Solving the Multi-Objective Travelling Salesman Problem with Real Data Application

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Abstract

The aim of this paper is building a mathematical model for Travelling salesman problem (TSP) with multi-objective; the model describes the problem of (TSP) with three objectives (cost, distance, time), Real data were collected with a sample of twenty states of United State of America, Three methods were used (Branch and Bound algorithm, Nearest neighbor and two-way exchange improvement heuristic), The comparison was conducted among results reached.

To solve the problem multi-objective of (TSP), The weighted model demonstrated the effectiveness and flexibility to solve real problems of multi-objective (TSP), where it can be said that it is impossible to solve this problem without resorting to multiple -objective mathematical models, In other words, the number of possible rout for the 20 town is $\{(n-1)!=19!=121645100408832000\}$, to find the optimal routs among these routs it takes very long time and a lot of effort, here stand out importance of two-way exchange improvement heuristic algorithm, where this rout is satisfactory to the decision maker in terms of cost, distance and time.
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Keywords: Traveling Salesman Problem (TSP), mathematical programming formulation, multi-objective model, weighted-sum method, Branch & Bound algorithm, nearest neighbor, two-way exchange improvement heuristic

1-Introduction

This paper has focused attention of study traveling salesman problem (TSP) when there are multi-objective, as this problem is one of the problems of the combinatorial optimization which has gained widespread reputation and interest from researchers so as to simple formulation and its important applications, This concern came from the actual need of many productive sectors and companies that distribute products locally or imported to customers or other industrial sectors.

The Traveling Salesman Problem (TSP) is the problem of finding minimum expensive to visit a set of cities, a particular sequence, beginning and the end at the same city, each city must be visited exactly one time. Since this problem was formulated mathematically, the essence of the problem was in the area of combinatorial optimization. There is an important difference can be made between the symmetric TSP and the asymmetric TSP, for the symmetric case all distances are equal $\{dij = dji\}$ no matter what it was if we travel from city (i) to city (j) or on the contrary because the distance is the same, in the second case the distances are not equal for all pairs of cities. This kind of problems arises when we do not transact with locative distances between cities but with the time and cost associated with travelling between locations.

2-Historical overview:

The problem (TSP) was first mentioned by German scientist Karl Menger in the book "The Successful Rover" in 1832. He was the first scientist wrote in this problem, where he wanted to find $\{l\,(\mathrm{c})\}$, where:

$$l\,(\mathrm{c}) = \sup \sum_{i=1}^{n-1} dist\,(\mathrm{x}_i , \mathrm{x}_{i+1})$$

Where sup (supremum) refer to the highest values, which is take it on every selection $\{x_1, x_2, ..., x_{n-1}\}$,on C, In the order placed by C , Karl Menger has to solve this problem is that can be examine all the final set X for C that is:

$$\{\exists n \in N : X \subset C, |X| = n\}$$

Then we take the minimum value for all ranks X, therefore, define each set X for metric space $\{S:\lambda(X)\}$ it is the length of the shortest path through which it passes, and it has proved the following:

$$l(\text{c}) = \sup_X \lambda(X)$$

In 1930 Karl Menger presented the problem more clearly and considered it as a separate problem, in the same year winter put the problem under the name (travelling salesman), in the period between 1950 and 1960, the problem of the traveling vendor began to spread in the scientific community, especially in Europe and the United States of America.

In the meantime, when the challenge among the pioneers of algorithms increased, several researchers, including Dantzic and Johnson), succeeded in linear programming method to development method of cutting plane, In this new method, it was possible to solve the problem and find a tour among 49 cities, and proved lack of a shorter trip.

In subsequent decades the problem was studied by many mathematicians, physicists, chemists and other scientists.

In 1972, Richard M. Karp indicated that the Hamiltonian cycle problem was NP-complete, which means implicitly the NP-cruelty of TSP. This has provided an explanation mathematically for arithmetic difficulty in finding the optimal tours. It is then scientists have since developed many methods to solve the problem directly, such as genetic algorithms and mixed linear programming. The possible forms of the problem (TSP) are as follows:

2-1- Single versus multiple warehouses

In the case of a single warehouse, all sellers start from and finish their tours at one point, on the other hand, if there are multiple warehouse with a number of sellers present in each warehouse, sellers can either return to the original warehouse after completing their tour or return to any warehouse with a restriction that the initial number of sellers in each warehouse is still the same after each travel, The first case is called "fixed destination case", and the second "non-fixed destination case".

2-2- Fixed charges

When the number of sellers is usually constant, each of them has a fixed cost incurred each time the seller is used in the solution. In this case, reducing the number of them that has been activated in the solution may also be a source of interest.

2-3- Time restriction

In this type, some cities need to visit at certain time intervals, this is great protraction of the multiple traveling salesman problem with time, because to have commonly used applications in the real life such as in school bus, and airline scheduling problems[3].

3-Formulating the Travelling Salesman Problem (TSP)

When $i = j$, x_{ij} does not exist so it is not included in the model. We now give the mathematical programming formulation of the asymmetric TSP [6].

$$Minimize: \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$(Minimize\, total\, \cos t\, of\, tour)$$

$$Subject To: \sum_{j=1}^{n} x_{ij} = 1; \quad i = 1,...,n \qquad (1)$$

$$(Leave\, each\, city\, exactly\, once)$$

$$\sum_{i=1}^{n} x_{ij} = 1; \quad j = 1,...,n \qquad (2)$$

$$(Visit\, each\, city\, exactly\, once)$$

$$x_{ij} \in \{0,1\}; \quad i, j = 1,...,n \qquad (3)$$

$$(x_{ij}\, is\, a\, binary\, decision\, \text{var}\, ible)$$

$$\{(i,j): x_{ij} = 1,\ i, j = 2, \ldots, n\}$$

$$\sum_{i,j \in S}^{n} x_{ij} = |S| - 1;\ S \subseteq \{2,...,n\},\ 2 \le |S| \le n-1 \qquad (4)$$

$$(Subtour\, \text{elimination}).$$

4-Methods of solving a TSP

In literatures there are many different ways to solve TSP, efficiency techniques and also results. Let us refer to a summary of the most widely used methods:

