Journal of Al-Nahrain University Vol. 9(1), June, 2006, pp.69-74 Science

An Adaptive Iterative Images Restoration using Projection onto Convex Sets Method

Ayad A. Al-Ani

Department of Physics, College of Science, Al-Nahrain University, Jadriyah, Baghdad, Iraq

E-mail: ayadalanii@yahoo.com

Abstract

The aim of this paper is to construct an iterative restoration filter for restoring degraded (blurred and noisy) images using projection onto convex sets algorithm. In this work, our adaptation was assumed that the weighting matrix is a Sobel operator. The reason of this modification was assumed that the optimization problem can be interpreted as finding the image with the least curvature among the solution of equation $\mathbf{g} = \mathbf{Hf} + \mathbf{n}$. Results using adaptive iterative filter were compared, qualitati-vely and quantitatively, using mean square error (MSE). Results shows that the adaptive filter has better performance for restoring the degraded image.

Introduction

Image degradation model due to linear imaging system is given by [1,2]:

$$g = Hf + n \qquad (1)$$

Where, g, f, and H are the image, the object, and the imaging operator, respectively. Here, H (blurring function) does not necessarily space invariant operator [1]. **n** is a white Gaussian noise.

Equation (1) takes the form of a matrix-vector equation for the case of sampled signal. We prefer this notation since it is compact and general.

The ultimate goal of restoration techniques is to improve a given degraded image using some priori knowledge of the degradation phenomenon. Thus, restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image.

In general, the restoration method of finding the best estimate $\hat{f}$ of the unkown object function f is usually formulated as a constrained optimization problem, such that [3]:

$$\left\| g - H\hat{f} \right\|^2 \le \varepsilon^2 \qquad (2)$$

Where $\hat{\mathbf{f}}$ is the restored image, ε is a positive parameter depending on the noise energy. $\|\ \|^2$ is the norm function.

The estimate of $\hat{f}$ is found by minimizing an error function $E(\hat{f})$ with respect to $\hat{f}$ [2], i.e.

$$E(\hat{f}) = \left\| g - H\hat{f} \right\|^2 + \alpha \left\| C\hat{f} \right\|^2 \qquad (3)$$

The solution is readily found by solving the equation:

$$\frac{\partial E(\hat{f})}{\partial \hat{f}} = 0 \qquad (4)$$

The solution of the above equation is given by [3]:

$$\hat{f} = \left[H^T H + \alpha C^T C \right]^{-1} H^T g \qquad (5)$$

Where, C is some suitable weighting matrix. The constrained problem can be transformed into an unconstrained one using the method of Lagrangian multipliers. α, is a regularization parameter. $[\ ]^{-1}$, is the inverse of matrix regularization. T is the transpose of a function.

Projection onto Convex sets Method

In this section, we derive the algorithm for the adaptive iterative restoration technique using Projection onto Convex sets method.

Projection onto Convex sets or briefly POCS method defined as follows [4].

A subset of a vector space is called convex if it contains the line segment between any bair of point in the set. Thus if $f_1, f_2 \in S$ and $f_3 = \alpha f_1 + (1-\alpha) f_2$, where $\alpha \in [0,1]$, then $f_3 \in S$. A simple example of a convex set is:

$C_1 = \{ f : \| g - Hf \| \langle\ \varepsilon \}$, that is the set of all vectors f satisfying the constraint in Eq.(2)

Now, we have:

$$\begin{aligned} \| g - Hf_3 \| &= \| g - H(\alpha f_1 + (1-\alpha) f_2) \| \\ &= \| \alpha (g - Hf_1) + (1-\alpha)(g - Hf_2) \| \\ &\le \alpha \| g - Hf_1 \| + (1-\alpha) \| (g - Hf_2) \| \\ &\langle\ \varepsilon \end{aligned}$$

