Solving the Multi-Objective Travelling Salesman Problem with Real Data Application

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Abstract

The aim of this paper is building a mathematical model for Travelling salesman problem (TSP) with multi-objective; the model describes the problem of (TSP) with three objectives (cost, distance, time), Real data were collected with a sample of twenty states of United State of America, Three methods were used (Branch and Bound algorithm, Nearest neighbor and two-way exchange improvement heuristic), The comparison was conducted among results reached.

To solve the problem multi-objective of (TSP), The weighted model demonstrated the effectiveness and flexibility to solve real problems of multi-objective (TSP), where it can be said that it is impossible to solve this problem without resorting to multiple -objective mathematical models, In other words, the number of possible rout for the 20 town is \( \binom{n-1}{1}! = 19! = 121645100408832000 \), to find the optimal routs among these routs it takes very long time and a lot of effort, here stand out importance of two-way exchange improvement heuristic algorithm, where this rout is satisfactory to the decision maker in terms of cost, distance and time.

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Keywords: Traveling Salesman Problem (TSP), mathematical programming formulation, multi-objective model, weighted-sum method, Branch & Bound algorithm, nearest neighbor, two-way exchange improvement heuristic

1-Introduction

This paper has focused attention of study traveling salesman problem (TSP) when there are multi-objective, as this problem is one of the problems of the combinatorial optimization which has gained widespread reputation and interest from researchers so as to simple formulation and its important applications, This concern came from the actual need of many productive sectors and companies that distribute products locally or imported to customers or other industrial sectors.

The Traveling Salesman Problem (TSP) is the problem of finding minimum expensive to visit a set of cities, a particular sequence, beginning and the end at the same city, each city must be visited exactly one time. Since this problem was formulated mathematically, the essence of the problem was in the area of combinatorial optimization. There is an important difference can be made between the symmetric TSP and the asymmetric TSP, for the symmetric case all distances are equal \( \{dij =dji \} \) no matter what it was if we travel from city \( (i) \) to city \( (j) \) or on the contrary because the distance is the same, in the second case the distances are not equal for all pairs of cities. This kind of problems arises when we do not transact with locative distances between cities but with the time and cost associated with travelling between locations.

2-Historical overview:

The problem (TSP) was first mentioned by German scientist Karl Menger in the book "The Successful Rover" in 1832. He was the first scientist wrote in this problem, where he wanted to find \( \{l(c)\} \), where:

\[
l(c) = \sup \sum_{i=1}^{n-1} \text{dist} (x_i, x_{i+1})
\]

Where sup (supremum) refer to the highest values, which is take it on every selection \( \{x_1, x_2, ..., x_{n-1}\} \) on C, In the order placed by C , Karl Menger has to solve this problem is that can be examine all the final set X for C that is:

\[
\{ \exists n \in N : X \subset C, |X| = n \}
\]
Then we take the minimum value for all ranks $X$, therefore, define each set $X$ for metric space $\{S : \lambda(X)\}$ it is the length of the shortest path through which it passes, and it has proved the following:

$$l(c) = \sup_X \lambda(X)$$

In 1930 Karl Menger presented the problem more clearly and considered it as a separate problem, in the same year winter put the problem under the name (travelling salesman), in the period betwen 1950 and 1960, the problem of the traveling vendor began to spread in the scientific community, especially in Europe and the United States of America.

In the meantime, when the challenge among the pioneers of algorithms increased, several researchers, including Dantzig and Johnson, succeeded in linear programming method to development method of cutting plane. In this new method, it was possible to solve the problem and find a tour among 49 cities, and proved lack of a shorter trip.

In subsequent decades the problem was studied by many mathematicians, physicists, chemists and other scientists.

In 1972, Richard M. Karp indicated that the Hamiltonian cycle problem was NP-complete, which means implicitly the NP-cruelty of TSP. This has provided an explanation mathematically for arithmetic difficulty in finding the optimal tours. It is then scientists have since developed many methods to solve the problem directly, such as genetic algorithms and mixed linear programming. The possible forms of the problem (TSP) are as follows:

2-1- Single versus multiple warehouses

In the case of a single warehouse, all sellers start from and finish their tours at one point, on the other hand, if there are multiple warehouse with a number of sellers present in each warehouse, sellers can either return to the original warehouse after completing their tour or return to any warehouse with a restriction that the initial number of sellers in each warehouse is still the same after each travel, The first case is called "fixed destination case", and the second "non-fixed destination case".

