Special Issus: 1st Scientific International Conference, College of Science, Al-Nahrain University, 21-22/11/2017, Part I, pp.150-156

The Generalized Backstepping Control Method for Stabilizing and Solving Systems of Multiple Delay Differential Equations

Fadhel S. Fadhel [1*] and Saja F. Noaman [2**]
[1*] Department of Mathematics and Computer Applications, College of Science, Al-Nahrain University, Baghdad-Iraq.
[2] M.Sc. Student, Department of Mathematics and Computer Applications, College of Science, Al-Nahrain University, Baghdad-Iraq.
[*] Corresponding author: fsf@sc.nahrainuniv.edu.iq , dr_fadhel67@yahoo.com
[**] Corresponding author: saja89faik@yahoo.com

Abstract

In this paper, a generalized approach of the backstepping method for stabilizing and solving system of ordinary differential equations will be applied for systems of differential equations with multiple delay by transforming the non-linear system of delay differential equations into a system of ordinary differential equations with cooperation of the method of steps. The basic idea of this approach is to find Lyapunov function for stabilizing the system of different time steps which is chosen as the definitions of the present approach. [DOI: 10.22401/ANJS.00.1.20]

Keyword: Backstepping control method, Delay differential equations, Boundary control problems, Method of steps, Stabilization of delay differential equations, Lyapunov functions.

1. Introduction

Over the last few decades, the focus have been carried over the areas of control theory and engineering control which has shifted from linear to nonlinear systems providing control algorithms for systems that are both more general and more realistic. Therefore, during recent years nonlinear controls has shown a strong presence in the academic curricula, industry and conferences, [6].

As it is known in literatures, the study of nonlinear systems is different than linear systems and hence linearization approach may be used to study the behavior of nonlinear system, but there are two basic limitations of linearization; the first of which since the linearization is an approximation in a neighborhood of an operating point and then it can only predict the "local" behavior of the nonlinear system in the vicinity of that point. It cannot predict the “nonlocal” behavior so far from the operating point and certainly not the "global" behavior throughout the state space. The second limitation is that the dynamics of nonlinear systems are much richer than the dynamics of the linear system, [2].

The backstepping method technique is based directly on the mathematical model governing the real life examined system. It is being developed by introducing new variables into it by using certain transformation techniques in a form depending on the state variables, controlling parameters and stabilizing functions. Those compensate exists for nonlinearities in the system, [12]. The advantage of the backstepping method is that it has the availability to avoid cancellations of useful nonlinearities appearing in the system and pursue the objectives of stabilization and tracking rather than that of linearization method. For tracking problem, backstepping always use the error between the actual and desired input to start the design process, [11].

The backstepping method that is adapted to the problem of globally and uniformly asymptotically stabilizing nonlinear systems in feedback form with a delay arbitrarily large in the input. The strategy of the design relies on the construction of a Lyapunov-Krasovskii functional which was given in 2003, [7], Krstic and Smyshlyaev in 2008 [5], where they developed the backstepping approach for the first-order hyperbolic PDEs and presented a design for the linear time invariant ordinary differential equations (ODEs) with time delays, these recovers the classical predictor designs for the finite spectrum assignment. Also, Mazenc and Ito in 2011, [8] worked on developing a technique to carry out a backstepping design of stabilizing control laws for a family of neutral nonlinear delayed systems.

Stabilization and robustness analysis for nonlinear control systems with input or state delays is a challenging field that has been addressed by many authors in a large number of their works. In some applications including network control, population dynamics, biological systems to cite only a few [9], when the delays are small enough that one can ignore them and still be assured of satisfactory performance of the controllers. However, there are many applications when the delays are too large to disregard. For instance, in bioreactors, there is often a long time lag between the time of the organisms in the fed bioreactor and the time taken by organisms to grow. One can often model bioreactors by control systems, where the food entering the bioreactor is the control input and the numbers of organisms in the reactor are the states of the system, and then we have a control system with long input delays, [3].

In the present paper, we will introduce and propose a generalized approach of backstepping control method for a systems of delay differential equations (DDEs) given in, [1]. This approach has its basic idea on transforming the DDEs into a system of ordinary differential equations (ODEs) in cooperation with a known method for solving DDEs, which is the method of steps and using the backstepping method to solve the resulting system which make our original system stable. This approach is more easy and powerful than other approaches.