Another example of a convex set is $C_2 = \{ f \mid f_i \geq 0, \forall_i \}$, that is the set of all vectors with nonnegative elements. In typical restoration problem the true solution is a member of C_1, since it satisfies the imaging equation (Eq.(1)), and is a member of C_2, that is an intensity vector. A projection operator P_i belonging to a convex set C_i, is the operator that maps a given vector to the closest point $P_i f$ in C_i, so that $\| f - P_i f \|$ is minimize. From Figure (1), the initial guess represents the object image, i.e. $\hat{f}_0 = g$. The operator P_1 projects $\hat{f}_0$ to $\hat{\mathbf{f}}_1$ on the boundary of C_1. The vector $\hat{f}_1$ is thus a solution of the optimization problem

$$\min \left\| \hat{f}^1 - \hat{f}^0 \right\| \qquad \text{and} \qquad \left\| g - Hf^1 \right\| \leq \varepsilon$$

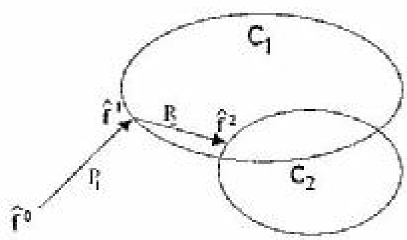


Fig.(1) Two steps in the POCS Method [4]

The operator P_2 projects $\hat{f}_1$ onto C_2. This operator sets negative components of $\hat{f}^1$ to zero. In the case of an arbitrary number of constraints, $C_1, C_2, \ldots\ldots C_N$, repeated application of the projection operators $P_1, P_2, \ldots\ldots P_N$ can be proven to converge to a point:

$$\hat{f} \in C_1 \cap C_2 \cap \ldots\ldots \cap C_N .$$

We will now use the POCS method to investigate a restoration method. The restoration method, formulated as a constrained optimization problem is given by [4]:

$$\min_{\Delta \hat{f}} \Delta \hat{f}^T C \Delta \hat{f} \qquad \text{and} \qquad \left\| g - Hf \right\| \leq \varepsilon$$

Where $\Delta \hat{f}$ is the correction vector or optimization step and C is a diagonal weighting matrix. The restoration method can also be formulated as an unconstrained problem,

$$\min_{\Delta \hat{f}} \left\| g - H \right\|^2 + \alpha \Delta \hat{f}^T C \Delta \hat{f}$$

Where, α is the regularization parameter. The restoration problem is solved iteratively using the recurrence equation [5, 6]:

$$\hat{f}^{k+1} = \hat{f}^{k} + \left[H^T H + \alpha C \right]^{-1} H^T \left(g - H \hat{f}^{k} \right) \text{(6)}$$

$$\hat{f}^{k+1} = \hat{f}^{k} + W \left(g - H \hat{f}^{k} \right) \ldots\ldots\ldots\ldots(7)$$

Where, $W = \left[H^T H - \alpha C \right]^{-1} H^T$, k is the number of iteration.

The conventional method assumed that C (weighting matrix) is a Laplacian operator. In this work, our adaptation was assumed that C is a Sobel operator " S(x,y) ", as given in equation [1].

$$S(x,y) = \begin{bmatrix} 1 & 0 & \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

This modification was assumed that the optimization problem can be interpreted as finding the image with the least curvature among the solution of equation (1).

Results and Discussion

A color image of 128*128 pixels size, **Stork bird-image, as shown in figure (3-a)**, was used to check the quality of the adaptive iterative restoration filter.

The degraded (blurred and noisy) images were simulated as follows:

1-The blurred images were simulated by convolving the original image with Circular function of Radius (R), one value of R=1 has been taken.

2-Random noise of Gaussian distribution with zero means was added to the blurred image (obtained in step 1). Different SNR = 10 dB, 20 dB, 30 dB, 40 dB, and 50 dB, have been taken.

