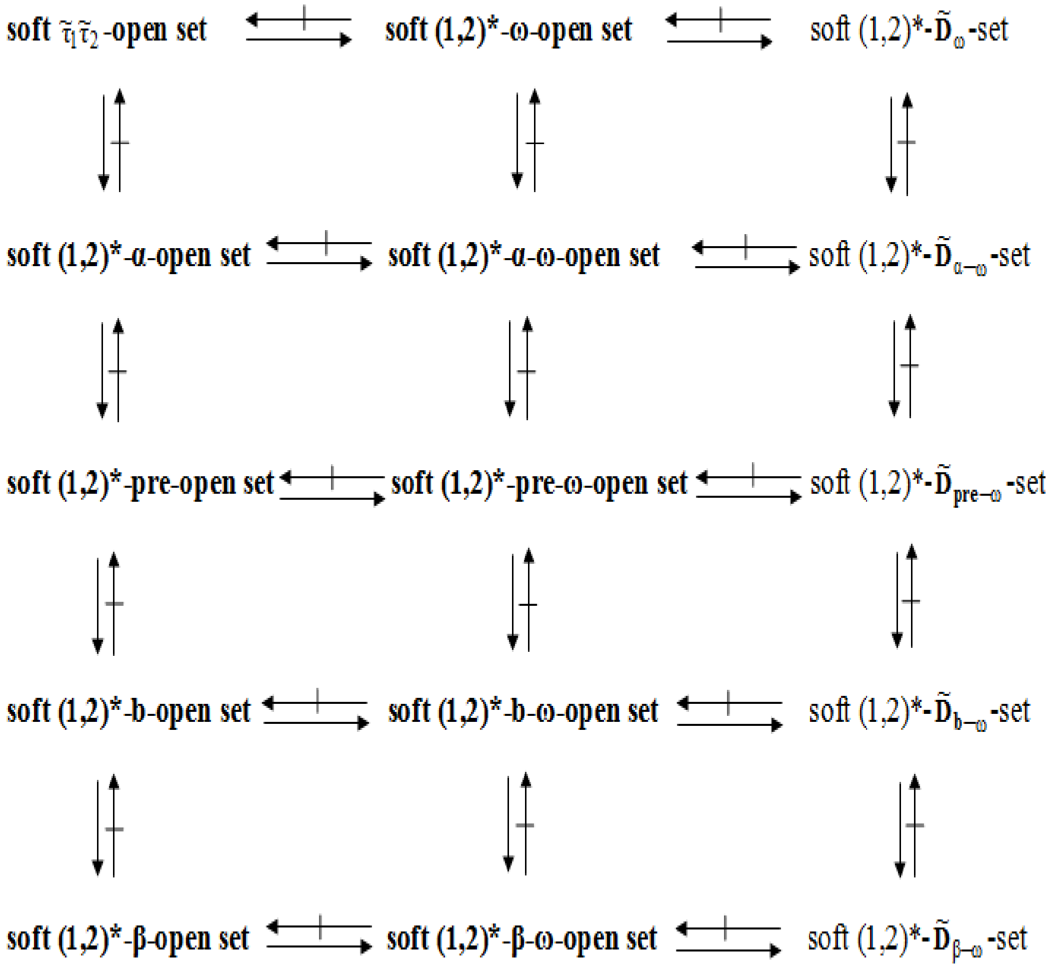
Remark (2.7): The converse of proposition (2.6) number (i),(ii) and (iii) in general may not be true. We see that in the following examples:

Example (2.8): Let $X=\mathfrak{R}$, $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\varphi},(U,P)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\varphi}\}$ be soft topologies over X, where $(U,P)=\{(p_1,\{2\}),(p_2,\{2\})\}$. The soft sets in $\{\tilde{X},\tilde{\varphi},(U,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Then $(A,P)=\{(p_1,\{1\}),(p_2,\{1\})\}$ is a soft (1,2)*-$\tilde{D}_{\omega}$-set, since $\exists$ $(A_1,P)=\{(p_1,\mathfrak{R}-\{2\}),(p_2,\mathfrak{R}-\{2\})\}$ and $(A_2,P)=\{(p_1,\mathfrak{R}-\{1\}),(p_2,\mathfrak{R}-\{1\})\}$ are soft (1,2)*-ω-open sets in $\tilde{X}$ such that $(A_1,P)\neq\tilde{\mathfrak{R}}$ and $(A,P)=(A_1,P)\setminus(A_2,P)$, but (A,P) is not soft (1,2)*-$\tilde{D}$-set.