2. The Generalized Backstepping Control Method

Consider the generalized nth order dynamical system, [10]:

$$\left.\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \dots, x_n) + u_1 \\ \dot{x}_2 &= f_2(x_1, x_2, \dots, x_n) + u_2 \\ \dot{x}_3 &= f_3(x_1, x_2, \dots, x_n) + u_3 \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \dots, x_n) + u_n \end{aligned}\right\} \quad \dots\dots (1)$$

where $x(t) \epsilon R^n$ is the state vector of the system, $f_i, i = 1,2, \dots, n$ are either linear or nonlinear functions and $u_i, i = 1,2, \dots, n$ are the controller input.

Backstepping control design of systems (1) is recursive method, which will guarantee the global asymptotic stable performance of the system. By using the backstepping control design at the i^{th} step, the i^{th} subsystem may be stabilized with respect to a certain Lyapunov function V_i, $i = 1,2, \dots, n$ which are designed, in addition to the design of the virtual controls α_i, $i = 1,2, \dots, n$ and a control input functions u_i,$i = 1,2, \dots, n$ that to make system (1) converge to zero with time increasing.

The analysis of this method may be summarized as in the following steps:

Step (1):

Consider the stability of the first equation of system (1)

$$\dot{x}_1 = f_1(x_1, x_2, \dots, x_n) + u_1 \quad \dots\dots (2)$$

Where x_2 is regarded as a virtual controller, then define $x_1 = z_1$and derive the dynamics of the new coordinates as:

$$\dot{z}_1 = \dot{x}_1 = f_1(z_1, x_2, \dots, x_n) + u_1 \quad \dots\dots (3)$$

Now, construct the first Lyapunov function in quadratic form as:

$$V_1 = {z_1}^T p_1 z_1 \quad \dots\dots (4)$$

The derivative of V_1 is

$$\dot{V}_1 = -{z_1}^T Q_1 z_1 < 0 \quad \dots\dots (5)$$

where Q_1 is a positive definite matrix. Then $\dot{V}_1$ is a negative definite function in R^n and thus by Lyapunov stability theory, system (2) is asymptotically stable. Clear that the virtual control $x_2 = \alpha_1(z_1)$ and the state feedback input u_1, which makes system (2) asymptotically stable. The function $\alpha_1(z_1)$ should be estimated while z_2 is considered as a controller.

Step (2):

Define the error between z_2 and $\alpha_1(z_1)$ to be defined as:

$$z_2 = x_2 - \alpha_1(z_1) \quad \dots\dots (6)$$

Then consider the $z_1, z_2 -$ subsystem:

$$\begin{aligned} \dot{z}_1 &= f_1(z_1, x_2, x_3 \dots, x_n) + u_1 \\ \dot{z}_2 &= f_1(z_1, z_2, x_3 \dots, x_n) - \dot{\alpha}_1(z_1) + u_2 \end{aligned} \quad \dots(7)$$

where x_3 is a virtual controller of subsystem (7), and assume that it is equal to $\alpha_1(z_1, z_2)$ which makes the subsystem (7) asymptotically stable. Consider the Lyapunov function defined by:

$$V_2(z_1, z_2) = V_1(z_1) + {z_2}^T p_2 z_2 \quad \dots\dots (8)$$

Therefore, the derivative of $V_2$is:

$$\dot{V}_2(z_1, z_2) = -{z_1}^T Q_1 z_1 - {z_2}^T Q_2 z_2 < 0 \dots (9)$$

where Q_1,Q_2 are positive definite matrices. Then $\dot{V}_2$ is a negative definite function in R^n and thus by Lyapunov stability theory, subsystem (7) is asymptotically stable. Similarly, the virtual control $x_3 = \alpha_2(z_1, z_2)$ may be defined and the state feedback input u_2 makes subsystem (7) asymptotically stable.