4-1- Branch and Bound algorithm

The B And B algorithm starts with the optimal solution associated with the allocation problem, If the solution represents a path, the process ends, otherwise we impose constraints to remove the sub-tour, this can create as many different branches as variables associated with one of the sub-tour, each branch represents put one of the variables for the sub-tour equal to zero [1]. Initially before solving the problem, that we specify the upper bound select any rout connected (that does not contain sub-tour), and preferably use intuitions because it produce a higher limit than any rout, then we solve the problem as a normal allocation problem If the solution represents a Hamiltonian cycle (that does not contain sub-tour), the solution will stop and we consider the resulting solution is the optimal solution, If the solution to the problem of allocation does not represent Hamiltonian cycle, we'll assign the resulting solution as a lower bound, and that any solution that produces greater or equal to the upper bound path will ignore. Then select one of the sub routs to branch it preferably the selected sub rout contains the minimum number of cities (node), because it creates fewer braches. Note that the basic idea of branching to smashing one of the sub routs and modifying organic variables for the other sub-rout automatically.

4-2 Nearest neighbor

Intuitive methods are defined as a guessing state for the priority of choosing a point for another within the solution for some objectives often intuitions can find good solutions to the problem but they may not be optimal solutions. A good solution can be found to the problem of a traveling salesman by starting from the city the specific node, and then connecting it with the nearest city that has not been visited before, and continues the process until the Hamiltonian cycle is formed [4].

1. Choose the city randomly.
2. Find the node closest to it and non-visited.
3. Is there a node that has not been visited? If the answer is yes, repeat step 2.
4. We return to the city from which we started.

Thus we get the Hamiltonian cycle with $\{O(n^2)\}$, this method is useful and highly efficient because there is only one path to be formed, but it may not reach to the objective well.

4-3 Two-way exchange improvement heuristic

This method is also called Two-optimal improvement heuristic; the basic principle of this method is to modify the solution to a better solution. By modifying the tour, two arcs are deleted and reconnected the paths in a different way which reduces the total distance between nodes of the network until no deleted pair of arcs is found [8].

5-The Main Features of Decision Making of Multi-Objective

It can be said that the decision-maker actually seeks to achieve several objective; therefore the traditional model (one- objective) is no longer appropriate for him. The traditional framework for analyzing decision-making, presumably assume that there are three elements, Decision maker (personal or organization defined as a single entity), a set of available choices, and finally specific criteria (objective). Specific criteria are used to associate them with a number of alternatives so it can be arranged in the form of a set to get the optimal value that can be achieved from the selected objectives, Decision makers often do not mind to organize a set of possible solutions that are subject to one (objective) criterion but prefer the presence of a centrist compromise solution involves several objectives[2].

6-Definition of efficient solution

A solution $\{x^1 \in X\}$ is called efficient if and only if there is no other solution $\{x \in X\}$ where

$$\left\{f_k(x) \geq f_k(x^1)\ \forall k\ (k=1,2,...,p)\right\}.$$

The inequality being strict for at least one $\left\{k\left(f_k(x) > f_k(x^1)\right)\right\}$. Each solution x has a point $\left\{F=\left(f_1(x),...,f_p(x)\right)\right\}$ as representation in the objective function space.

7- Optimizing a Weighted-Sum of the Objective Functions

The process of computation of (efficient/ non-dominated) solutions more utilized consists in solving a scalar problem in which the objective function is a weighted-sum of the p original objective functions with positive weights ω_k :

$$\text{minimize (or maximize) } F_W = \sum_{k=1}^{p} \omega_k f_k(\text{x})$$
$$Subject\,To: \qquad x \in X \qquad (5)$$

$if \quad x^1 \in X$ is a solution to the problem $\min_{x \in X} \sum_{k=1}^{p} \omega_k f_k(\text{x})$ for $W = (\omega_1, ..., \omega_p)$, where $\omega_k > 0,\ k = 1, ..., p,$ and $\sum_{k=1}^{p} \omega_k = 1,$ then x^1 is an efficient solution to the multi-objective problem. The truthfulness of this proposition can be shown as follows. Suppose that x^1 is not efficient. Then, there is an $x^2 \in X$ such that $f_k(x^2) \geq f_k(x^1),\ k = 1, ..., p,$ and the inequality is strict for at least one k . But x^1 was obtained by optimizing a weighted-sum objective function with strictly positive weights then $\sum_{k=1}^{p} \omega_k f_k(x^2) > \sum_{k=1}^{p} \omega_k f_k(x^1),$ which contradicts the hypothesis that x^1 minimizes the weighted-sum objective function [7].

8-Data type

Before building the mathematical model of the problem, we must identify our data and statement qualitatively; therefore we will define the model data type, the data related to the problem are concerned with objectives placed by the decision maker and these objectives are defined according to the following indicators:

- Choose the route that achieves the least time it takes to reach between any two cities in the tour; this objective is expressed by indicator (time).
- Choose the shortest route possible connecting between any two cities in the tour; this objective is expressed by indicator (distance).
- Choose the route that achieves the lowest cost to reach between any two cities in the tour; this objective is expressed by indicator (cost). Data problem (cost, distance, and time) was obtained by the web sites [9].

Table (1-A)
represent the data collected belong to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes.

