Journal of Al-Nahrain University Vol. 9(2), December, 2006, pp.111-115 Science

Determination of the Hubble Constant in the Presence of Shear and Non-Isothermal Models

[1]Ahmed M. A. Al-Ani, [2]Ayad A. Al-Ani*, and [1]Mohamed A. Salih

[1]Department of Astronomy, College of Science, Baghdad University

[2]Department of Physics, College of Science Al-Nahrain, University,Jadriyah, Baghdad, Iraq

Abstract

In our present work two models have been adopted for estimating the Hubble constant. These models are the effect of shear γ in the lens model and also the non isothermal model. These models have been applied for six gravitationally lensed systems. These systems are: B 0218+357 / B-A, Q 0957+561 / B-A, PG 1115+080 / A-B, PG 1115−080 / C-B, B 1600+434 / B-A, and PKS 1830-211 / B-A . The results shows that an external shear γ changes the time delay by an amount proportional to the shear strength, and affects 4-image lenses more than 2-image lenses. Furthermore, we find, that such small core radius changes the time delays negligibly ≤ 1%.

The effect of shear on the lens model

The environment of a galaxy, including any cluster surrounding the primary lens, will in general contribute both convergence and shear. The effective potential due to the local environment should be take in care in some gravitationally lensed observed systems.

In the principal axes system of the external shear, where the convergence and shear are locally independent of the deflection angle of lensing. An external shear breaks the circular symmetry of a lens and therefore, it often has the same effect as introducing ellipticity in the lens. It is frequently possible to model the same system either with an elliptical potential or with a circular potential plus an external shear.

Time Delay in the Presence of Shear

In many gravitational lenses the images cannot be fit without inclusion of a tidal perturbation from objects near the lens galaxy or along the line of sight [1,2]. The lowest order of the perturbation " ϕ_γ " can be modeled as an external shear with potential [3]:

$$\phi_\gamma = -\frac{1}{2}\gamma r^2 \cos 2(\theta - \theta_\gamma) = -\frac{1}{2}\left[\gamma_1 (x^2 - y^2) + 2\gamma_2 x y\right] \quad (1)$$

Where γ is the strength of the shear and θ_γ is the direction with respect to the lens galaxy's major axis, while:

$\gamma_1 = \gamma \cos 2\theta_\gamma$ and $\gamma_2 = \gamma \sin 2\theta_\gamma$.

The total potential is then;

$\Phi_{tot} = \Phi + \Phi_\gamma$,

and the lens equation can be written as

$$\xi = x - \phi_x + \gamma_1 x + \gamma_2 y \quad , \quad \eta = y - \phi_y + \gamma_2 x + \gamma_1 y \quad (2)$$

The relative time delay " $\Delta t(x,y)$ " between images in gravitational lens system is proportional to the scale of the universe and thus, is inversely proportional to the Hubble parameter, and is given by [4]:

$$\Delta t(x,y) = \frac{D}{2c}(1+z_l)\left[\xi^2 + \eta^2 - r^2 - r^2\gamma \cos 2(\theta - \theta_\gamma)\right] \quad (3)$$

Where $D = \frac{D_l D_s}{D_{ls}}$

While, the relative time delay between two images **i** and **j** in the presence of shear is given by **[4]**:

$$\Delta t_{ij} = \frac{D}{2c}(1+z_l)\left\{\begin{array}{l}(r_j^2 - r_i^2) - \\ \gamma\left[r_j^2 \cos 2(\theta_j - \theta_\gamma) - r_i^2 \cos 2(\theta_i - \theta_\gamma)\right]\end{array}\right\} \quad (4)$$

This time delay depends on the shear amplitude and direction and therefore cannot be determined without detailed modeling. Nevertheless, we can make several remarks. The change in the time delay (relative to the no-shear case) is proportional to γ . For two-image lenses that have images at different distances $r_i \neq r_j$ and a small shear, the shear should have a small effect on the time delay. However, when the images lie at approximately the same distance $r_i \approx r_j$, such as in some four-image lenses, the shear term may be significant.