Step (n):

So on, proceeding similarly as in steps (1) and (2), define the error variable z_n by:

$$z_n = x_n - \alpha_{n-1}(z_1, z_2, \dots, z_n) \quad \text{.................} (10)$$

and consider the $z_1, z_2, \dots, z_n$-subsystem given by:

$$\left.\begin{array}{l} \dot{z}_1 = f_1(z_1, x_2, x_3 \dots, x_n) + u_1 \\ \dot{z}_2 = f_1(z_1, z_2, x_3 \dots, x_n) - \dot{\alpha}_1(z_1) + u_2 \\ \dot{z}_3 = f_3(z_1, z_2, z_3, \dots, x_n) - \dot{\alpha}_2(z_1, z_2) + u_3 \\ \vdots \\ \dot{z}_n = f_n(z_1, z_2, \dots, z_n) - \dot{\alpha}_{n-1}(z_1, z_2, \dots, z_n) + u_n \end{array}\right\} \quad \text{................................} (11)$$

Therefore, the Lyapunov function is defined by:

$$V_n(z_1, z_2, \dots, z_n) = V_{n-1}(z_1, z_2, \dots, z_{n-1}) + {z_n}^T p_n z_n \quad \text{..} (12)$$

Hence, the derivative of V_n is:

$$\dot{V}_n = -{z_1}^T Q_1 z_1 - {z_2}^T Q_2 z_2 - \dots - {z_n}^T Q_n z_n < 0 \quad \text{................................} (13)$$

where Q_1,Q_2,Q_3, … ,Q_n are positive definite matrices. Then $\dot{V}_n$ is a negative definite function in $R^n$and similarly by Lyapunov stability theory the subsystem (11) is asymptotically stable. The virtual control $x_n = \alpha_{n-1}(z_1, z_2, \dots, z_{n-1})$ and the state feedback input u_n are then evaluated which makes subsystem (11) is asymptotically stable. Thus as a result of the above steps, system (1) is globally exponentially stable for all initial conditions $x_i(0) \epsilon R^n$.

3. General Strict-Feedback Delay System

The nonlinear multiple delay systems that will be considered in this paper has the form, [4]:

$$\left.\begin{array}{l} \dot{x}_1(t) = f_1(t, x_1(t), x_1(t-\tau_1), \dot{x}_1(t-\tau_1), \dots, x_1(t-\tau_m), \\ \quad \dot{x}_1(t-\tau_m), x_2(t), x_2(t-\tau_1), \dot{x}_2(t-\tau_1), \dots, \\ \quad x_2(t-\tau_m), \dot{x}_2(t-\tau_m), \dots, x_n(t-\tau_m), \dot{x}_n(t- \\ \quad \tau_m)) + u_1(t) \\ \dot{x}_2(t) = f_2(t, x_1(t), x_1(t-\tau_1), \dot{x}_1(t-\tau_1), \dots, x_1(t-\tau_m), \\ \quad \dot{x}_1(t-\tau_m), x_2(t), x_2(t-\tau_1), \dot{x}_2(t-\tau_1), \dots, \\ \quad x_2(t-\tau_m), \dot{x}_2(t-\tau_m), \dots, x_n(t-\tau_m), \dot{x}_n(t- \\ \quad \tau_m)) + u_2(t) \\ \quad \vdots \\ \dot{x}_n(t) = f_n(t, x_1(t), x_1(t-\tau_1), \dot{x}_1(t-\tau_1), \dots, x_1(t-\tau_m), \\ \quad \dot{x}_1(t-\tau_m), x_2(t), x_2(t-\tau_1), \dot{x}_2(t-\tau_1), \dots, \\ \quad x_2(t-\tau_m), \dot{x}_2(t-\tau_m), \dots, x_n(t-\tau_m), \dot{x}_n(t- \\ \quad \tau_m)) + u_n(t) \end{array}\right\} \quad \text{...........} (14)$$

$t \geq t_0$; with initial conditions:

$$x_1(t) = \varphi_{10}(t), x_2(t) = \varphi_{20}(t), \dots, x_n(t) = \varphi_{n0}(t), \quad \text{for} \quad t_0 - \tau_1 \leq t \leq t_0$$

$$x_1(t) = \varphi_{11}(t), x_2(t) = \varphi_{21}(t), \dots, x_n(t) = \varphi_{n1}(t), \quad \text{for} \quad t_0 - \tau_2 \leq t \leq t_0 - \tau_1$$

$$\vdots$$

$$x_1(t) = \varphi_{1n}(t), x_2(t) = \varphi_{2n}(t), \dots, x_n(t) = \varphi_{nn}(t), \quad \text{for} \quad t_0 - \tau_m \leq t \leq t_0 - \tau_{m-1}$$

where x_i, $i = 1, \dots, n$ are the system state, $\tau_i \geq 0$, $i = 1,2, \dots, m$ are the multiple time delay which are supposed to be of ascending order and $m \leq n$, f_i are continuous functions, u_i are the control function and $\varphi_{ij}(t)$ are continuous functions over time $t, i = 1,2, \dots, n, j = 0,1,2, \dots, n$.