Example (2.9): Let $X=\mathfrak{R}$, $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\varphi},(U,P)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\varphi}\}$ be soft topologies over X, where $(U,P)=\{(p_1,\{3\}),(p_2,\{3\})\}$. The soft sets in $\{\tilde{X},\tilde{\varphi},(U,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Then $(A,P)=\{(p_1,(0,3]),(p_2,(0,3])\}$ is a soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set, since (A,P) is a soft (1,2)*-α-ω-open set, but (A,P) is not soft (1,2)*-$\tilde{D}_{\omega}$-set.

Example (2.10): Let $X=\mathfrak{R}$, $P=\{p_1,p_2\}$ and let $\tilde{\tau}_1=\{\tilde{X},\tilde{\varphi},(U_1,P)\}$ and $\tilde{\tau}_2=\{\tilde{X},\tilde{\varphi},(U_2,P)\}$ be soft topologies over X, where $(U_1,P)=\{(p_1,\{4\}),(p_2,\{4\})\}$ and $(U_2,P)=\{(p_1,\mathfrak{R}-\{4\}),(p_2,\mathfrak{R}-\{4\})\}$. The soft sets in $\{\tilde{X},\tilde{\varphi},(U_1,P),(U_2,P)\}$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Then $(A,P)=\{(p_1,(0,4)),(p_2,(0,4))\}$ is a soft (1,2)*-$\tilde{D}_{pre-\omega}$-set, since (A,P) is a soft (1,2)*-pre-ω-open set, but (A,P) is not soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set.

The following diagram shows the relation between the types of soft open sets and each of weak soft (1,2)*-ω-open sets and weak soft (1,2)*-$\tilde{\mathbf{D}}_{\omega}$-sets.



Theorem (2.11):

If $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) surjective function and (A,P) is a soft (1,2)*-$\tilde{D}_\omega$-set (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set, soft (1,2)*-$\tilde{D}_{pre-\omega}$-set, soft (1,2)*-$\tilde{D}_{b-\omega}$-set, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set) in $\tilde{Y}$, then the inverse image of (A,P) is a soft (1,2)*-$\tilde{D}$-set in $\tilde{X}$.

***Proof*:** Let (A,P) be a soft (1,2)*-$\tilde{D}_\omega$-set in $\tilde{Y}$, then there are two soft (1,2)*-ω-open sets (A_1,P) and (A_2,P) in $\tilde{Y}$ such that $(A_1,P)\neq\tilde{Y}$ and $(A,P)=(A_1,P)\setminus(A_2,P)$. Since f is strongly soft (1,2)*-ω-continuous, then by definition (1.15), $f^{-1}((A_1,P))$ and $f^{-1}((A_2,P))$ are soft $\tilde{\tau}_1\tilde{\tau}_2$-open sets in $\tilde{X}$. Since $(A_1,P)\neq\tilde{Y}$ and f is surjective, then $f^{-1}((A_1,P))\neq\tilde{X}$. Hence $f^{-1}((A,P))=f^{-1}((A_1,P))\setminus f^{-1}((A_2,P))$ is a soft (1,2)*-$\tilde{D}$-set in $\tilde{X}$. By the same way we can prove other cases.

Theorem (2.12):

If $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ is a strongly soft (1,2)*-ω-open (resp. strongly soft (1,2)*-α-ω-open, strongly soft (1,2)*-pre-ω-open, strongly soft (1,2)*-b-ω-open, strongly soft (1,2)*-β-ω-open) bijective function and (A,P) is a soft (1,2)*-$\tilde{D}_\omega$-set (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set, soft (1,2)*-$\tilde{D}_{pre-\omega}$-set, soft (1,2)*-$\tilde{D}_{b-\omega}$-set, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set) in $\tilde{X}$, then $f((A,P))$ is a soft (1,2)*-$\tilde{D}$-set in $\tilde{Y}$.

***Proof*:** Let (A,P) be a soft (1,2)*-$\tilde{D}_\omega$-set in $\tilde{X}$, then there are two soft (1,2)*-ω-open sets (A_1,P) and (A_2,P) in $\tilde{X}$ such that

$(A_1,P)\neq\tilde{X}$ and $(A,P)=(A_1,P)\setminus(A_2,P)$. Since f is strongly soft (1,2)*-ω-open, then by definition (1.16), $f((A_1,P))$ and $f((A_2,P))$ are soft $\tilde{\sigma}_1\tilde{\sigma}_2$-open sets in $\tilde{Y}$. Since $(A_1,P)\neq\tilde{X}$ and f is injective, then $f((A_1,P))\neq\tilde{Y}$. Since f is bijective, then $f((A,P))=f((A_1,P))\setminus f((A_2,P))$ is a soft (1,2)*-$\tilde{D}$-set in $\tilde{Y}$. By the same way we can prove other cases.