City	Washington 1			California 2			Arizona 3			Colorado 4			New Mexico 5		
	c1	d1	t1	c2	d2	t2	c3	d3	t3	c4	d4	t4	c5	d5	t5
Washington 1		∞		469	953	245	332	144	285	182	1298	232	264	1483	340
California 2	469	953	245		∞		229	735	81	132	1117	111	114	1001	122
Arizona 3	332	144	285	229	735	81		∞		135	645	76	227	433	44
Colorado 4	182	1298	232	132	1117	111	135	645	76		∞		194	424	52
New Mexico 5	264	1483	340	114	1001	122	227	433	44	194	424	52		∞	
Nevada 6	159	876	950	82	555	718	126	677	810	48	803	850	162	924	920
Texas 7	82	1937	1800	62	1406	1180	78	826	780	50	744	690	183	450	400
Oklahoma 8	228	2015	1740	222	1502	1240	224	934	850	49	792	660	282	567	480
Iowa 9	248	1729	1500	227	1846	1500	126	1363	1260	116	774	650	276	996	910
Montana 10	176	664	700	174	1305	1170	244	1182	1240	158	780	700	318	1155	1020
Minnesota 11	179	1470	1250	122	1991	1680	119	1667	1620	92	991	930	286	1277	1220
Indiana 12	267	2159	1835	183	2260	1860	294	1660	1500	66	1188	990	345	1293	1090
Missouri 13	314	1983	1740	104	1843	1500	219	1275	1120	77	886	750	268	907	760
Louisiana 14	128	2439	2160	148	1902	1680	197	1334	1190	106	1164	1060	329	968	830
Alabama 15	379	2607	2280	252	2162	1920	226	1593	1400	231	1492	1326	354	1228	1050
Kentucky 16	153	2386	2100	257	2309	1980	302	1740	1500	260	1272	1060	342	1373	1200
Georgia 17	252	2719	2400	155	2453	2160	78	1884	1680	118	1623	1415	484	1517	1322
Florida 18	379	3061	2580	232	2785	2400	212	2148	1920	327	1965	1740	230	1770	1560
New York 19	152	2794	2460	349	2914	2520	204	2333	2160	156	1842	1560	216	1965	1740
Virginia 20	126	2734	2400	302	2647	2340	339	2078	1920	212	1668	1500	341	1711	1500

Table (1-B)
represent the data collected belong to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes.

City	Nevada 6			Texas 7			Oklahoma 8			Iowa 9			Montana 10		
	c6	d6	t6	c7	d7	t7	c8	d8	t8	c9	d9	t9	c10	d10	t10
Washington 1	159	876	950	82	1937	1800	228	2015	1740	248	1729	1500	176	664	700
California 2	82	555	718	62	1406	1180	222	1502	1240	227	1846	1500	174	1305	1170
Arizona 3	126	677	810	78	826	780	224	934	850	126	1363	1260	244	1182	1240
Colorado 4	48	803	850	50	744	690	49	792	660	116	774	650	158	780	700
New Mexico 5	162	924	920	183	450	400	282	567	480	276	996	910	318	1155	1020
Nevada 6	∞			76	1329	1260	213	1425	1320	86	1499	1380	240	879	990
Texas 7	76	1329	1260	∞			45	341	300	194	916	750	242	1435	1230
Oklahoma 8	213	1425	1320	45	341	300	∞			276	633	520	406	1432	1190
Iowa 9	86	1499	1380	194	916	750	276	633	520	∞			358	1159	1000
Montana 10	240	879	990	242	1435	1230	406	1432	1190	358	1159	1000	∞		
Minnesota 11	123	1565	1500	82	1278	1117	286	994	830	135	404	350	282	799	740
Indiana 12	126	1938	1740	188	1110	920	399	805	660	187	463	380	346	1488	1300
Missouri 13	358	1643	1500	155	725	590	345	419	350	114	325	320	258	1399	1240
Louisiana 14	108	1825	1680	88	568	460	286	466	430	230	940	850	396	1854	1680
Alabama 15	231	2085	1980	192	828	680	412	736	650	278	940	800	418	2022	1800
Kentucky 16	468	2029	1860	274	1114	920	348	884	720	278	666	540	428	1715	1500
Georgia 17	94	2380	2160	214	1157	930	306	1005	820	276	1053	880	396	2136	1920
Florida 18	108	2626	2460	92	1369	1130	166	1277	1100	166	1396	1150	378	2478	2220
New York 19	180	2566	2400	101	1769	1560	290	1477	1250	252	1093	950	332	2110	1920
Virginia 20	288	2425	2280	175	1404	1170	291	1199	1010	276	1020	890	416	2063	1860

Table (1-C)
represent the data collected belong to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes

City	Minnesota 11			Indiana 12			Missouri 13			Louisiana 14			Alabama 15		
	c11	d11	t11	c12	d12	t12	c13	d13	t13	c14	d14	t14	c15	d15	t15
Washington 1	179	1470	1250	267	2159	1835	314	1983	1740	128	2439	2160	379	2607	2280
California 2	122	1991	1680	183	2260	1860	104	1843	1500	148	1902	1680	252	2162	1920
Arizona 3	119	1667	1620	294	1660	1500	219	1275	1120	197	1334	1190	226	1593	1400
Colorado 4	92	991	930	66	1188	990	77	886	750	106	1164	1060	231	1492	1326
New Mexico 5	286	1277	1220	345	1293	1090	268	907	760	329	968	830	354	1228	1050
Nevada 6	123	1565	1500	126	1938	1740	358	1643	1500	108	1825	1680	231	2085	1980
Texas 7	82	1278	1117	188	1110	920	155	725	590	88	568	460	192	828	680
Oklahoma 8	286	994	830	399	805	660	345	419	350	286	466	430	412	736	650
Iowa 9	135	404	350	187	463	380	114	325	320	230	940	850	278	940	800
Montana 10	282	799	740	346	1488	1300	258	1399	1240	396	1854	1680	418	2022	1800
Minnesota 11		∞		270	733	620	187	724	660	156	1301	1160	151	1327	1110
Indiana 12	270	733	620		∞		190	389	320	192	886	760	235	614	510
Missouri 13	187	724	660	190	389	320		∞		199	584	600	152	609	560
Louisiana 14	156	1301	1160	192	886	760	199	584	600		∞		278	388	370
Alabama 15	151	1327	1110	235	614	510	152	609	560	278	388	370		∞	
Kentucky 16	256	959	800	250	243	200	259	470	380	245	825	710	296	500	420
Georgia 17	92	1441	1190	194	727	590	160	821	660	84	653	600	135	277	260
Florida 18	138	1783	1560	76	1069	870	76	1163	960	74	836	690	154	566	490
New York 19	187	1368	1180	172	738	630	201	1062	910	174	1380	1256	190	1063	890
Virginia 20	275	1307	1199	221	588	520	238	866	740	221	1014	890	210	670	570

Table (1-D)

Represent the data collected belong to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes.