In particular, perturbation theory reveals that in the presence of a shear the distance of an image from the critical curve scales ($\alpha\ \gamma$) to lowest order [5], so the two terms in equation (4) can be comparable. Although, we cannot determine the time delay without knowing the shear amplitude and direction [6,7]. If the angle between the shear and the lens galaxy major axis is allowed to take on any value, then the time delay is bounded by:

$$T_{i,j}^{(-)} \le \Delta t_{i,j} \le T_{i,j}^{(+)} \qquad \text{.............................(5)}$$

Where $T_{i,j}^{(\pm)}$ is given by[3]:

$$T_{i,j}^{(\pm)} = \frac{D}{2c}(1+z_d)\left\{ \begin{array}{l} (r_j^2 - r_i^2) \pm \\ \gamma\left[r_i^4 + r_j^4 - 2r_j^2 r_i^2 \cos 2(\theta_i - \theta_j)\right]^{n} \end{array} \right\} \quad (6)$$

If the images are directly opposite each other $|\theta_i - \theta_j| = 180$, as in a circular lens, the bounds will be [5]:

$$T_{i,j}^{(\pm)} = (1 \pm \gamma)\,\Delta t_{i,j}^{0} \qquad \text{.....................(7)}$$

Where Δt^0_{ij} is the time delay incase of no-shear. This means that, for example, a 10% shear (γ = 0.1) leads to a 10% uncertainty in the time delay, and hence there is no information about the angle between the shear and the lens galaxy major axis. Applying equations (5), (6) and (7) in order to obtain bounds for specific lenses, then substituting the value of the time delay $\Delta T_{ij}^{(\pm)}$ (under the effect of shear) instead of Δt^0_{ij}, then we get Hubble constant as [5]:

$$H_o = \frac{\{q_o z_s + (q_o - 1)(G_s - 1)\}\{q_o z_d - (q_o - 1)(G_d - 1)\}}{\Delta t_{ij}} \cdot \frac{\{(r_j^2 - r_i^2) \pm \gamma\left[r_i^4 + r_j^4 - 2r_j^2 r_i^2 \cos 2(\theta_i - \theta_j)\right]^{n}\}}{q_o^2} \quad (8)$$

Results and Discussion

Time delay in in the presence of Shear model

In this section we calculate the expected values for H_0 by using observations from table (1) [3] for everyone from the six observed systems, as shown in Figure (1) [3].

Lens / Components	z_l	z_s	$\Delta t_{(i,j)}$ days	r_j (")	r_i (")	$\theta_i - \theta_j$
B 0218+357 / B-A	0.96	0.68	10.5±0.2	0.24±0.06	0.1±0.06	176.4
Q 0957+561 / B-A	1.41	0.36	417±3	5.2275±0.0035	1.034±0.0035	154.2
PG 1115+080 / A-B	1.72	0.31	11.7±1.2	1.147±0.025	0.95±0.004	115.5
PG 1115+080 / C-B	1.72	0.31	25±1.6	1.397±0.004	0.95±0.004	114.6
B 1600+434 / B-A	1.59	0.42	47±6	1.14±0.05	0.25±0.05	179.4
PKS 1830-211 / B-A	2.51	0.89	26±5	0.67±0.08	0.32±0.08	160.5

Table 1

The observational data for six time delay lenses

Where z_l, z_s are the red shifts of the lens, source. Δt is the time delay in days between the two observed images. (r_i", r_j") represents the angular position of the tow images. ($\theta_i - \theta_j$) are the separation angle between the two images A and B [3]

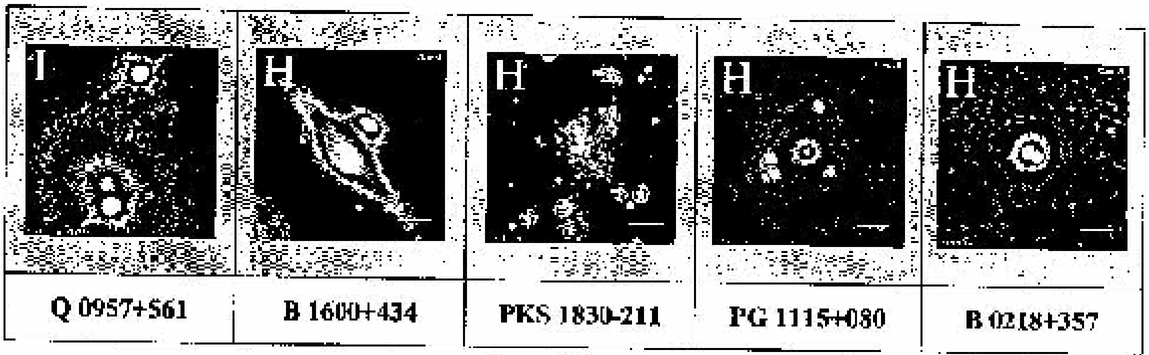

Figure (1)
The Images for the six time delay lenses [3]