Hence in connection with the method of steps, we have the initial conditions which are given for a time step intervals with length equal to $\tau_i = min\{\tau_i\}_{i=1,2,..,m}$ and then to find the solution for all $t \geq t_0$ divided into steps with length τ_i.

For the first time step $[t_0, t_0 + \tau_1]$, we have:

$$\begin{array}{l} \dot{x}_1(t) = f_1(t, x_1(t), \varphi_{10}(t-\tau_1), \dot{\varphi}_{10}(t-\tau_1), \dots, \\ \quad \varphi_{1n}(t-\tau_m), \dot{\varphi}_{1n}(t-\tau_m), x_2(t), \\ \quad \varphi_{20}(t-\tau_1), \dot{\varphi}_{20}(t-\tau_1), \dots, \\ \quad \varphi_{2n}(t-\tau_m), \dot{\varphi}_{2n}(t-\tau_m), \dots, \\ \quad \varphi_{nn}(t-\tau_m), \dot{\varphi}_{nn}(t-\tau_m)) + u_1(t) \\ \dot{x}_2(t) = f_2(t, x_1(t), \varphi_{10}(t-\tau_1), \dot{\varphi}_{10}(t-\tau_1), \dots, \\ \quad \varphi_{1n}(t-\tau_m), \dot{\varphi}_{1n}(t-\tau_m), x_2(t) \\ \quad \varphi_{20}(t-\tau_1), \dot{\varphi}_{20}(t-\tau_1), \dots, \\ \quad \varphi_{2n}(t-\tau_m), \dot{\varphi}_{2n}(t-\tau_m), \dots, \\ \quad \varphi_{nn}(t-\tau_m), \dot{\varphi}_{nn}(t-\tau_m)) + u_2(t) \end{array}$$

$\dot{x}_n(t) = f_n(t, x_1(t), \varphi_{10}(t-\tau_1), \dot{\varphi}_{10}(t-\tau_1), \ldots,$
$\varphi_{1n}(t-\tau_m), \dot{\varphi}_{1n}(t-\tau_m), x_2(t),$
$\varphi_{20}(t-\tau_1), \dot{\varphi}_{20}(t-\tau_1), \ldots,$
$\varphi_{2n}(t-\tau_m), \dot{\varphi}_{2n}(t-\tau_m), \ldots,$
$\varphi_{nn}(t-\tau_m), \dot{\varphi}_{nn}(t-\tau_m)) + u_n(t)$
.... (15)

Hence, the resulting system of differential equations will be solved by using the backstepping control method, as it is discussed previously. The solution of the obtained system of ODEs may be found by any classical approximate or numerical method for solving systems of non-linear ODEs and solving the resulting system may produce a discrete set numerical results that may be interpolated using Lagrange interpolation polynomials as the initial conditions for the next time steps, i.e.,