City	Kentucky 16			Georgia 17			Florida 18			New York 19			Virginia 20		
	c16	d16	t16	c17	d17	t17	c18	d18	t18	c19	d19	t19	c20	d20	t20
Washington 1	153	2386	2100	252	2719	2400	379	3061	2580	152	2794	2460	126	2734	2400
California 2	257	2309	1980	155	2453	2160	232	2785	2400	349	2914	2520	302	2647	2340
Arizona 3	302	1740	1500	78	1884	1680	212	2148	1920	204	2333	2160	339	2078	1920
Colorado 4	260	1272	1060	118	1623	1415	327	1965	1740	156	1842	1560	212	1668	1500
New Mexico 5	342	1373	1200	484	1517	1322	230	1770	1560	216	1965	1740	341	1711	1500
Nevada 6	468	2029	1860	94	2380	2160	108	2626	2460	180	2566	2400	288	2425	2280
Texas 7	274	1114	920	214	1157	930	92	1369	1130	101	1769	1560	175	1404	1170
Oklahoma 8	348	884	720	306	1005	820	166	1277	1100	290	1477	1250	291	1199	1010
Iowa 9	278	666	540	276	1053	880	166	1396	1150	252	1093	950	276	1020	890
Montana 10	428	1715	1500	396	2136	1920	378	2478	2220	332	2110	1920	416	2063	1860
Minnesota 11	256	959	800	92	1441	1190	138	1783	1560	187	1368	1180	275	1307	1199
Indiana 12	250	243	200	194	727	590	76	1069	870	172	738	630	221	588	520
Missouri 13	259	470	380	160	821	660	76	1163	960	201	1062	910	238	866	740
Louisiana 14	245	825	710	84	653	600	74	836	690	174	1380	1256	221	1014	890
Alabama 15	296	500	420	135	277	260	154	566	490	190	1063	890	210	670	570
Kentucky 16	∞			268	519	420	213	861	700	219	700	600	250	419	380
Georgia 17	268	519	420	∞			56	366	340	110	916	760	170	511	460
Florida 18	213	861	700	56	366	340	∞			174	1143	940	172	805	690
New York 19	219	700	600	110	916	760	174	1143	940	∞			156	388	400
Virginia 20	250	419	380	170	511	460	172	805	690	156	388	400	∞		

9-The Practical Part

A multi-objective linear programming model will be built to solve the problem of a traveling sales man in the United States by formulating the (TSP) which is mentioned in paragraph (3) and using the data shown in the above tables, as follows:

9-1 Decision Variables

Let x_{ij} represent the binary variable or in other words

$$x_{ij} = \begin{cases} 1, & \text{if the arc from } i \text{ to } j \text{ is selected} \\ \text{for all } i \neq j, i, j = 1,2,...,n & \\ 0, & \text{otherwise} \end{cases}$$

9-2 Objective functions

The three objective functions (cost, distance and time) $\{f_1, f_2, f_3\}$ are formulated respectively as follows:

Firstly, the objective function of achieving the maximum reduction of the total cost

$$f_1(\text{x}) \sum_{i=1}^{20} \sum_{j=1}^{20} c_{ij} x_{ij}$$

$$\underset{\text{Total cost}}{Minimize}\ f_1(\text{x}) = 469x_{1,2} + 332x_{1,3} + 182x_{1,4} + 246x_{1,5}$$
$$+159x_{1,6} + 82x_{1,7} + 228x_{1,8} + 248x_{1,9}$$
$$+179x_{1,11} + 267x_{1,12} + 314x_{1,13} + 128x_{1,14}$$
$$+379x_{1,15} + 153x_{1,16} + 176x_{1,10} + 252x_{1,17}$$
$$+379x_{1,18} + 152x_{1,19} + 126x_{1,20} + ...$$
$$+126x_{20,1} + 302x_{20,2} + 339x_{20,3} + 212x_{20,4}$$
$$+341x_{20,5} + 288x_{20,6} + 175x_{20,7} + 291x_{20,8}$$
$$+276x_{20,9} + 416x_{20,10} + 275x_{20,11} + 221x_{20,12}$$
$$+238x_{20,13} + 221x_{20,14} + 210x_{20,15} + 250x_{20,16}$$
$$+170x_{20,17} + 172x_{20,18} + 156x_{20,19} \qquad (6)$$

Secondly, the objective function of achieving the maximum reduction of the total distances

$$f_2(\text{x}) \sum_{i=1}^{20} \sum_{j=1}^{20} d_{ij} x_{ij}$$

$$\underset{\text{Total distance}}{Minimize}\ f_2(\text{x}) = 953x_{1,2} + 144x_{1,3} + 1298x_{1,4} + 1483x_{1,5}$$
$$+876x_{1,6} + 1937x_{1,7} + 1015x_{1,8} + 1729x_{1,9}$$
$$+1470x_{1,11} + 2159x_{1,12} + 1983_{1,13} + 2439x_{1,14}$$
$$+2607x_{1,15} + 2386x_{1,16} + 664x_{1,10} + 2719x_{1,17}$$
$$+3061x_{1,18} + 2794x_{1,19} + 2734x_{1,20} \ ... + 2734x_{20,1}$$
$$+2647x_{20,2} + 2078x_{20,3} + 1668x_{20,4} + 1711x_{20,5}$$
$$+2425x_{20,6} + 1404x_{20,7} + 1199x_{20,8} + 1020x_{20,9}$$
$$+2063x_{20,10} + 1307x_{20,11} + 588x_{20,12} + 866x_{20,13}$$
$$+1014x_{20,14} + 670x_{20,15} + 419x_{20,16} + 511x_{20,17}$$
$$805x_{20,18} + 388x_{20,19} \qquad (7)$$