Figure (2) to figure (7), shows the limits between H_0 and γ from the strong Lensing in the gravitationally lensed system for B 0218+357 / B-A, Q 0957+561 / B-A, PG 1115+080 / A-B, PG 1115+080 / C-B, B 1600+434 / B-A, and PKS 1830-211 / B-A. We plotted Hubble constant "H_0" against the strength of the shear "γ", for $0 \leq \gamma \leq 0.5$, for each cosmological model and we neglect the error bars in the observations. The bounds are computed using equations (6) and (8). The two sets of bounds refer to the two different time delays. All results are computed with different adopted cosmological models representing by decelerating parameter as follows:

- q_0 = 0.01 for the closed universe model (red curves)
- q_0 = 0.5 for the flat universe model (green curves)
- q_0 = 2 for the open universe model (blue curves).

These figures do not include statistical uncertainties due to observational errors.

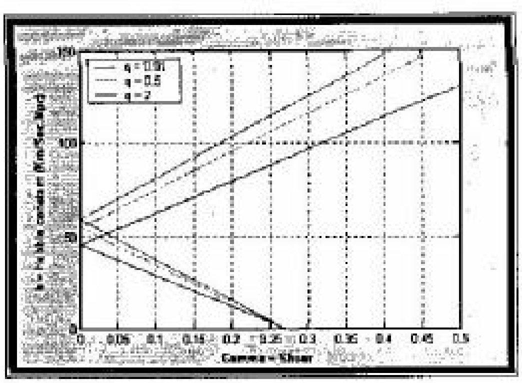

Figure (2)
Limits between H_0 and γ from the strong lensing in the gravitationally lensed system PG 1115+080 A-B for (q_0 = 0.01, 0.5 and 2)

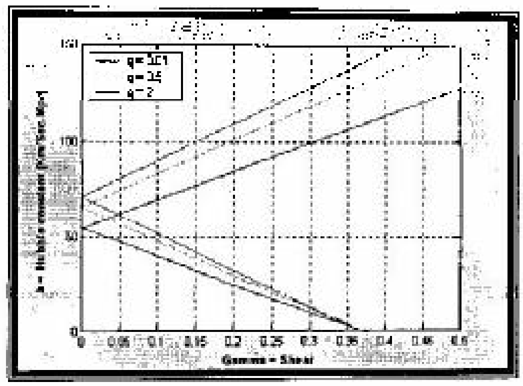

Figure (3)
Limits between H_0 and γ from the strong lensing in the gravitationally lensed system PG 1115+080 C-B for (q_0 = 0.01, 0.5 and 2)

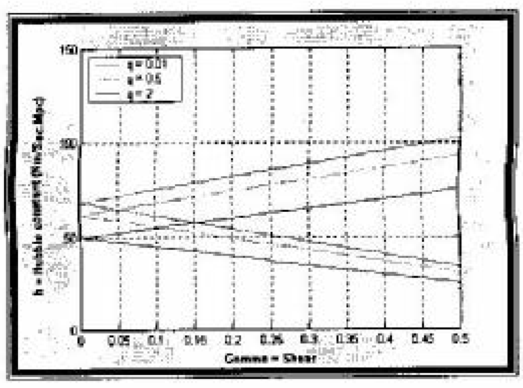

Figure (4)
Limits between H_0 and γ from the strong lensing in the gravitationally lensed system B 1600+434 B-A for (q_0 = 0.01, 0.5 and 2)

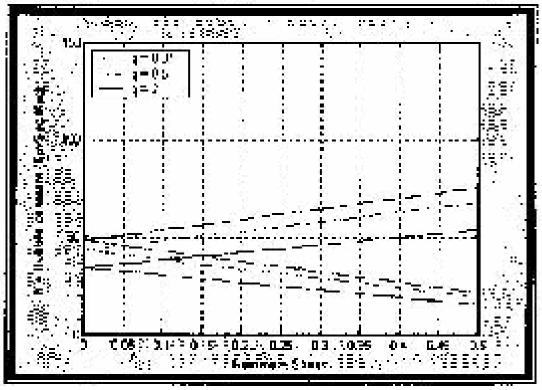

Figure (5)
Limits between H_0 and γ from the strong lensing in the gravitationally lensed system B 0218+357 B-A for (q_0 = 0.01, 0.5 and 2)