2-2- Fixed charges

When the number of sellers is usually constant, each of them has a fixed cost incurred each time the seller is used in the solution. In this case, reducing the number of them that has been activated in the solution may also be a source of interest.

2-3- Time restriction

In this type, some cities need to visit at certain time intervals, this is great protraction of the multiple traveling salesman problem with time, because to have commonly used applications in the real life such as in school bus, and airline scheduling problems[3].

3-Formulating the Travelling Salesman Problem (TSP)

When $i = j$, $x_{ij}$ does not exist so it is not included in the model. We now give the mathematical programming formulation of the asymmetric TSP [6].

Minimize: $\sum_{i \neq j} c_{ij} x_{ij}$

( Minimize total cost of tour)

SubjectTo: $\sum_{j \neq i} x_{ij} = 1; \ i = 1,...,n$ (1)

( Leave each city exactly once)

$\sum_{i \neq j} x_{ij} = 1; \ j = 1,...,n$ (2)

(Visit each city exactly once)

$x_{ij} \in \{0,1\}; \ i,j = 1,...,n$ (3)

($x_i$ is a binary decision variable)

$\{(i,j): x_{ij} = 1, \ i,j = 2,...,n\}$

$\sum_{i,j \in S} x_{ij} = |S| - 1; S \subseteq \{2,...,n\}, 2 \leq |S| \leq n-1$ (4)

( Subtour elimination).

4-Methods of solving a TSP

In literatures there are many different ways to solve TSP, efficiency techniques and also results. Let us refer to a summary of the most widely used methods:

4-1- Branch and Bound algorithm

The B And B algorithm starts with the optimal solution associated with the allocation
problem, If the solution represents a path, the process ends, otherwise we impose constraints to remove the sub-tour, this can create as many different branches as variables associated with one of the sub-tour, each branch represents put one of the variables for the sub-tour equal to zero [1]. Initially before solving the problem, that we specify the upper bound select any rout connected (that does not contain sub-tour), and preferably use intuitions because it produce a higher limit than any rout, then we solve the problem as a normal allocation problem If the solution represents a Hamiltonian cycle (that does not contain sub-tour), the solution will stop and we consider the resulting solution is the optimal solution, If the solution to the problem of allocation does not represent Hamiltonian cycle, we’ll assign the resulting solution as a lower bound, and that any solution that produces greater or equal to the upper bound path will ignore. Then select one of the sub routs to branch it preferably the selected sub rout contains the minimum number of cities (node), because it creates fewer braches. Note that the basic idea of branching to smashing one of the sub routs and modifying organic variables for the other sub-rout automatically.

4-2 Nearest neighbor

Intuitive methods are defined as a guessing state for the priority of choosing a point for another within the solution for some objectives often intuitions can find good solutions to the problem but they may not be optimal solutions. A good solution can be found to the problem of a traveling salesman by starting from the city the specific node, and then connecting it with the nearest city that has not been visited before, and continues the process until the Hamiltonian cycle is formed [4].

1. Choose the city randomly.
2. Find the node closest to it and non-visited.
3. Is there a node that has not been visited?
   If the answer is yes, repeat step 2.
4. We return to the city from which we started.

Thus we get the Hamiltonian cycle with \(O(n^2)\), this method is useful and highly efficient because there is only one path to be formed, but it may not reach to the objective well.

4-3 Two-way exchange improvement heuristic

This method is also called Two-optimal improvement heuristic; the basic principle of this method is to modify the solution to a better solution. By modifying the tour, two arcs are deleted and reconnected the paths in a different way which reduces the total distance between nodes of the network until no deleted pair of arcs is found [8].