Thirdly, the objective function of achieving the maximum reduction of the total time

$$f_3(\text{x}) \sum_{i=1}^{20} \sum_{j=1}^{20} t_{ij} x_{ij}$$

$$\underset{\text{Total time}}{Minimize}\ f_3(\text{x}) = 245x_{1,2} + 285x_{1,3} + 232x_{1,4} + 340x_{1,5} + 950x_{1,6} + 1800x_{1,7} + 1740x_{1,8} + 1500x_{1,9} + 700x_{1,10}$$
$$+1250x_{1,11} + 1835x_{1,12} + 1740_{1,13} + 2160x_{1,14} + 2280x_{1,15} + 2100x_{1,16} + 2400x_{1,17} + 2580x_{1,18}$$
$$+2460x_{1,19} + 2400x_{1,20} \ ... + 2400x_{20,1} + 2340x_{20,2} + 1920x_{20,3} + 1500x_{20,4} + 1500x_{20,5} +$$
$$2280x_{20,6} + 1170x_{20,7} + 1010x_{20,8} + 890x_{20,9} + 1860x_{20,10} + 1199x_{20,11} + 520x_{20,12} + 740x_{20,13} +$$
$$890x_{20,14} + 570x_{20,15} + 380x_{20,16} + 460x_{20,17} + 690x_{20,18} + 400x_{20,19} \qquad (8)$$

Subject To :

The constraints of the multi-objective problem can be represented in the following mathematical formula:

$$\left.\begin{array}{l} \sum_{j=1}^{20} x_{(i=1)j} = 1, \ \sum_{j=1}^{20} x_{(i=2)j} = 1, \ \sum_{j=1}^{20} x_{(i=3)j} = 1, \\ \sum_{j=1}^{20} x_{(i=4)j} = 1, \sum_{j=1}^{20} x_{(i=5)j} = 1, \sum_{j=1}^{20} x_{(i=6)j} = 1, \\ \sum_{j=1}^{20} x_{(i=7)j} = 1, \sum_{j=1}^{20} x_{(i=8)j} = 1, \sum_{j=1}^{20} x_{(i=9)j} = 1, \\ \sum_{j=1}^{20} x_{(i=10)j} = 1, \sum_{j=1}^{20} x_{(i=11)j} = 1, \sum_{j=1}^{20} x_{(i=12)j} = 1, \end{array}\right\} (9)$$

$$\left.\begin{array}{l}\sum_{j=1}^{20} x_{(i=13)j}=1, \sum_{j=1}^{20} x_{(i=14)j}=1, \sum_{j=1}^{20} x_{(i=15)j}=1,\\ \sum_{j=1}^{20} x_{(i=16)j}=1, \sum_{j=1}^{20} x_{(i=17)j}=1, \sum_{j=1}^{20} x_{(i=18)j}=1,\\ \sum_{j=1}^{20} x_{(i=19)j}=1, \sum_{j=1}^{20} x_{(i=20)j}=1.\end{array}\right\}$$

$$\left.\begin{array}{l}\sum_{i=1}^{20} x_{i(j=1)}=1, \sum_{i=1}^{20} x_{i(j=2)}=1,\\ \sum_{i=1}^{20} x_{i(j=3)}=1, \sum_{i=1}^{20} x_{i(j=4)}=1,\\ \sum_{i=1}^{20} x_{i(j=5)}=1, \sum_{i=1}^{20} x_{i(j=6)}=1,\\ \sum_{i=1}^{20} x_{i(j=7)}=1, \sum_{i=1}^{20} x_{i(j=8)}=1,\\ \sum_{i=1}^{20} x_{i(j=9)}=1, \sum_{i=1}^{20} x_{i(j=10)}=1,\\ \sum_{i=1}^{20} x_{i(j=11)}=1, \sum_{i=1}^{20} x_{i(j=12)}=1,\\ \sum_{i=1}^{20} x_{i(j=13)}=1, \sum_{i=1}^{20} x_{i(j=14)}=1,\\ \sum_{i=1}^{20} x_{i(j=15)}=1, \sum_{i=1}^{20} x_{i(j=16)}=1,\\ \sum_{i=1}^{20} x_{i(j=17)}=1, \sum_{i=1}^{20} x_{i(j=18)}=1,\\ \sum_{j=1}^{20} x_{i(j=19)}=1, \sum_{j=1}^{20} x_{i(j=20)}=1.\end{array}\right\} \quad (10)$$

$$x_{ij}=0 \;\; or \;\; 1, \quad i \neq j, i, j=1,2,\ldots,20 \qquad (11)$$

The above model is a problem of achieving optimization of multi-objective, when we solved it, the results of this model includes a conflict among three objectives and cannot achieve the maximum reduction of objective simultaneously without increasing one of the objectives, Therefore, the method of weights-sum was used to solve the conflict, This method depends mainly on the experience of the decision-maker in the development of weights according to the importance of each objective and this will be explained in the next paragraph.

9-4 Build a weighted mathematical model to solve the multi-objective of (TSP)

The weighted mathematical model of the multi-objective of (TSP) can be represented as follows:

$$\left.\begin{array}{l}\min \; F_W=\sum_{k=1}^{3}\omega_k f_k(\mathrm{x})\\ =\omega_1 f_1(\mathrm{x})+\omega_2 f_2(\mathrm{x})+\omega_3 f_3(\mathrm{x})\\ \text{Subject To:}\\ \text{The same constraints } (6, 7 \text{ and } 8)\\ \text{as are mentioned above}\end{array}\right\} \quad (12)$$

The model is solved as follows:

Let us consider that the weights given by the decision maker are $\left(W=\{\omega_1=0.3,\ \omega_2=0.5,\ \omega_3=0.2\}\right)$, the total of these weights is $\left\{\sum_{k=1}^{3}\omega_k=1\right\}$

The weights given are the product of the decision maker's experience. In his view, the cost and time are dependent on the distance. Whenever the distance low, result the cost and time are low, vice versa, so after substituted the weights in the model (9), we get the following new model:

$$\min \; F_{W=\{0.3,0.5,0.2\}}=$$
$$0.3\; f_1(\mathrm{x})+0.5\; f_2(\mathrm{x})+0.2\; f_3(\mathrm{x})$$
Subject To:

$$\left.\text{The same constraints } (6, 7 \text{ and } 8) \text{ as are mentioned above}\right\} \quad (13)$$