3. Weak Soft (1,2)*-$\tilde{D}_\omega$-Separation Axioms

Now, we define and study new types of soft separation axioms in soft bitopological spaces, namely, soft (1,2)*-ω-$\tilde{D}_j$-spaces, soft (1,2)*-α-ω-$\tilde{D}_j$-spaces, soft (1,2)*-pre-ω-$\tilde{D}_j$-spaces, soft (1,2)*-b-ω-$\tilde{D}_j$-spaces, and soft (1,2)*-β-ω-$\tilde{D}_j$-spaces, for $j=0,1,2$. Further we study the relations between these types of soft separation axioms and other types of soft separation axioms.

Definitions (3.1): A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-$\tilde{D}_0$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_0$-space, soft (1,2)*-pre-ω-$\tilde{D}_0$-space, soft (1,2)*-b-ω-$\tilde{D}_0$-space, soft (1,2)*-β-ω-$\tilde{D}_0$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there exists a soft (1,2)*-$\tilde{D}_\omega$-set (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-set, soft (1,2)*-$\tilde{D}_{pre-\omega}$-set, soft (1,2)*-$\tilde{D}_{b-\omega}$-set, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-set) in $\tilde{X}$ containing one of the soft points but not the other.

Definitions (3.2): A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-$\tilde{D}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_1$-space, soft (1,2)*-pre-ω-$\tilde{D}_1$-space, soft (1,2)*-b-ω-$\tilde{D}_1$-space, soft (1,2)*-β-ω-$\tilde{D}_1$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft (1,2)*-$\tilde{D}_\omega$-sets (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-sets, soft (1,2)*-$\tilde{D}_{pre-\omega}$-sets, soft (1,2)*-$\tilde{D}_{b-\omega}$-sets, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-sets) (U,P) and (V,P) in $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,P)$, $\tilde{y}\tilde{\notin}(U,P)$ and $\tilde{y}\tilde{\in}(V,P)$, $\tilde{x}\tilde{\notin}(V,P)$.

Definitions (3.3): A soft bitopological space $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is called a soft (1,2)*-ω-$\tilde{D}_2$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_2$-space, soft (1,2)*-pre-ω-$\tilde{D}_2$-space, soft (1,2)*-b-ω-$\tilde{D}_2$-space, soft (1,2)*-β-ω-$\tilde{D}_2$-space) if for any two distinct soft points $\tilde{x}$ and $\tilde{y}$ of $\tilde{X}$, there are two soft (1,2)*-$\tilde{D}_\omega$-sets (resp. soft (1,2)*-$\tilde{D}_{\alpha-\omega}$-sets, soft (1,2)*-$\tilde{D}_{pre-\omega}$-sets, soft (1,2)*-$\tilde{D}_{b-\omega}$-sets, soft (1,2)*-$\tilde{D}_{\beta-\omega}$-sets) (U,P) and (V,P) in $\tilde{X}$ such that $\tilde{x}\tilde{\in}(U,P)$, $\tilde{y}\tilde{\in}(V,P)$ and $(U,P)\tilde{\cap}(V,P)=\varphi$.

Theorem (3.4):

(i) Every soft (1,2)*-$\tilde{T}_j$-space (resp. soft (1,2)*-ω-$\tilde{T}_j$-space, soft (1,2)*-α-ω-$\tilde{T}_j$-space, soft (1,2)*-pre-ω-$\tilde{T}_j$-space, soft (1,2)*-b-ω-$\tilde{T}_j$-space, soft (1,2)*-β-ω-$\tilde{T}_j$-space) is a soft (1,2)*-$\tilde{D}_j$-space (resp. soft (1,2)*-ω-$\tilde{D}_j$-space, soft (1,2)*-α-ω-$\tilde{D}_j$-space, soft (1,2)*-pre-ω-$\tilde{D}_j$-space, soft (1,2)*-b-ω-$\tilde{D}_j$-space, soft (1,2)*-β-ω-$\tilde{D}_j$-space), $j=0,1,2$.

(ii) Every soft (1,2)*-ω-$\tilde{D}_j$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_j$-space, soft (1,2)*-pre-ω-$\tilde{D}_j$-space, soft (1,2)*-b-ω-$\tilde{D}_j$-space, soft (1,2)*-β-ω-$\tilde{D}_j$-space) is a soft (1,2)*-ω-$\tilde{D}_{j-1}$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_{j-1}$-space, soft (1,2)*-pre-ω-$\tilde{D}_{j-1}$-space, soft (1,2)*-b-ω-$\tilde{D}_{j-1}$-space, soft (1,2)*-β-ω-$\tilde{D}_{j-1}$-space), $j=1,2$.

(iii) Every soft (1,2)*-$\tilde{D}_j$-space is a soft (1,2)*-ω-$\tilde{D}_j$-space, $j=0,1,2$.

(iv) Every soft (1,2)*-ω-$\widetilde{D}_j$-space is a soft (1,2)*-α-ω-$\widetilde{D}_j$-space, $j=0,1,2$.