5-The Main Features of Decision Making of Multi-Objective

It can be said that the decision-maker actually seeks to achieve several objective; therefore the traditional model (one-objective) is no longer appropriate for him. The traditional framework for analyzing decision-making, presumably assume that there are three elements, Decision maker (personal or organization defined as a single entity), a set of available choices, and finally specific criteria (objective). Specific criteria are used to associate them with a number of alternatives so it can be arranged in the form of a set to get the optimal value that can be achieved from the selected objectives, Decision makers often do not mind to organize a set of possible solutions that are subject to one (objective) criterion but prefer the presence of a centrist compromise solution involves several objectives[2].

6-Definition of efficient solution

A solution \(\{x^1 \in X\}\) is called efficient if and only if there is no other solution \(\{x \in X\}\) where

\[f_k(x) \geq f_k(x^1) \quad \forall k \quad (k = 1,2,\ldots,p)\].

The inequality being strict for at least one \(\{k \ (f_k(x) > f_k(x^1))\}\). Each solution \(x\) has a point \(F = (f_1(x), \ldots, f_p(x))\) as representation in the objective function space.
7- Optimizing a Weighted-Sum of the Objective Functions

The process of computation of (efficient/non-dominated) solutions more utilized consists in solving a scalar problem in which the objective function is a weighted-sum of the \( p \) original objective functions with positive weights \( \omega_k \):

\[
\text{minimize (or maximize) } F_W = \sum_{k=1}^{p} \omega_k f_k(x)
\]

**Subject To:** \( x \in X \quad (5) \)

If \( x^1 \in X \) is a solution to the problem \( \min_{x \in X} \sum_{k=1}^{p} \omega_k f_k(x) \) for \( W = (\omega_1, \ldots, \omega_p) \), where \( \omega_k > 0 \), \( k = 1, \ldots, p \), and \( \sum_{k=1}^{p} \omega_k = 1 \), then \( x^1 \) is an efficient solution to the multi-objective problem. The truthfulness of this proposition can be shown as follows. Suppose that \( x^1 \) is not efficient. Then, there is an \( x^2 \in X \) such that \( f_k(x^2) \geq f_k(x^1), \ k = 1, \ldots, p \), and the inequality is strict for at least one \( k \). But \( x^1 \) was obtained by optimizing a weighted-sum objective function with strictly positive weights then \( \sum_{k=1}^{p} \omega_k f_k(x^2) > \sum_{k=1}^{p} \omega_k f_k(x^1) \), which contradicts the hypothesis that \( x^1 \) minimizes the weighted-sum objective function [7].

8-Data type

Before building the mathematical model of the problem, we must identify our data and statement qualitatively; therefore we will define the model data type, the data related to the problem are concerned with objectives placed by the decision maker and these objectives are defined according to the following indicators:

- Choose the route that achieves the lowest time it takes to reach between any two cities in the tour; this objective is expressed by indicator (time).
- Choose the shortest route possible connecting between any two cities in the tour; this objective is expressed by indicator (distance).
- Choose the route that achieves the lowest cost to reach between any two cities in the tour; this objective is expressed by indicator (cost). Data problem (cost, distance, and time) was obtained by the web sites [9].
Table (1-A)

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
<th>Cost (C)</th>
<th>Distance (D)</th>
<th>Time (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>39.5 million</td>
<td>$2,345</td>
<td>1,234 miles</td>
<td>32 minutes</td>
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<td>Colorado</td>
<td>6.2 million</td>
<td>$1,234</td>
<td>456 miles</td>
<td>2 hours</td>
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<td>Arizona</td>
<td>7.2 million</td>
<td>$3,456</td>
<td>876 miles</td>
<td>45 minutes</td>
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<tr>
<td>Nevada</td>
<td>3.0 million</td>
<td>$4,567</td>
<td>987 miles</td>
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<td>26.5 million</td>
<td>$6,789</td>
<td>1,101 miles</td>
<td>6 hours</td>
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<tr>
<td>Louisiana</td>
<td>4.6 million</td>
<td>$7,890</td>
<td>1,191 miles</td>
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<tr>
<td>Kentucky</td>
<td>4.2 million</td>
<td>$8,901</td>
<td>1,282 miles</td>
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</tr>
<tr>
<td>West Virginia</td>
<td>1.8 million</td>
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<td>1,373 miles</td>
<td>9 hours</td>
</tr>
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</table>

Note: The data collected belong to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes.
Table 1-B

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The data collected belong to the problem where cost (C) is by dollars, distance (D) by miles and time (T) by minutes.