Figure (3-b to 3-f) shows the original image after degraded with Circular blurring function of radius (R=1) and with additive Gaussian noise with signal to noise ratio (SNR) are 10 dB, 20 dB, 30 dB, 40 dB, and 50 dB, respectively. The figure shows, also, the Mean Square Error (MSE) of the degraded image with respect to the original image.

3- To restore the above simulated degraded images, we construct the procedure as follows:

Step 1: $\hat{f}^1 = g$

Step 2: Simulate H (which was assumed to be as a real one

Step 3: Calculating W using the formula:

$$W = \left[H^T H + \alpha C\right]^{-1} H^T$$

The above equation was solved using the inverse method [7]. Here, we note that the best value of α was found equal to one. C was adapted as to be Sobel operator, as shown in **Fig.(2)**.

Step 4: Calculating $\hat{f}^{k+1}$ as given in equation (7), also by using the inverse method

Step 5: Comparing between $\hat{f}^{k+1}$ and $\hat{f}^k$, i.e.

$if \left| \hat{f}^{k+1} - \hat{f}^k \right| \leq \psi$, then end else go to step 4

Note that, by experience and for programming simplicity, we fixed the no. of iteration to 30. since the convergence is done before 20th iteration. But, for more safety and certainty we fixed number of iteration equal to 30.

Figure (4) shows the restored images and the corresponding MSE for the degraded image that blurred with circular blur of R=1 and SNR=10 dB, after 1 iteration, 10 iterations, 20 iterations. and 30 iterations, respectively.

Figure (5) shows the restored images and the corresponding MSE for the degraded image that blurred with circular blur of R=1 and SNR=20 dB, after 1 iteration, 10 iterations, 20 iterations, and 30 iterations, respectively. **Figure (6)** shows the restored images and the corresponding MSE for the degraded image that blurred with circular blur of R=1 and with SNR=30 dB, after 1 iteration, 10 iterations, 20 iterations, and 30 iterations, respectively. **Figure (7)** shows the restored images and the corresponding MSE for the degraded image that blurred with circular blur of R=1 and with SNR=40 dB, after 1 iteration, 10 iterations, 20 iterations, and 30 iterations, respectively. **Figure (8)** shows the restored images and the corresponding MSE for the degraded image that blurred with circular blur of R=1 and with SNR=50 dB, after 1 iteration. 10 iterations, 20 iterations, and 30 iterations, respectively

Figure (9) shows the MSE for the restored images versus number of iterations for the degraded image blurred with circular blur of R=1 and with SNR=10 dB, 20 dB, 30 dB, 40 dB, and 50 dB, respectively.

Conclusion

The aim of this method is to construct an iterative restoration filter for restoring degraded (blurred and noisy) images using projection onto convex sets algorithm.

From the above Figures, we can conclude that the adapted iterative restoration filter is an efficient method for restoring the degraded images. We also, conclude that the mean square error of the restored images decreases with the increasing the number of iterations until the result convergent. Moreover, the convergence, mostly, happens after 15 iterations. Finally. the adapted filter is given better performance for small values of SNR.

References

[1] R.C. Gonzalez and P. Wintz, "Digital Image Processing", 2nd ed., Addison Wisely Publishing Company (1989)

[2] H.C. Andrews and B.R. Hunt, "Digital Image Restoration", Prentice - Hill - Jersey (1977).

[3] W.K.Pratt, Digital image Processing, 3rd Ed., New York; John-Wiley and sons (2001)

[4] Olle Seger, Model building and restoration with application in confocal microscopy, Linkoping University (1993)

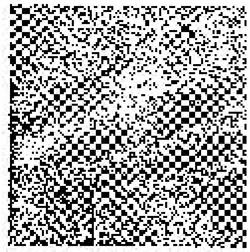
[5] D.G.Manolakis, Statistical and Adaptive signal processing: Spectral estimation, signal modeling, adaptive filtering and array processing, Boston, McGraw-Hill, (2000)

[6] M.Sonka, V.Halavac, and R.Boyle, Image Processing, Analysis, and Machine Vision, PWS Publishing (1999)