(v) Every soft (1,2)*-α-ω-$\widetilde{D}_j$-space is a soft (1,2)*-pre-ω-$\widetilde{D}_j$-space, $j=0,1,2$.

(vi) Every soft (1,2)*-pre-ω-$\widetilde{D}_j$-space is a soft (1,2)*-b-ω-$\widetilde{D}_j$-space, $j=0,1,2$.

(vii) Every soft (1,2)*-b-ω-$\widetilde{D}_j$-space is a soft (1,2)*-β-ω-$\widetilde{D}_j$-space, $j=0,1,2$.

***Proof*:**

(i) Follows from Remark (2.3).

(ii) It is obvious.

(iii),(iv),(v),(vi),(vii) Follows from proposition (2.6).

Remark (3.5): The converse of theorem (3.4), no. (i) in general may not be true. We see that by the following examples:

Example (3.6): Let $X=\{a,b,c\}$ and $P=\{p\}$ and let $\widetilde{\tau}_1=\{\widetilde{X},\widetilde{\varphi},(U_1,P),(U_2,P)\}$ and $\widetilde{\tau}_2=\{\widetilde{X},\widetilde{\varphi},(U_3,P)\}$ be soft topologies over X, where $(U_1,P)=\{(p,\{a\})\}$, $(U_2,P)=\{(p,\{a,b\})\}$ and $(U_3,P)=\{(p,\{a,c\})\}$. The soft sets in $\{\widetilde{X},\widetilde{\varphi},(U_1,P),(U_2,P),(U_3,P)\}$ are soft $\widetilde{\tau}_1\widetilde{\tau}_2$-open sets. Thus $(X,\widetilde{\tau}_1,\widetilde{\tau}_2,P)$ is a soft (1,2)*-$\widetilde{D}_j$-space, but is not soft (1,2)*-$\widetilde{T}_j$-space, $j=1,2$.

Example (3.7): Let $X=\mathfrak{R}$ and $P=\{p_1,p_2\}$ and let $\widetilde{\tau}_1=\{(U,P)\widetilde{\subseteq}\widetilde{X}:(U,P)^c$ is finite$\}\widetilde{\cup}\{\widetilde{\varphi}\}$ and $\widetilde{\tau}_2=\{\widetilde{X},\widetilde{\varphi}\}$ be soft topologies over X. Thus $(X,\widetilde{\tau}_1,\widetilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\widetilde{D}_2$-space (resp. soft (1,2)*-α-ω-$\widetilde{D}_2$-space, soft (1,2)*-pre-ω-$\widetilde{D}_2$-space, soft (1,2)*-b-ω-$\widetilde{D}_2$-space, soft (1,2)*-β-ω-$\widetilde{D}_2$-space), but is not soft (1,2)*-β-ω-$\widetilde{T}_2$-space.

Remark (3.8): The converse of theorem (3.4), no. (iii) in general may not be true. We see that by the following examples:

Example (3.9): Let $X=\{a,b\}$ and $P=\{p_1,p_2\}$ and let $\widetilde{\tau}_1=\{\widetilde{X},\widetilde{\varphi},(U_1,P)\}$ and $\widetilde{\tau}_2=\{\widetilde{X},\widetilde{\varphi},(U_2,P)\}$ be soft topologies over X, where $(U_1,P)=\{(p_1,\{a\}),(p_2,\{a\})\}$ and $(U_2,P)=\{(p_1,\{b\}),(p_2,\{b\})\}$. The soft sets in $\{\widetilde{X},\widetilde{\varphi},(U_1,P),(U_2,P)\}$ are soft $\widetilde{\tau}_1\widetilde{\tau}_2$ -open. Thus $(X,\widetilde{\tau}_1,\widetilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\widetilde{D}_j$-space, but is not soft (1,2)*-$\widetilde{D}_j$-space, $j=0,1,2$.

Theorem (3.10): A soft bitopological space $(X,\widetilde{\tau}_1,\widetilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\widetilde{D}_j$-space (resp. soft (1,2)*-α-ω-$\widetilde{D}_j$-space, soft (1,2)* -pre-ω-$\widetilde{D}_j$-space, soft (1,2)*-b-ω-$\widetilde{D}_j$-space, soft (1,2)*-β-ω-$\widetilde{D}_j$-space) if and only if it is a soft (1,2)*-ω-$\widetilde{T}_j$-space (resp. soft (1,2)*-α-ω-$\widetilde{T}_j$-space, soft (1,2)*-pre-ω-$\widetilde{T}_j$-space, soft (1,2)*-b-ω-$\widetilde{T}_j$-space, soft (1,2)*-β-ω-$\widetilde{T}_j$-space), for $j=0,1$.