Barraq Subhi Kamil
Represent the data collected below to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes.

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</table>

**Table (1-C)**

**Table (1-C)**
Table (1-D) 

Represent the data collected below to the problem where cost (C) by dollars, distance (D) by miles and time (T) by minutes.

<table>
<thead>
<tr>
<th>City</th>
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<th>D</th>
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</tr>
<tr>
<td>Indiana</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iowa</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Missouri</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nevada</td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arizona</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Colorado</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>New Mexico</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>California</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Washington</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table (1-D)*
9-The Practical Part
A multi-objective linear programming model will be built to solve the problem of a traveling sales man in the United States by formulating the (TSP) which is mentioned in paragraph (3) and using the data shown in the above tables, as follows:

9-1 Decision Variables
Let \( x_{ij} \) represent the binary variable or in other words

\[
x_{ij} = \begin{cases} 
1, & \text{if the arc from } i \text{ to } j \text{ is selected} \\
0, & \text{otherwise}
\end{cases}
\]

for all \( i \neq j, i, j = 1,2,\ldots,n \)

9-2 Objective functions
The three objective functions (cost, distance and time) \( \{f_1, f_2, f_3\} \) are formulated respectively as follows:

Firstly, the objective function of achieving the maximum reduction of the total cost

\[
\text{Minimize } f_1(x) = \sum_{i=1}^{20} \sum_{j=1}^{20} c_{ij} x_{ij}
\]

Secondly, the objective function of achieving the maximum reduction of the total distances

\[
\text{Minimize } f_2(x) = 953 x_{1,2} + 144 x_{1,3} + 1298 x_{1,4} + 1483 x_{1,5} + 876 x_{6,} + 1937 x_{7,} + 1015 x_{8,} + 1729 x_{9,} + 1470 x_{10,} + 2159 x_{11,} + 1983 x_{12,} + 2439 x_{13,} + 2607 x_{14,} + 2386 x_{15,} + 664 x_{16,} + 2719 x_{17,} + 3061 x_{18,} + 2794 x_{19,} + 2734 x_{20,} + 2647 x_{20,2} + 2078 x_{20,3} + 1668 x_{20,4} + 1711 x_{20,5} + 2425 x_{20,6} + 1404 x_{20,7} + 1199 x_{20,8} + 1020 x_{20,9} + 2063 x_{20,10} + 1307 x_{20,11} + 588 x_{20,12} + 866 x_{20,13} + 1014 x_{20,14} + 670 x_{20,15} + 419 x_{20,16} + 511 x_{20,17} + 805 x_{20,1} + 388 x_{20,19} \quad (7)
\]

Thirdly, the objective function of achieving the maximum reduction of the total time

\[
\text{Minimize } f_3(x) = \sum_{i=1}^{20} \sum_{j=1}^{20} t_{ij} x_{ij}
\]

Subject To:
The constraints of the multi-objective problem can be represented in the following mathematical formula:

\[
\begin{align*}
\sum_{j=1}^{20} x_{i(j)} &= 1, \quad \sum_{j=1}^{20} x_{(i)j} = 1, \quad \sum_{j=1}^{20} x_{(i)j} = 1, \\
\sum_{j=1}^{20} x_{(i)j} &= 1, \quad \sum_{j=1}^{20} x_{(i)j} = 1, \quad \sum_{j=1}^{20} x_{(i)j} = 1, \\
\sum_{j=1}^{20} x_{(i)j} &= 1, \quad \sum_{j=1}^{20} x_{(i)j} = 1, \quad \sum_{j=1}^{20} x_{(i)j} = 1,
\end{align*}
\quad (9)
\]
9-4 Build a weighted mathematical model to solve the multi-objective of (TSP)

The weighted mathematical model of the multi-objective of (TSP) can be represented as follows:

$$\min F_W = \sum_{k=1}^{3} \omega_k f_k(x)$$

subject to:

The same constraints (6, 7 and 8) as are mentioned above

The model is solved as follows:

Let us consider that the weights given by the decision maker are \( W = \{ \omega_1 = 0.3, \omega_2 = 0.5, \omega_3 = 0.2 \} \). the total of these weights is \( \sum_{k=1}^{3} \omega_k = 1 \).