$\dot{x}_1(t) = f_1(t, x_1(t), x_1(t-\tau_1), \dot{x}_1(t-\tau_1), \ldots,$
$x_1(t-\tau_m), \dot{x}_1(t-\tau_m), x_2(t), x_2(t-\tau_1),$
$\dot{x}_2(t-\tau_1), \ldots, x_2(t-\tau_m), \dot{x}_2(t-\tau_m), \ldots,$
$x_n(t-\tau_m), \dot{x}_n(t-\tau_m)) + u_1(t)$
$\dot{x}_2(t) = f_2(t, x_1(t), x_1(t-\tau_1), \dot{x}_1(t-\tau_1), \ldots,$
$x_1(t-\tau_m), \dot{x}_1(t-\tau_m), x_2(t), x_2(t-\tau_1),$
$\dot{x}_2(t-\tau_1), \ldots, x_2(t-\tau_m), \dot{x}_2(t-\tau_m), \ldots,$
$x_n(t-\tau_m), \dot{x}_n(t-\tau_m)) + u_2(t)$
$\dot{x}_n(t) = f_n(t, x_1(t), x_1(t-\tau_1), \dot{x}_1(t-\tau_1), \ldots,$
$x_1(t-\tau_m), \dot{x}_1(t-\tau_m), x_2(t), x_2(t-\tau_1),$
$\dot{x}_2(t-\tau_1), \ldots, x_2(t-\tau_m), \dot{x}_2(t-\tau_m), \ldots,$
$x_n(t-\tau_m), \dot{x}_n(t-\tau_m)) + u_n(t)$
.... (16)

with the initial conditions:
$x_1(t) = x_{11}(t), x_2(t) = x_{21}(t), \ldots,$
$x_n(t) = x_{n1}(t)$ for $t_0 \le t \le t_0 + \tau_1$
$x_1(t) = \varphi_{10}(t), x_2(t) = \varphi_{20}(t), \ldots,$
$x_n(t) = \varphi_{n0}(t)$ for $t_0 - \tau_1 \le t \le t_0$
$\vdots$
$x_1(t) = \varphi_{1n-1}(t), x_2(t) = \varphi_{2n-1}(t), \ldots,$
$x_n(t) = \varphi_{nn-1}(t)$
for $t_0 - \tau_{m-1} \le t \le t_0 - \tau_{m-2}$
where $x_{i1}(t) = x_i(t)$, $\forall\, i = 1,2,\ldots,n$ are the interpolation functions of the previous solutions of time step interval.

Again, applying the backstepping control method to stabilize and solve the system of DDEs for the next time step, and so on proceed to the next time steps.

4. Illustrative Example

Consider the following nonlinear system of multiple DDEs for all $t \geq 0$:

$$\left.\begin{aligned} \dot{x}_1(t) &= x_1(t)x_1(t-1) + x_2(t-2) + u_1(t) \\ \dot{x}_2(t) &= x_2(t)\dot{x}_2(t-2) - 2x_1(t)x_3(t-3) + u_2(t) \\ \dot{x}_3(t) &= x_3(t)x_3(t-1) + x_2^2(t)\dot{x}_1(t-3) + u_3(t) \end{aligned}\right\} \quad \text{........... (17)}$$

with the initial conditions:
$x_1(t) = \varphi_{10}(t) = 4t + 2$, $x_2(t) = \varphi_{20}(t) = 2t + 1$, $x_3(t) = \varphi_{30}(t) = t - 1$, for $-1 \le t \le 0$
$x_1(t) = \varphi_{11}(t) = 2t$, $x_2(t) = \varphi_{21}(t) = 4t + 3$, $x_3(t) = \varphi_{31}(t) = 2t$, for $-2 \le t \le -1$
$x_1(t) = \varphi_{12}(t) = t - 2$, $x_2(t) = \varphi_{22}(t) = 2t - 1$, $x_3(t) = \varphi_{32}(t) = 3t + 2$, for $-3 \le t \le -2$

To find the solution over the first time-step interval [0, 1], apply the method of steps, will give:
$\dot{x}_1(t) = x_1(t)\varphi_{10}(t-1) + \varphi_{21}(t-2) + u_1(t)$
$\dot{x}_2(t) = x_2(t)\dot{\varphi}_{21}(t-2) - 2x_1(t)\varphi_{32}(t-3) + u_2(t)$
$\dot{x}_3(t) = x_3(t)\varphi_{30}(t-1) + x_2^2(t)\dot{\varphi}_{12}(t-3) + u_3(t)$
So, the resulting system of ODEs has the form:

$$\left.\begin{aligned} \dot{x}_1(t) &= (4t-2)x_1(t) + (4t-5) + u_1(t) \\ \dot{x}_2(t) &= 4x_2(t) - 2(3t-7)x_1(t) + u_2(t) \\ \dot{x}_3(t) &= (t-2)x_3(t) + x_2^2(t) + u_3(t) \end{aligned}\right\} \quad \text{.......... (18)}$$