***Proof*:** Follows from proposition (1.14) and theorem (3.4), no. (i).

Theorem (3.11): A soft bitopological space $(X,\widetilde{\tau}_1,\widetilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\widetilde{D}_1$-space (resp. soft (1,2)*-α-ω-$\widetilde{D}_1$-space, soft (1,2)* -pre-ω-$\widetilde{D}_1$-space, soft (1,2)*-b-ω-$\widetilde{D}_1$-space, soft (1,2)*-β-ω-$\widetilde{D}_1$-space) if and only if it is a soft (1,2)*-ω-$\widetilde{D}_2$-space (resp. soft (1,2)*-α-ω-$\widetilde{D}_2$-space, soft (1,2)*-pre-ω-$\widetilde{D}_2$-space, soft (1,2)*-b-ω-$\widetilde{D}_2$-space, soft (1,2)*-β-ω-$\widetilde{D}_2$-space).

***Proof*:** Sufficiency. Follows from theorem (3.4), no. (ii).

Necessity. Let $\widetilde{x},\widetilde{y}\widetilde{\in}\widetilde{X}$ such that $\widetilde{x}\neq\widetilde{y}$. Since $(X,\widetilde{\tau}_1,\widetilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\widetilde{D}_1$-space, then there exists soft (1,2)*-$\widetilde{D}_\omega$-sets (U,P) and (V,P) in $\widetilde{X}$ such that $\widetilde{x}\widetilde{\in}(U,P)$, $\widetilde{y}\widetilde{\notin}(U,P)$ and $\widetilde{y}\widetilde{\in}(V,P)$, $\widetilde{x}\widetilde{\notin}(V,P)$. Let $(U,P)=(U_1,P)\setminus(U_2,P)$ and $(V,P)=$

$(U_3,P)\setminus(U_4,P)$, where $(U_1,P),(U_2,P),(U_3,P),(U_4,P)$ are soft (1,2)*-ω-open sets in $\tilde{X}$ and $(U_1,P)\neq\tilde{X}$, $(U_3,P)\neq\tilde{X}$. By $\tilde{x}\tilde{\notin}(V,P)$ we have two cases:

(i) $\tilde{x}\tilde{\notin}(U_3,P)$

(ii) $\tilde{x}\tilde{\in}(U_3,P)$ and $\tilde{x}\tilde{\in}(U_4,P)$.

In case (i): $\tilde{x}\tilde{\notin}(U_3,P)$. By $\tilde{y}\tilde{\notin}(U,P)$ we have two subcases:

(a) $\tilde{y}\tilde{\in}(U_1,P)$ and $\tilde{y}\tilde{\in}(U_2,P)$

(b) $\tilde{y}\tilde{\notin}(U_1,P)$.

Subcase (a): $\tilde{y}\tilde{\in}(U_1,P)$ and $\tilde{y}\tilde{\in}(U_2,P)$. We have $\tilde{x}\tilde{\in}(U_1,P)\setminus(U_2,P)$, $\tilde{y}\tilde{\in}(U_2,P)$ and $((U_1,P)\setminus(U_2,P))\tilde{\cap}(U_2,P)=\tilde{\varphi}$. Observe that $(U_2,P)\neq\tilde{X}$ since $(U,P)\neq\tilde{\varphi}$, thus by Remarks (2.3), (U_2,P) is a soft (1,2)*-$\tilde{D}_\omega$-set.

Subcase (b): $\tilde{y}\tilde{\notin}(U_1,P)$. Since $\tilde{x}\tilde{\in}(U_1,P)\setminus(U_2,P)$ and $\tilde{x}\tilde{\notin}(U_3,P)$, then $\tilde{x}\tilde{\in}(U_1,P)\setminus((U_2,P)\tilde{\cup}(U_3,P))$. Since $\tilde{y}\tilde{\in}(U_3,P)\setminus(U_4,P)$ and $\tilde{y}\tilde{\notin}(U_1,P)$, then $\tilde{y}\tilde{\in}(U_3,P)\setminus((U_4,P)\tilde{\cup}(U_1,P))$. Observe that $(U_2,P)\tilde{\cup}(U_3,P)$ and $(U_4,P)\tilde{\cup}(U_1,P)$ are soft (1,2)*-ω-open sets in $\tilde{X}$. Hence $\tilde{x}\tilde{\in}(U_1,P)\setminus((U_2,P)\tilde{\cup}(U_3,P))$, $\tilde{y}\tilde{\in}(U_3,P)\setminus((U_4,P)\tilde{\cup}(U_1,P))$ and $(U_1,P)\setminus((U_2,P)\tilde{\cup}(U_3,P))$ $\tilde{\cap}$ $(U_3,P)\setminus((U_4,P)\tilde{\cup}(U_1,P))=\tilde{\varphi}$.