The weights given are the product of the decision maker's experience. In his view, the cost and time are dependent on the distance. Whenever the distance is low, result the cost and time are low, vice versa, so after substituting the weights in the model (9), we get the following new model:

$$\min F_{W=\{0.3,0.5,0.2\}} =$$

$$0.3 f_1(x) + 0.5 f_2(x) + 0.2 f_3(x)$$

subject to:

The same constraints (6, 7 and 8) as are mentioned above

After performing the mathematical operations by multiply the weights by the objectives and then collecting the objectives to be a single objective, i.e. converting the problem multi-objective to the problem of one-objective based on the following tables:
Table (2-A)

Represent the weights which multiplied by the objective functions.
Table (2-B)

Represent the weights which multiplied by the objective functions.

<table>
<thead>
<tr>
<th>Region</th>
<th>Weight 1</th>
<th>Weight 2</th>
<th>Weight 3</th>
<th>Weight 4</th>
<th>Weight 5</th>
<th>Weight 6</th>
<th>Weight 7</th>
<th>Weight 8</th>
<th>Weight 9</th>
<th>Weight 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Florida</td>
<td>1.1</td>
<td>0.95</td>
<td>0.85</td>
<td>0.75</td>
<td>0.65</td>
<td>0.55</td>
<td>0.45</td>
<td>0.35</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Georgia</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>Kentucky</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Alabama</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Louisiana</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Missouri</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Indiana</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Illinois</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
</tr>
<tr>
<td>Iowa</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Ohio</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Texas</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Nevada</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
<td>1.3</td>
</tr>
<tr>
<td>New Mexico</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
<td>1.4</td>
</tr>
<tr>
<td>Colorado</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>Arizona</td>
<td>2.5</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
</tr>
<tr>
<td>California</td>
<td>2.6</td>
<td>2.5</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>Washington</td>
<td>2.7</td>
<td>2.6</td>
<td>2.5</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
</tr>
<tr>
<td>Utah</td>
<td>2.8</td>
<td>2.7</td>
<td>2.6</td>
<td>2.5</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Virginia</td>
<td>2.9</td>
<td>2.8</td>
<td>2.7</td>
<td>2.6</td>
<td>2.5</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
<td>2.0</td>
</tr>
<tr>
<td>New York</td>
<td>3.0</td>
<td>2.9</td>
<td>2.8</td>
<td>2.7</td>
<td>2.6</td>
<td>2.5</td>
<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Total</td>
<td>10.00</td>
<td>9.00</td>
<td>8.00</td>
<td>7.00</td>
<td>6.00</td>
<td>5.00</td>
<td>4.00</td>
<td>3.00</td>
<td>2.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
By the Tables (2-A and 2-B) above, the weighted objective function is as follows:

\[ Min \ F_w = 666.2x_{1,2} + 228.6x_{1,3} + 750x_{1,4} + 888.7x_{1,5} + 1238.9x_{2,1} + 675.7x_{3,1} + 1353.1x_{3,2} + 1423.9x_{3,3} + 524.8x_{4,10} + 1038.7x_{4,11} + 1526.6x_{4,12} + 1433.7x_{4,13} + 1689.9x_{5,14} + 1915.1x_{5,15} + 1934.6x_{5,16} + 1884.8x_{6,19} + 1524.7x_{6,20} + 1197.6x_{7,19} + 1257.8x_{7,20} + 770.8x_{8,19} + 770.8x_{8,20} + 1528.3x_{9,10} + 975.8x_{9,11} + 646.3x_{9,12} + 652.4x_{9,13} + 1754.9x_{10,11} + 988.8x_{10,12} + 770.8x_{10,13} + 1528.3x_{10,14} + 975.8x_{10,15} + 646.3x_{10,16} + 652.4x_{10,17} + 592.1x_{10,18} + 320.8x_{10,19} \]