Now, the resulting system of ODEs with non-constant coefficients (18) may be stabilized and solved by backstepping method, as in the following steps:

Step (1):

In the first step, consider the stability of the first equation of system (18) and let $z_1(t) = x_1(t)$ with its derivative:
$\dot{z}_1(t) = \dot{x}_1(t) = (4t-2)z_1(t) + (4t-5) + u_1(t)$
where $x_2(t)$ is regarded as the virtual controller. The first control Lyapunov function is considered to be:
$V_1 = \frac{1}{2} z_1^2(t)$
and the time derivative of which becomes:
$\dot{V}_1 = z_1(t)((4t-2)z_1(t) + (4t-5) + u_1(t))$
$= -(2+4t)z_1^2(t) + z_1(t)(8tz_1(t) + (4t-5) + u_1(t))$

Assume the controller $x_2(t) = \alpha_1(z_1)$. If $u_1(t) = -8tz_1(t) - (4t-5)$ and $\alpha_1(z_1) = 0$, then:
$\dot{V}_1 = -(2+4t)z_1^2(t)$
which is negative definite function. The recursive feedback control $u_1(t)$ and $\alpha_1(z_1)$ makes the first equation of system (18) asymptotically stable, where $\alpha_1(z_1)$ is an estimating function when $x_2(t)$ is considered as a controller.

Step (2):

Define the error between $x_2(\text{t})$ and $\alpha_1(z_1)$ as $z_2(t) = x_2(t) - \alpha_1(z_1)$. Then:

$\dot{z}_2(t) = 4z_2(t) - 2(3\text{t} - 7)x_1(t) + u_2(t)$

where $x_3(t)$ is regarded as the virtual controller. We consider the second control Lyapunov function is defined by:

$V_2 = V_1 + \frac{1}{2}z_2^2(t)$

and the time derivative of which becomes:

$$\dot{V}_2 = -(2+4t)z_1^2(t) + z_2(t)(4z_2(t) - 2(3\text{t}-7)x_1(t) + u_2(t))$$
$$= -(2+4t)z_1^2(t) - 4z_2^2(t) + z_2(t)(8z_2(t) - 2(3\text{t}-7)x_1(t) + u_2(t))$$

Assume the controller $x_3(t) = \alpha_2(z_1, z_2)$.

If $u_2(t) = -8z_2(t) + 2(3\text{t}-7)x_1(t)$ and $\alpha_2(z_1, z_2) = 0$, then:

$\dot{V}_2 = -(2+4t)z_1^2(t) - 4z_2^2(t)$

which is Also negative definite function. The recursive feedback control $u_2(\text{t})$ and $\alpha_2(z_1, z_2)$ makes the second equation of system (18) asymptotically stable, where $\alpha_2(z_1, z_2)$ is an estimating function when $x_3(\text{t})$ is considered as a controller.

Step (3):

Define the error between $x_3(t)$ and $\alpha_2(z_1, z_2)$ as $z_3(t) = x_3(t) - \alpha_2(z_1, z_2)$. Then:

$\dot{z}_3(t) = (t-2)z_3(t) + x_2^2(t) + u_3(t)$

Consider the control third control Lyapunov function defined by:

$V_3 = V_2 + \frac{1}{2}z_3^2(t)$

The derivative of V_3 is

$$\dot{V}_3 = -(2+4t)z_1^2(t) - 4z_2^2(t) + z_3(t)((t-2)z_3(t) + x_2^2(t) + u_3(t))$$
$$= -(2+4t)z_1^2(t) - 4z_2^2(t) - (2+t)z_3^2(t) + z_3(t)(2tz_3(t) + x_2^2(t) + u_3(t))$$

If $u_3(t) = -2\text{t}z_3(t) - x_2^2(t)$, then:

$$\dot{V}_3 = -(2+4t)z_1^2(t) - 4z_2^2(t) - (2+t)z_3^2(t)$$

which is negative definite function. The recursive feedback control $u_3(t)$ makes the third equation of system (18) asymptotically stable.