After performing the mathematical operations by multiply the weights by the objectives and then collecting the objectives to be a single objective, i.e. converting the problem multi-objective to the problem of one-objective based on the following tables:

Table (2-A)
represent the weights which multiplied by the objective functions

City	Washington1	California2	Arizona3	Colorado4	New Mexico5	Nevada 6	Texas 7	Oklahoma 8	Iowa 9	Montana 10
Washington1	∞	666.2	228.6	750	888.7	675.7	1353.1	1423.9	1238.9	524.8
California2	666.2	∞	452.4	620.3	559.1	445.7	957.6	1065.6	1291.1	938.7
Arizona3	228.6	452.4	∞	378.2	293.4	538.3	592.4	704.2	971.3	912.2
Colorado4	750	620.3	378.2	∞	280.6	585.9	525	542.7	551.8	577.4
New Mexico5	888.7	559.1	293.4	280.6	∞	694.6	359.9	464.1	762.8	876.9
Nevada 6	675.7	445.7	538.3	585.9	694.6	∞	939.3	1040.4	1051.3	709.5
Texas 7	1353.1	957.6	592.4	525	359.9	939.3	∞	244	666.2	1036.1
Oklahoma 8	1423.9	1065.6	704.2	542.7	464.1	1040.4	244	∞	503.3	1075.8
Iowa 9	1238.9	1291.1	971.3	551.8	762.8	1051.3	666.2	503.3	∞	886.9
Montana 10	524.8	938.7	912.2	577.4	876.9	709.5	1036.1	1075.8	886.9	∞
Minnesota 11	1038.7	1368.1	1193.2	709.1	968.3	1119.4	887	748.8	312.5	632.1
Indiana 12	1526.6	1556.9	1218.2	811.8	968	1354.8	795.4	654.2	363.6	1107.8
Missouri 13	1433.7	1252.7	927.2	616.1	685.9	1228.9	527	383	260.7	1024.9
Louisiana 14	1689.9	1331.4	964.1	825.8	748.7	1280.9	402.4	404.8	709	1381.8
Alabama 15	1873.2	1540.6	1144.3	1080.5	930.2	1507.8	607.6	621.6	713.4	1496.4
Kentucky 16	1658.9	1627.6	1260.6	926	1029.1	1526.9	823.2	690.4	524.4	1285.9
Georgia 17	1915.1	1705	1301.4	1129.9	1168.1	1650.2	828.7	758.3	785.3	1570.8
Florida 18	2160.2	1942.1	1521.6	1428.6	1266	1837.4	938.1	908.3	977.8	1796.4
New York 19	1934.6	2065.7	1659.7	1279.8	1395.3	1817	1226.8	1075.5	812.1	1538.6
Virginia 20	1884.8	1882.1	1524.7	1197.6	1257.8	1754.9	988.5	888.8	770.8	1528.3

Table (2-B)
Represent the weights which multiplied by the objective functions.

City	Minnesota 11	Indiana 12	Missouri 13	Louisiana 14	Alabama 15	Kentucky 16	Georgia 17	Florida 18	New York 19	Virginia 20
Washington1	1038.7	1526.6	1433.7	1689.9	1873.2	1658.9	1915.1	2160.2	1934.6	1884.8
California2	1368.1	1556.9	1252.7	1331.4	1540.6	1627.6	1705	1942.1	2065.7	1882.1
Arizona3	1193.2	1218.2	927.2	964.1	1144.3	1260.6	1301.4	1521.6	1659.7	1524.7
Colorado4	709.1	811.8	616.1	825.8	1080.5	926	1129.9	1428.6	1279.8	1197.6
New Mexico5	968.3	968	685.9	748.7	930.2	1029.1	1168.1	1266	1395.3	1257.8
Nevada 6	1119.4	1354.8	1228.9	1280.9	1507.8	1526.9	1650.2	1837.4	1817	1754.9
Texas 7	887	795.4	527	402.4	607.6	823.2	828.7	938.1	1226.8	988.5
Oklahoma 8	748.8	654.2	383	404.8	621.6	690.4	758.3	908.3	1075.5	888.8
Iowa 9	312.5	363.6	260.7	709	713.4	524.4	785.3	977.8	812.1	770.8
Montana 10	632.1	1107.8	1024.9	1381.8	1496.4	1285.9	1570.8	1796.4	1538.6	1528.3
Minnesota 11	∞	571.5	550.1	929.3	930.8	716.3	986.1	1244.9	976.1	975.8
Indiana 12	571.5	∞	315.5	652.6	479.5	236.5	539.7	731.3	546.6	464.3
Missouri 13	550.1	315.5	∞	471.7	462.1	388.7	590.5	796.3	773.3	652.4
Louisiana 14	929.3	652.6	471.7	∞	351.4	628	471.7	578.2	993.4	751.3
Alabama 15	930.8	479.5	462.1	351.4	∞	422.8	231	427.2	766.5	512
Kentucky 16	716.3	236.5	388.7	628	422.8	∞	423.9	634.4	535.7	360.5
Georgia 17	986.1	539.7	590.5	471.7	231	423.9	∞	267.8	643	398.5
Florida 18	1244.9	731.3	796.3	578.2	427.2	634.4	267.8	∞	811.7	592.1
New York 19	976.1	546.6	773.3	993.4	766.5	535.7	643	811.7	∞	320.8
Virginia 20	975.8	464.3	652.4	751.3	512	360.5	398.5	592.1	320.8	∞

By the Tables (2-A and 2-B) above, the weighted objective function is as follows:

$$Min\ F_w = 666.2x_{1,2} + 228.6x_{1,3} + 750x_{1,4} + 888.7x_{1,5} + 1238.9x_{1,9} + 675.7x_{1,6} + 1353.1x_{1,7} + 1423.9x_{1,8} + 524.8x_{1,10} + 1038.7x_{1,11} + 1526.6x_{1,12} + 1433.7x_{1,13} + 1689.9x_{1,14} + 1873.2x_{1,15} + 1658.9x_{1,16} + 1915.1x_{1,17} + 2160.2x_{1,18} + 1934.6x_{1,19} + 1884.8x_{1,20} + \ldots + 1884.8x_{20,1} + 1882.1x_{20,2} + 1524.7x_{20,3} + 1197.6x_{20,4} + 1257.8x_{20,5} + 1754.9x_{20,6} + 988.5x_{20,7} + 888.8x_{20,8} + 770.8x_{20,9} + 1528.3x_{20,10} + 975.8x_{20,11} + 464.3x_{20,12} + 652.4x_{20,13} + 751.3x_{20,14} + 512x_{20,15} + 360.5x_{20,16} + 398.5x_{20,17} + 592.1x_{20,18} + 320.8x_{20,19} \quad (14)$$