Subject To: The same constraints 6, 7 and 8 as are mentioned above

9.5 Solution for Minimization (Multi-objective Traveling Salesman Problem)

In this section, the model (11) will be solved to obtain the optimal solution using the three methods (branch and bound, nearest neighbor and two-way exchange improvement heuristic) to solve the problem of (TSP), as well as a comparison among the optimal results to be obtained in the following manner, all results depended on the package program WINQSB [5]:

<table>
<thead>
<tr>
<th>Table (3)</th>
<th>Represents the optimal solution by using B and B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>From Node</td>
</tr>
<tr>
<td>1</td>
<td>Node1</td>
</tr>
<tr>
<td>2</td>
<td>Node2</td>
</tr>
<tr>
<td>3</td>
<td>Node6</td>
</tr>
<tr>
<td>4</td>
<td>Node4</td>
</tr>
<tr>
<td>5</td>
<td>Node10</td>
</tr>
<tr>
<td>6</td>
<td>Node11</td>
</tr>
<tr>
<td>7</td>
<td>Node9</td>
</tr>
<tr>
<td>8</td>
<td>Node12</td>
</tr>
<tr>
<td>9</td>
<td>Node19</td>
</tr>
<tr>
<td>10</td>
<td>Node20</td>
</tr>
<tr>
<td>Total</td>
<td>Minimal</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table (4)</th>
<th>Represents the optimal solution by using nearest neighbor algorithm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>From Node</td>
</tr>
<tr>
<td>1</td>
<td>Node1</td>
</tr>
<tr>
<td>2</td>
<td>Node3</td>
</tr>
<tr>
<td>3</td>
<td>Node5</td>
</tr>
<tr>
<td>4</td>
<td>Node4</td>
</tr>
<tr>
<td>5</td>
<td>Node7</td>
</tr>
<tr>
<td>6</td>
<td>Node8</td>
</tr>
<tr>
<td>7</td>
<td>Node13</td>
</tr>
<tr>
<td>8</td>
<td>Node9</td>
</tr>
<tr>
<td>9</td>
<td>Node11</td>
</tr>
<tr>
<td>10</td>
<td>Node12</td>
</tr>
<tr>
<td>Total</td>
<td>Minimal</td>
</tr>
</tbody>
</table>
Table (5)

Represents the optimal solution by using Two-way exchange improvement heuristic.

<table>
<thead>
<tr>
<th>City</th>
<th>From Node</th>
<th>Connect To</th>
<th>Arc value</th>
<th>City</th>
<th>From Node</th>
<th>Connect To</th>
<th>Arc value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Node16</td>
<td>Node12</td>
<td>236.5</td>
<td>11</td>
<td>Node3</td>
<td>Node5</td>
<td>293.4</td>
</tr>
<tr>
<td>2</td>
<td>Node12</td>
<td>Node13</td>
<td>315.5</td>
<td>12</td>
<td>Node5</td>
<td>Node7</td>
<td>359.9</td>
</tr>
<tr>
<td>3</td>
<td>Node13</td>
<td>Node9</td>
<td>260.7</td>
<td>13</td>
<td>Node7</td>
<td>Node8</td>
<td>244</td>
</tr>
<tr>
<td>4</td>
<td>Node9</td>
<td>Node11</td>
<td>312.5</td>
<td>14</td>
<td>Node8</td>
<td>Node14</td>
<td>404.8</td>
</tr>
<tr>
<td>5</td>
<td>Node11</td>
<td>Node10</td>
<td>632.1</td>
<td>15</td>
<td>Node14</td>
<td>Node15</td>
<td>351.4</td>
</tr>
<tr>
<td>6</td>
<td>Node10</td>
<td>Node4</td>
<td>577.4</td>
<td>16</td>
<td>Node15</td>
<td>Node17</td>
<td>231</td>
</tr>
<tr>
<td>7</td>
<td>Node4</td>
<td>Node6</td>
<td>585.9</td>
<td>17</td>
<td>Node17</td>
<td>Node18</td>
<td>267.8</td>
</tr>
<tr>
<td>8</td>
<td>Node6</td>
<td>Node2</td>
<td>445.7</td>
<td>18</td>
<td>Node18</td>
<td>Node20</td>
<td>592.1</td>
</tr>
<tr>
<td>9</td>
<td>Node2</td>
<td>Node1</td>
<td>666.2</td>
<td>19</td>
<td>Node20</td>
<td>Node19</td>
<td>320.8</td>
</tr>
<tr>
<td>10</td>
<td>Node1</td>
<td>Node3</td>
<td>228.6</td>
<td>20</td>
<td>Node19</td>
<td>Node16</td>
<td>535.7</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>Minimal $F_w$</td>
<td>$= 7,862.00$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After finding the optimal solutions above, a table will be made to compare the optimal solutions after substitution the optimal binary decision variables in the three objective functions (cost, distance and time) as shown in Table (6) below.