[7] C.R.Vogel, Computational Methods for inverse problems. SIAM, Philadelphia (2002).

a

original



b

MSE = 2472

c

MSE =359

d

MSE =144

e

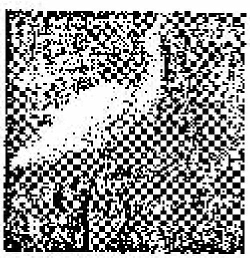
MSE =122

f

MSE =120

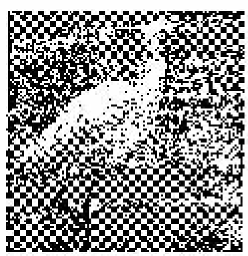
Fig. (3)

(a) original bird image of size 128x128 pixels
(b) degraded image of (a) with Circular function of R = 1 and SNR = 10
(c) degraded image of (a) with Circular function of R = 1 and SNR = 20
(d) degraded image of (a) with Circular function of R = 1 and SNR = 30
(e) degraded image of (a) with Circular function of R = 1 and SNR = 40
(f) degraded image of (a) with Circular function of R = 1 and SNR = 50



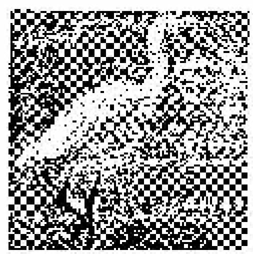
a

MSE =148.66



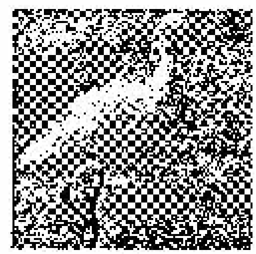
b

MSE =127.56



c

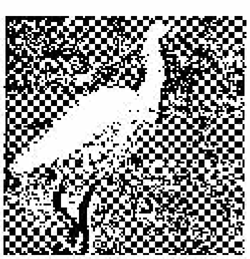
MSE =120.20



d

MSE =115.61

Fig. (4)
Restored images of the degraded image shown in Fig. (3-b)
(degraded with circular function of R=1 and SNR=10) after :
(a) 1 iteration (b) 10 iteration (a) 20 iteration (b) 30 iteration

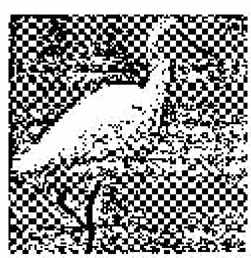


a

MSE = 60.90

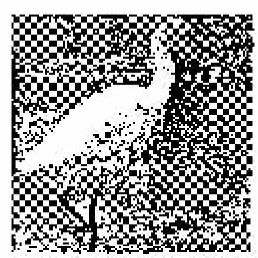
b

MSE = 55.11



c

MSE = 54.96



d

MSE = 55.02

Fig. (5)
Restored images of the degraded image shown in Fig. (3-c)
(degraded with circular function of R=1 and SNR=20) after :
(a) 1 iteration (b) 10 iteration (a) 20 iteration (b) 30 iteration

a	b	c	d
MSE = 34.39	MSE = 26.10	MSE = 25.78	MSE = 25.70

Fig. (6)
Restored images of the degraded image shown in Fig. (3-d)
(degraded with circular function of R=1 and SNR=30) after :
(a) 1 iteration (b) 10 iteration (a) 20 iteration (b) 30 iteration

a	b	c	d
MSE = 14.67	MSE = 7.34	MSE = 7.11	MSE = 7.05

Fig. (7)
Restored images of the degraded image shown in Fig. (3-e)
(degraded with circular function of R=1 and SNR=40) after :
(a) 1 iteration (b) 10 iteration (a) 20 iteration (b) 30 iteration

a	b	c	d
MSE = 2.83	MSE = 0.942	MSE = 0.92	MSE = 0.91

Fig. (8)
Restored images of the degraded image shown in Fig. (3-f)
(degraded with circular function of R=1 and SNR=50) after :
(a) 1 iteration (b) 10 iteration (a) 20 iteration (b) 30 iteration