Subject To: The same constraints 6, 7 and 8 as are mentioned above

9-5 Solution for Minimization (Multi-objective Traveling Salesman Problem)

In this section, the model (11) will be solved to obtain the optimal solution using the three methods (branch and bound, nearest neighbor and two-way exchange improvement heuristic) to solve the problem of (TSP), as well as a comparison among the optimal results to be obtained in the following manner, all results depended on the package program WINQSB [5]:

Table (3)
Represents the optimal solution by using B and B.

City	From Node	Connect To	Arc value	City	From Node	Connect To	Arc value
1	Node1	Node2	666.2	11	Node16	Node13	388.7
2	Node2	Node6	445.7	12	Node13	Node15	462.1
3	Node6	Node4	585.9	13	Node15	Node17	231
4	Node4	Node10	577.4	14	Node17	Node18	267.8
5	Node10	Node11	632.1	15	Node18	Node14	578.2
6	Node11	Node9	312.5	16	Node14	Node8	404.8
7	Node9	Node12	363.6	17	Node8	Node7	244
8	Node12	Node19	546.6	18	Node7	Node5	359.9
9	Node19	Node20	320.8	19	Node5	Node3	293.4
10	Node20	Node16	360.5	20	Node3	Node1	228.6
	Total	Minimal	F_W	=	8,269.80		
	(Result	from	Branch	and	Bound	Method)	

Table (4)
Represents the optimal solution by using nearest neighbor algorithm.

City	From Node	Connect To	Arc value	City	From Node	Connect To	Arc value
1	Node1	Node3	228.6	11	Node16	Node20	360.5
2	Node3	Node5	293.4	12	Node20	Node19	320.8
3	Node5	Node4	280.6	13	Node19	Node17	643
4	Node4	Node7	525	14	Node17	Node15	231
5	Node7	Node8	244	15	Node15	Node14	351.4
6	Node8	Node13	383	16	Node14	Node18	578.2
7	Node13	Node9	260.7	17	Node18	Node10	1796.4
8	Node9	Node11	312.5	18	Node10	Node6	709.5
9	Node11	Node12	571.5	19	Node6	Node2	445.7
10	Node12	Node16	236.5	20	Node2	Node1	666.2
	Total	Minimal	F_W	=	9,438.50		
	(Result	from	Nearest	Neighbor	Heuristic)		

Table (5)
Represents the optimal solution by using Two-way exchange improvement heuristic.

City	From Node	Connect To	Arc value	City	From Node	Connect To	Arc value
1	Node16	Node12	236.5	11	Node3	Node5	293.4
2	Node12	Node13	315.5	12	Node5	Node7	359.9
3	Node13	Node9	260.7	13	Node7	Node8	244
4	Node9	Node11	312.5	14	Node8	Node14	404.8
5	Node11	Node10	632.1	15	Node14	Node15	351.4
6	Node10	Node4	577.4	16	Node15	Node17	231
7	Node4	Node6	585.9	17	Node17	Node18	267.8
8	Node6	Node2	445.7	18	Node18	Node20	592.1
9	Node2	Node1	666.2	19	Node20	Node19	320.8
10	Node1	Node3	228.6	20	Node19	Node16	535.7
	Total	Minimal	F_W	=	7,862.00		
	(Result	from	Two-way	Exchange	Improvement	Heuristic)	

After finding the optimal solutions above, a table will be made to compare the optimal solutions after substation the optimal binary decision variables in the three objective functions (cost, distance and time) as shown in Table (6) below.

Table (6)
Represents a comparison of optimal solutions with the given weights.

Index	Methods (TSP)	Objective fn1. (cost)	Objective fn2. (distance)	Objective fn3. (time)
1	Branch & Bound	3688	10694	9082
2	Nearest neighbor	4134	12299	10244
3	Two-way exchange improvement	3817	10009	8562
Weight		0.3	0.5	0.2

The objectives as important in terms of the weighted preference of the decision maker can be summarized as follows:

The highest weight (0.5) is for distance that is the second objective, heuristic algorithm gave the maximum reduction.

The middle weight (0.3) is for cost that is the first objective; B&B algorithm gave the

The lowest weight (0.2) is for time that is the third objective, heuristic algorithm gave the maximum reduction.

Since the decision-maker is looking for reduce the distance to adopt the time and cost, The Heuristic algorithm is the best solution for the problem and the optimal path is as shown in Table (10).

Table (7)
Represents the optimal path (rout) by Heuristic algorithm.

Index	City (From - To)	Optimal Route	Cost\$	Distance\mile	Time\minute
1	Washington - Arizona	Start Travel (1) - ٣	332	144	285
2	Arizona - New Mexico	٣ - ٥	227	433	44
3	New Mexico - Texas	٥ - ٧	183	450	400
4	Texas - Oklahoma	٧ - ٨	45	341	300
5	Oklahoma - Louisiana	٨ - 14	286	466	430
6	Louisiana - Alabama	14 - 15	278	388	370
7	Alabama - Georgia	15 - 17	135	277	260
8	Georgia - Florida	17 - 18	56	366	340
9	Florida - Virginia	18 - 20	172	805	690
10	Virginia - New York	20 - 19	156	388	400
11	New York - Kentucky	19 - 16	219	700	600
12	Kentucky - Indiana	16 - 12	250	243	200
13	Indiana - Missouri	12 - 13	190	389	320
14	Missouri - Iowa	13 - 9	114	325	320
15	Iowa - Minnesota	9 - 11	135	404	350
16	Minnesota - Montana	11 - 10	282	799	740
17	Montana - Colorado	10 - 4	158	780	700
18	Colorado - Nevada	4 - 6	48	803	850
19	Nevada - California	6 - 2	82	555	718
20	California - Washington	2 – End Travel (1)	469	953	245
Total cost, distance and time			**3817**	**10009**	**8562**