In case (ii): $\tilde{x}\tilde{\in}(U_3,P)$ and $\tilde{x}\tilde{\in}(U_4,P)$. We have $\tilde{y}\tilde{\in}(U_3,P)\setminus(U_4,P)$, $\tilde{x}\tilde{\in}(U_4,P)$ and $((U_3,P)\setminus(U_4,P))\tilde{\cap}(U_4,P)=\tilde{\varphi}$. Observe that $(U_4,P)\neq\tilde{X}$ since $(V,P)\neq\tilde{\varphi}$, thus by Remarks (2.3), (U_4,P) is a soft (1,2)*-$\tilde{D}_\omega$-set. Hence $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{D}_2$-space. Similarly, we can prove other cases.

Proposition (3.12):

(i) Every soft (1,2)*-α-ω-$\tilde{D}_j$-space is a soft (1,2)*-ω-$\tilde{D}_j$-space, $j=0,1,2$.

(ii) Every soft (1,2)*-pre-ω-$\tilde{D}_j$-space is a soft (1,2)*-α-ω-$\tilde{D}_j$-space, $j=0,1,2$.

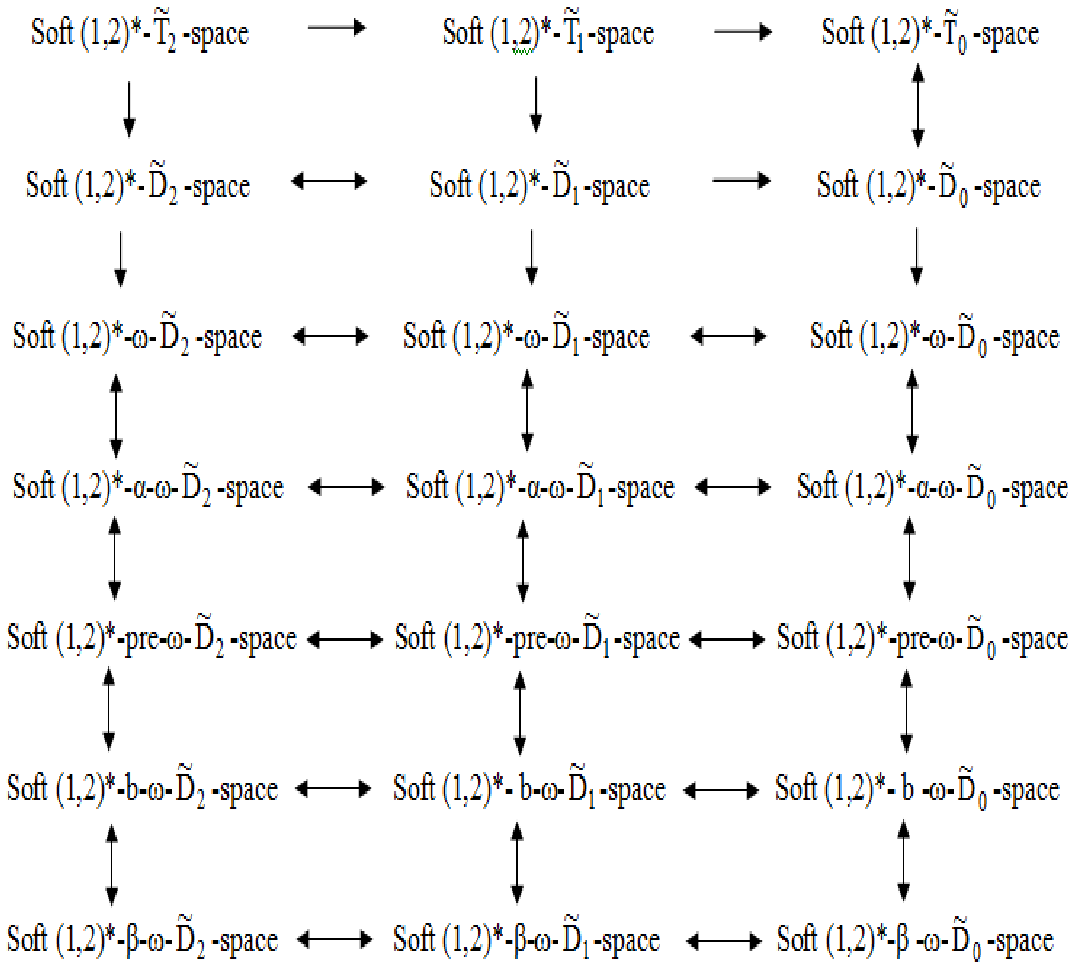
(iii) Every soft (1,2)*-b-ω-$\tilde{D}_j$-space is a soft (1,2)*-pre-ω-$\tilde{D}_j$-space, $j=0,1,2$.

(iv) Every soft (1,2)*-β-ω-$\tilde{D}_j$-space is a soft (1,2)*-b-ω-$\tilde{D}_j$-space, $j=0,1,2$.