As a results of the above three steps, the following controller functions are obtained:

$u_1(t) = -8\text{t}x_1(t) - (4t-5)$
$u_2(t) = -8x_2(t) + 2(3\text{t}-7)x_1(t)$
$u_3(\text{t}) = -2\text{t}x_3(t) - x_2^2(t)$

Substitution $u_1(\text{t})$, $u_2(\text{t})$ and $u_3(\text{t})$ in system (18) will give the following linear system of ODEs:

$$\left.\begin{aligned} \dot{x}_1(t) &= -(2+4\text{t})x_1(t) \\ \dot{x}_2(t) &= -4x_2(t) \\ \dot{x}_3(t) &= -(2+\text{t})x_3(t) \end{aligned}\right\} \text{.....................(19)}$$

with initial conditions:

$x_1(0) = 2$, $x_2(0) = 1$, $x_3(0) = -1$

Now system (19) may be solved using numerical methods for solving systems of ODEs and we get the following results presented in Table(1) and their plots in Fig.(1).

Table (1)
The numerical solution of system (19) for the first time step [0,1].

t	$x_1(t)$	$x_2(t)$	$x_3(t)$
0	2	1	−1
0.2	1.237567	0.449329	−0.657047
0.4	0.65256	0.201896	−0.414783
0.6	0.293214	0.090718	−0.251579
0.8	0.11227	0.040762	−0.146607
1	0.036631	0.018316	−0.082085

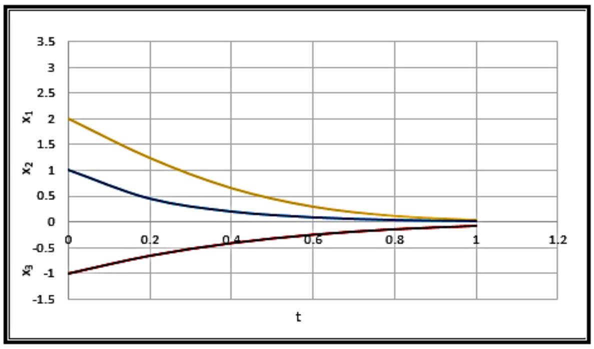

Fig. (1) The solutions $x_1(t)$, $x_2(t)$ and $x_3(t)$ of system (19) over [0,1].

Now, for the second time step interval [1,2] first we will find the Lagrange interpolation polynomials of degree 5 which interpolate the discrete results of the first time step given in Table (1) and then considered as the initial solution for the second time step, where:

$$P1_5(t) = 2 - 3.986315\, t - 0.204898t^2 + 6.528625t^3 - 6.114731\, t^4 + 1.813958t^5$$

$$P2_5(t) = 1 - 3.955322t + 7.458742t^2 - 8.198099\, t^3 + 5.031458\, t^4 - 1.318464t^5$$

$P3_5(t) = -1 + 2.001292t - 1.514768t^2 + 0.393786t^3 + 0.096068\,t^4 - 0.058464t^5$

Now, consider system (17) with the initial conditions:

$x_1(t) = x_{11}(t)$
$= 2 - 3.986315\,t - 0.204898t^2 + 6.528625t^3 - 6.114731\,t^4 + 1.813958t^5$,

$x_2(t) = x_{21}(t)$
$= 1 - 3.955322t + 7.458742t^2 - 8.198099\,t^3 + 5.031458\,t^4 - 1.318464t^5$,

$x_3(t) = x_{31}(t)$
$= -1 + 2.001292t - 1.514768t^2 + 0.393786t^3 + 0.096068\,t^4 - 0.058464t^5$

for $0 \leq t \leq 1$

Similarly, the solution of the original system (17), of this example for the time step intervals [1,2] and [2,3] are collected together and sketched in Fig.(2). The controller functions u_1(t), u_2(t) and u_3(t) are presented in Fig.(3).

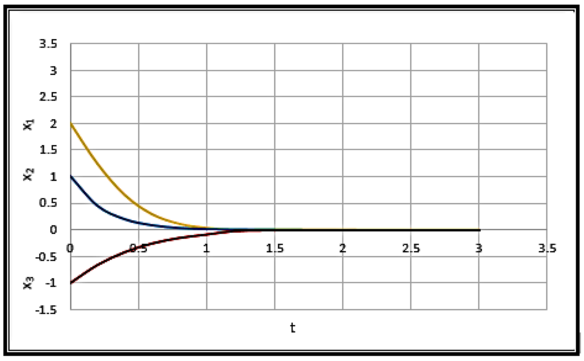

Fig.(2) The solutions $x_1(t)$, $x_2(t)$ and $x_3(t)$ of system (17) over [0,3].