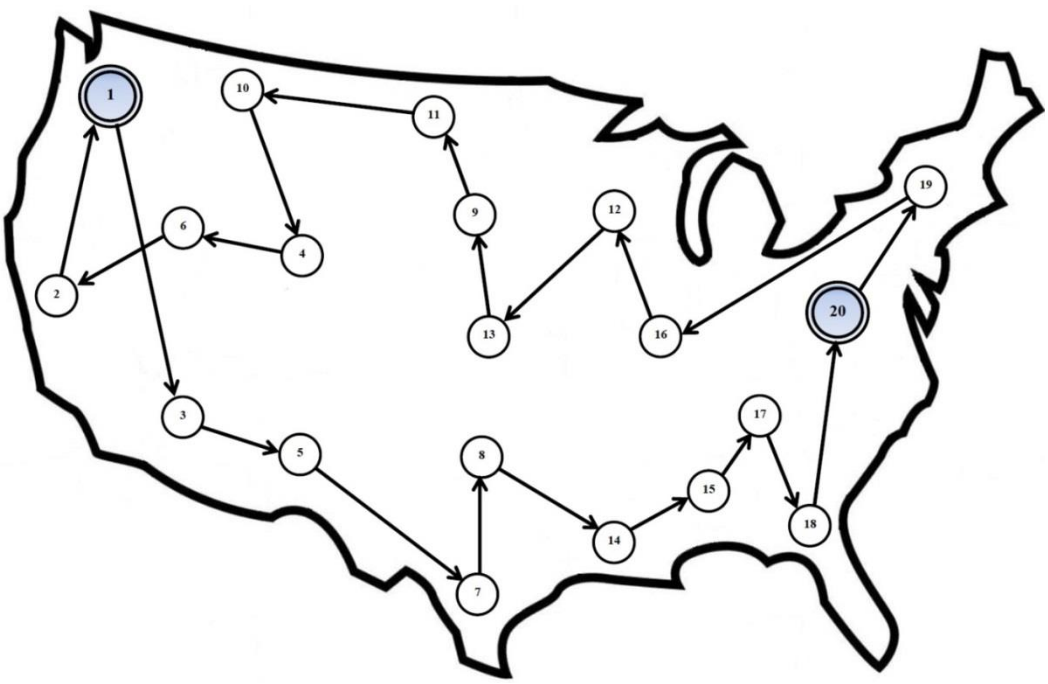
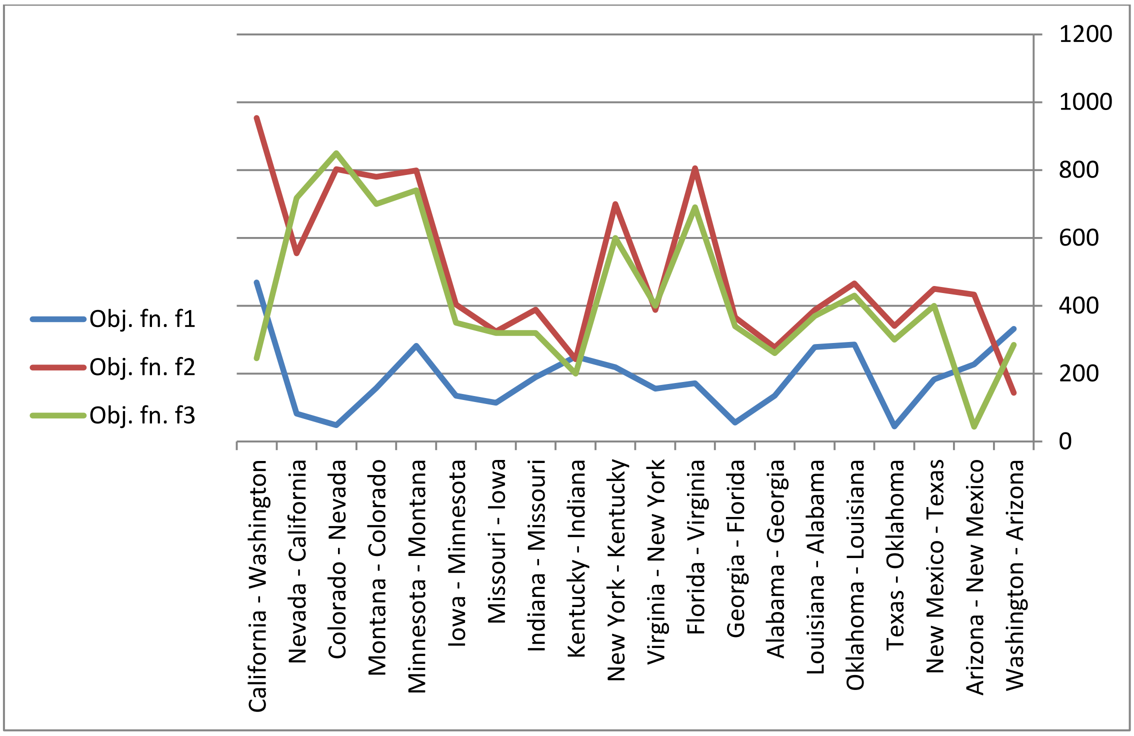


Fig.(1): Illustrates the optimal rout.



Optimal Route

Fig.(2): Chart illustrate optimal solution with objective functions f_1, f_2, f_3.

Conclusions

The weighted model demonstrated the effectiveness and flexibility to solve real problems of multi- objective (TSP), where it can be said that it is impossible to solve this problem without resorting to multiple-objective mathematical models, In other words, the number of possible rout for the 20 US states is $\{(n-1)! = 19! = 121645100408832000\}$, to find the optimal routs among these routs it takes very long time and a lot of effort, here stand out importance of two-way exchange improvement heuristic algorithm, where this rout is satisfactory to the decision maker in terms of cost, distance and time.

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