***Proof*: (i)** If $j=0$, then let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since $\tilde{X}-\{\tilde{x}\}$ is a soft (1,2)*-ω-open set in $\tilde{X}$, then by Remark (2.3), $\tilde{X}-\{\tilde{x}\}$ is a soft (1,2)*-$\tilde{D}_\omega$-set in $\tilde{X}$ which contains $\tilde{y}$, but not $\tilde{x}$. Hence $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{D}_0$-space. If $j=1$, then let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since $\tilde{X}-\{\tilde{x}\}$ and $\tilde{X}-\{\tilde{y}\}$ are soft (1,2)*-ω-open sets in $\tilde{X}$, then by Remark (2.3), $\tilde{X}-\{\tilde{x}\}$ and $\tilde{X}-\{\tilde{y}\}$ are soft (1,2)*-$\tilde{D}_\omega$-sets in $\tilde{X}$ such that $\tilde{X}-\{\tilde{y}\}$ containing $\tilde{x}$, but not $\tilde{y}$ and $\tilde{X}-\{\tilde{x}\}$ containing $\tilde{y}$, but not $\tilde{x}$. Therefore $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{D}_1$-space. If $j=2$, then let $\tilde{x},\tilde{y}\tilde{\in}\tilde{X}$ such that $\tilde{x}\neq\tilde{y}$. Since $\{\tilde{x}\}=(\tilde{X}-\{\tilde{y}\})\setminus(\tilde{X}-\{\tilde{x}\})$ and $\{\tilde{y}\}=(\tilde{X}-\{\tilde{x}\})\setminus(\tilde{X}-\{\tilde{y}\})$ are disjoint soft (1,2)*-$\tilde{D}_\omega$-sets in $\tilde{X}$ containing $\tilde{x}$ and $\tilde{y}$ respectively, therefore $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{D}_2$-space.

(ii),(iii), and (iv) similar to (i).

The following diagram show the relations between the soft (1,2)*-$\tilde{\mathbf{T}}_{\mathbf{j}}$-spaces and each of soft (1,2)*-$\tilde{\mathbf{D}}_{\mathbf{j}}$-spaces, soft (1,2)*-ω-$\tilde{\mathbf{D}}_{\mathbf{j}}$-spaces, soft (1,2)*-α-ω-$\tilde{\mathbf{D}}_{\mathbf{j}}$-spaces, soft (1,2)*-pre-ω-$\tilde{\mathbf{D}}_{\mathbf{j}}$-spaces, soft (1,2)*-b-ω-$\tilde{\mathbf{D}}_{\mathbf{j}}$-spaces, and soft (1,2)* -β-ω-$\tilde{\mathbf{D}}_{\mathbf{j}}$-spaces, for $\mathbf{j}=0,1,2$.



Definition (3.13): Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space. A soft point $\tilde{x}\tilde{\in}\tilde{X}$ which has $\tilde{X}$ as the only soft (1,2)*-ω-neighborhood (resp. soft (1,2)*-α-ω-neighborhood, soft (1,2)*-pre-ω-neighborhood, soft (1,2)*-b-ω-neighborhood, soft (1,2)*-β-ω-neighborhood) is called a soft (1,2)*-ω-neat (resp. soft (1,2)*-α-ω-neat, soft (1,2)*-pre-ω-neat, soft (1,2)*-b-ω-neat, soft (1,2)*-β-ω-neat) point.

Theorem (3.14): Let $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ be a soft bitopological space, then the following are equivalent:

(i) $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{D}_1$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_1$-space, soft (1,2)*-pre-ω-$\tilde{D}_1$-space, soft (1,2)*-b-ω-$\tilde{D}_1$-space, soft (1,2)*-β-ω-$\tilde{D}_1$-space).

(ii) $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ has no soft (1,2)*-ω-neat (resp. soft (1,2)*-α-ω-neat, soft (1,2)*-pre-ω-neat, soft (1,2)*-b-ω-neat, soft (1,2)*-β-ω-neat) point.

***Proof*:**

(i)$\Rightarrow$(ii). Since $(X,\tilde{\tau}_1,\tilde{\tau}_2,P)$ is a soft (1,2)*-ω-$\tilde{D}_1$-space, then each soft point $\tilde{x}\tilde{\in}\tilde{X}$ is contained in a soft (1,2)*-$\tilde{D}_\omega$-set $(U,P)=(U_1,P)\setminus(U_2,P)$, where (U_1,P) and (U_2,P) are soft (1,2)*-ω-open sets and thus in (U_1,P). By definition (2.1), $(U_1,P)\neq\tilde{X}$. This implies that $\tilde{x}$ is not a soft (1,2)*-ω-neat point.

(ii)$\Rightarrow$(i). Follows from proposition (1.14) and theorem (3.4), no. (i).