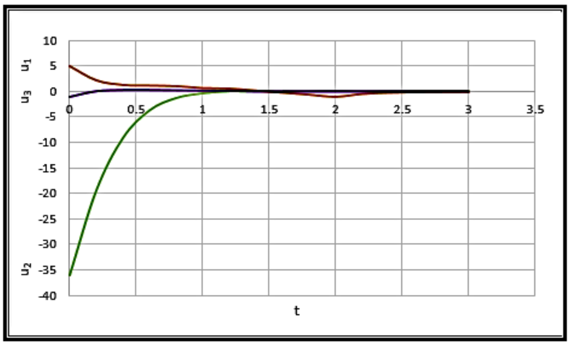

Fig.(3) The control function of $u_1(t)$, $u_2(t)$ and $u_3(t)$.

5. Conclusions

In the present paper, the generalized backstepping control method has been applied for stabilizing and solving certain type of a system of differential equations with multiple delays. This approach, in connection with the method of steps is very high efficient for stabilizing and solving systems of differential equations in general and systems with multiple delays as it is shown in Figs(1)-(3).

References

[1]. Fadhel S. and Noaman S., “Stabilizability and Solvability of Delay Differential Equations Using The Backstepping Method”, International Journal of Pure and Applied Mathematics (IJPAM), ISSN: 1311-8080, Vol. 1017, No. 2, 2017,(To a appear).

[2]. Fadhel S. F. and Yousif A. A., “Stabilization and Solution of Two Dimensions Nonlinear Hyperbolic Partial Differential Equations Using the Discretized Backstepping Method”, Control Theory and Informatics, Vol.6, No.3, PP. 30–37, 2016.

[3]. Karafyllis I., Malisoff M., Mazenc F. and Pepe P., “Recent Results on Nonlinear Delay Control Systems”, Editors, Springer International Publishing Switzerland, 2016.

[4]. Karafyllis I., “Finite-time Global Stabilization by Means of Time-varying Distributed Delay Feedback”, Society for Industrial and Applied Mathematics, Vol. 45, No. 1, pp. 320–342, 2006.

[5]. Krstic M. and Smyshlyaev A., “Adaptive boundary control for unstable parabolic PDEs-Part I: Lyapunov design”, IEEE Transactionson Automatic Control, Vol. 53, pp. 1575–1591, 2008.

[6]. Lotfazar A., Eghtesad M. and Mohseni M., “Integrator backstepping control of a 5 dof robot manipulator with cascaded dynamics”, IJE Transactions B: Applications, Vol. 16, No. 4, pp.373–383, 2003.

[7]. Mazenc F. and Bliman P. -A., “Backstepping Design for Time-Delay Nonlinear Systems”, IEEE Trans, Vol.5, pp.4551–4556, 2003.

[8]. Mazenc F. and Ito H.,“Lyapunov technique and backstepping for nonlinear neutral

systems", IEEE Trans,Vol.58, No.2, pp.512–517, 2013.

[9]. Mazenc F., Niculescu S. -I. and Bekaik M. "Stabilization of nonlinear systems with delay in the input through backstepping", IEEE Conference on Decision and Control and European Control Conference (CDC-ECC), pp.7605–7610, 2011.

[10]. Rasappan S. and Vaidyanathan S., "Hybrid synchronization of n-scroll Chua and Lur'echaotic system via backstepping control with novel feedback", Archives of Control Sciences, Vol.22, No.3, pp.343–365, 2012.

[11]. Rozali S. Md., Kamarudin M. N., Rahmat M. F. and Husain A. R., "Asymptotic Tracking Position Control for Nonlinear Systems using Backstepping Technique", Elsevier Ltd, Vol. 53, pp.255–263, 2013.

[12]. Rozali S. M., Rahmat M. F., Husain A. R. and Kamarudin M. N., "Design of adaptive backstepping with gravitational search algorithm for nonlinear system", Journal of Theoretical and Applied Information Technology, Vol. 59 No.2, pp. 460–468, 2014.