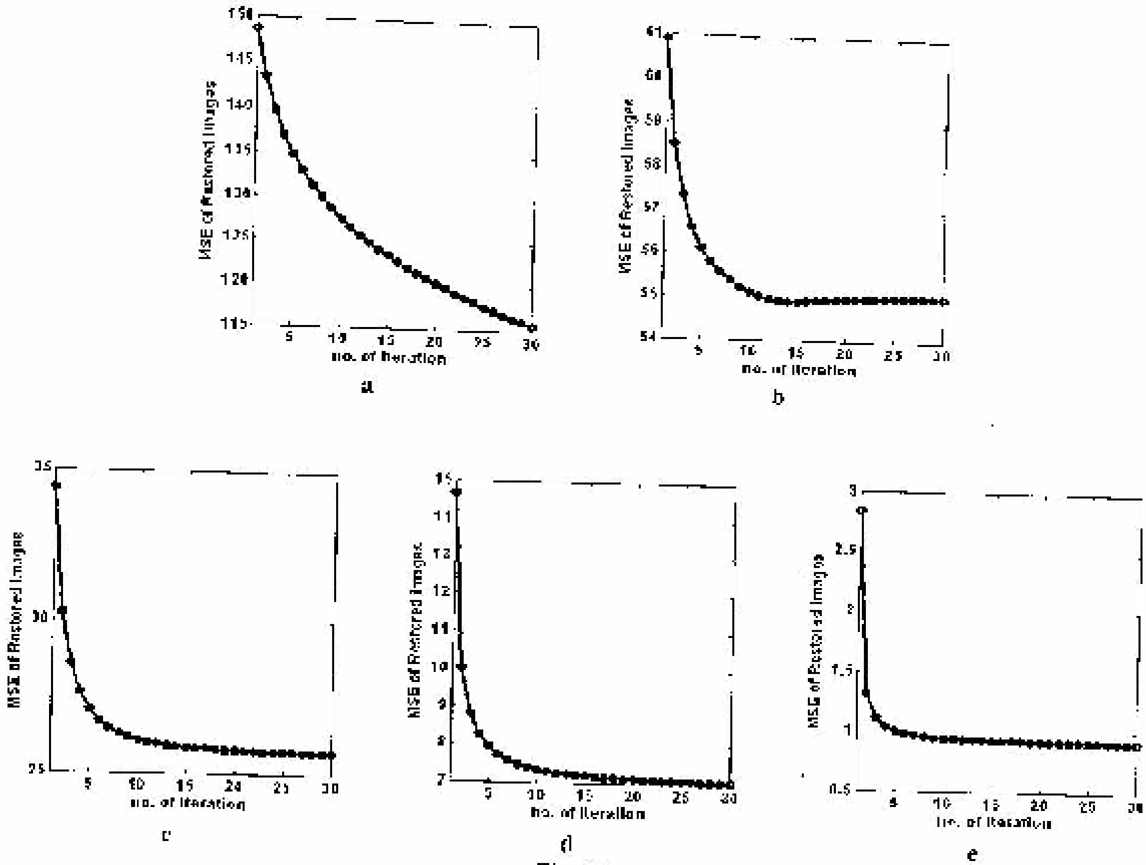


Fig. (9)

Mean Square Errors (MSE) for the restored images Versus no. of iterations for:

(a) degraded bird image, with Circular function of R = 1 and SNR = 10
(b) degraded bird image, with Circular function of R = 1 and SNR = 20
(c) degraded bird image, with Circular function of R = 1 and SNR = 30
(d) degraded bird image, with Circular function of R = 1 and SNR = 40
(e) degraded bird image, with Circular function of R = 1 and SNR = 50