Theorem (3.15):

Let $f:(X,\tilde{\tau}_1,\tilde{\tau}_2,P)\to(Y,\tilde{\sigma}_1,\tilde{\sigma}_2,P)$ be a strongly soft (1,2)*-ω-continuous (resp. strongly soft (1,2)*-α-ω-continuous, strongly soft (1,2)*-pre-ω-continuous, strongly soft (1,2)*-b-ω-continuous, strongly soft (1,2)*-β-ω-continuous) bijective function. If $\tilde{Y}$ is a soft (1,2)*-ω-$\tilde{D}_j$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_j$-space, soft (1,2)*-pre-ω-$\tilde{D}_j$-space, soft (1,2)*-

b-ω-$\tilde{D}_j$-space, soft (1,2)*-β-ω-$\tilde{D}_j$-space), then $\tilde{X}$ is a soft (1,2)*-$\tilde{D}_j$-space, $j = 0,1,2$.

***Proof*:** Suppose that $(Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)$ is a soft (1,2)*-ω-$\tilde{D}_2$-space. Let $\tilde{x}_1, \tilde{x}_2 \tilde{\in} \tilde{X}$ such that $\tilde{x}_1 \neq \tilde{x}_2$. Since f is injective and $(Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)$ is a soft (1,2)*-ω-$\tilde{D}_2$-space, then there exists disjoint soft (1,2)*-D_ω-sets (A_1, P) and (A_2, P) in $\tilde{Y}$ such that $f(\tilde{x}_1) \tilde{\in} (A_1, P)$ and $f(\tilde{x}_2) \tilde{\in} (A_2, P)$. By theorem (2.11), $f^{-1}((A_1, P))$ and $f^{-1}((A_2, P))$ are soft (1,2)*-$\tilde{D}$-sets in $\tilde{X}$. Since $\tilde{x}_1 \tilde{\in} f^{-1}((A_1, P))$, $\tilde{x}_2 \tilde{\in} f^{-1}((A_2, P))$ and $f^{-1}((A_1, P)) \tilde{\cap} f^{-1}((A_2, P)) = \tilde{\phi}$. Thus $(X, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-$\tilde{D}_2$-space. Similarly, we can prove other cases.

Theorem (3.16):

Let $f : (X, \tilde{\tau}_1, \tilde{\tau}_2, P) \to (Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)$ be a strongly soft (1,2)*-ω-open (resp. strongly soft (1,2)*-α-ω-open, strongly soft (1,2)*-pre-ω-open, strongly soft (1,2)*-b-ω-open, strongly soft (1,2)*-β-ω-open) bijective function. If $\tilde{X}$ is a soft (1,2)*-ω-$\tilde{D}_j$-space (resp. soft (1,2)*-α-ω-$\tilde{D}_j$-space, soft (1,2)*-pre-ω-$\tilde{D}_j$-space, soft (1,2)*-b-ω-$\tilde{D}_j$-space , soft (1,2)*-β-ω-$\tilde{D}_j$-space), then $\tilde{Y}$ is a soft (1,2)*-$\tilde{D}_j$-space, $j = 0,1,2$.

***Proof*:** Suppose that $(X, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{D}_2$-space. Let $\tilde{y}_1, \tilde{y}_2 \tilde{\in} \tilde{Y}$ such that $\tilde{y}_1 \neq \tilde{y}_2$. Since f is surjective, then there exists $\tilde{x}_1, \tilde{x}_2 \tilde{\in} \tilde{X}$ such that $f(\tilde{x}_1) = \tilde{y}_1$ and $f(\tilde{x}_2) = \tilde{y}_2$. But f is a soft function, then $\tilde{x}_1 \neq \tilde{x}_2$, since $(X, \tilde{\tau}_1, \tilde{\tau}_2, P)$ is a soft (1,2)*-ω-$\tilde{D}_2$-space, then there exists disjoint soft (1,2)*-$\tilde{D}_\omega$-sets (A_1, P) and (A_2, P) in $\tilde{X}$ such that $\tilde{x}_1 \tilde{\in} (A_1, P)$ and $\tilde{x}_2 \tilde{\in} (A_2, P)$. By theorem (2.12), $f((A_1, P))$ and $f((A_2, P))$ are soft (1,2)*-$\tilde{D}$-sets in $\tilde{Y}$ such that $f(\tilde{x}_1) = \tilde{y}_1 \tilde{\in} f((A_1, P))$ and $f(\tilde{x}_2) = \tilde{y}_2 \tilde{\in} f((A_2, P))$. Since f is injective, then $f((A_1, P)) \tilde{\cap} f((A_2, P)) = \tilde{\phi}$. Hence $(Y, \tilde{\sigma}_1, \tilde{\sigma}_2, P)$ is a soft (1,2)*-$\tilde{D}_2$-space. Similarly, we can prove other cases.

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