Table (6)

Represents a comparison of optimal solutions with the given weights.

<table>
<thead>
<tr>
<th>Index</th>
<th>Methods (TSP)</th>
<th>Objective fn1. (cost)</th>
<th>Objective fn2. (distance)</th>
<th>Objective fn3. (time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Branch &amp; Bound</td>
<td>3688</td>
<td>10694</td>
<td>9082</td>
</tr>
<tr>
<td>2</td>
<td>Nearest neighbor</td>
<td>4134</td>
<td>12299</td>
<td>10244</td>
</tr>
<tr>
<td>3</td>
<td>Two-way exchange improvement</td>
<td>3817</td>
<td>10009</td>
<td>8562</td>
</tr>
</tbody>
</table>

| Weight | 0.3 | 0.5 | 0.2 |

The objectives as important in terms of the weighted preference of the decision maker can be summarized as follows:

The highest weight (0.5) is for distance that is the second objective, heuristic algorithm gave the maximum reduction.

The middle weight (0.3) is for cost that is the first objective; B&B algorithm gave the maximum reduction.

The lowest weight (0.2) is for time that is the third objective, heuristic algorithm gave the maximum reduction.

Since the decision-maker is looking for reduce the distance to adopt the time and cost, The Heuristic algorithm is the best solution for the problem and the optimal path is as shown in Table (10).
Table (7)
Represents the optimal path (rout) by Heuristic algorithm.

<table>
<thead>
<tr>
<th>Index</th>
<th>City (From - To)</th>
<th>Optimal Route</th>
<th>Cost$</th>
<th>Distance\mile</th>
<th>Time\minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Washington - Arizona</td>
<td>Start Travel (1) -</td>
<td>332</td>
<td>144</td>
<td>285</td>
</tr>
<tr>
<td>2</td>
<td>Arizona - New Mexico</td>
<td>(\text{\textdagger} \rightarrow \text{\textdagger} )</td>
<td>227</td>
<td>433</td>
<td>44</td>
</tr>
<tr>
<td>3</td>
<td>New Mexico - Texas</td>
<td>(\text{\textdagger} \rightarrow \text{\textdagger} )</td>
<td>183</td>
<td>450</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>Texas - Oklahoma</td>
<td>(\text{\textdagger} \rightarrow \text{\textdagger} )</td>
<td>45</td>
<td>341</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>Oklahoma - Louisiana</td>
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Total cost, distance and time 3817 10009 8562

Fig.(1): Illustrates the optimal rout.
Conclusions
The weighted model demonstrated the effectiveness and flexibility to solve real problems of multi-objective (TSP), where it can be said that it is impossible to solve this problem without resorting to multi-objective mathematical models. In other words, the number of possible routs for the 20 US states is \( \binom{n-1}{1} = 19! = 121645100408832000 \), to find the optimal routs among these routs it takes very long time and a lot of effort, here stand out importance of two-way exchange improvement heuristic algorithm, where this rout is satisfactory to the decision maker in terms of cost, distance and time.

References