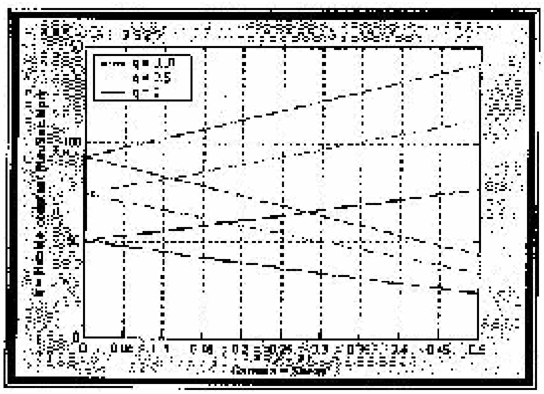

Figure (6)
Limits between H_0 and γ from the strong Lensing in the gravitationally lensed system PKS 1830-211 B-A for (q_0 = 0.01, 0.5 and 2)

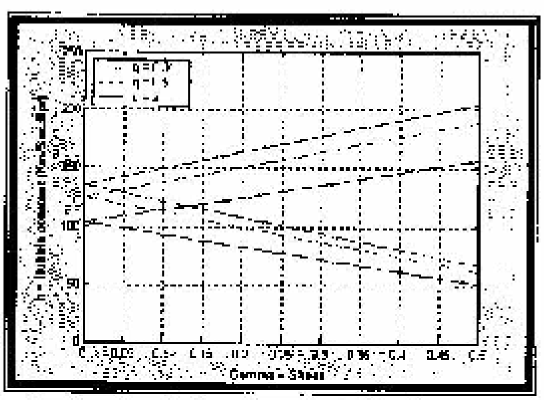

Figure (7)
Limits between H_0 and γ from the strong lensing in the gravitationally lensed system Q0957+561 B-A for (q_0 = 0.01, 0.5 and 2)

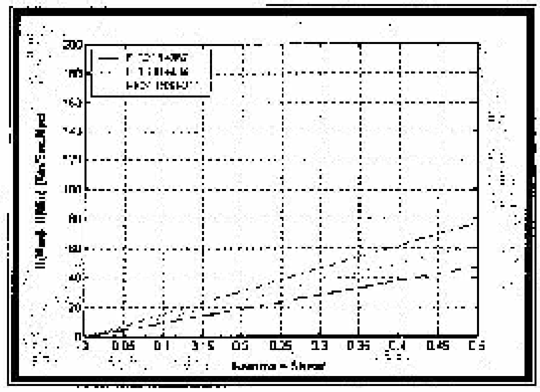

Figure (8)
The effect of γ on the 4-Image lenses

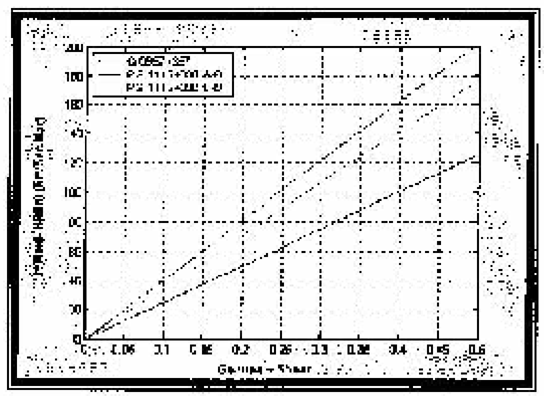

Figure (9)
The effect of γ on the 2-Image lenses

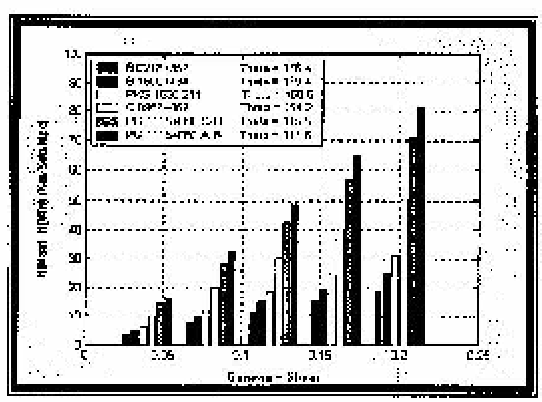

Figure (10)
The effect of γ on the different Gravitational lenses

Time delay in non-isothermal models

Observed lenses seem to be consistent with isothermal galaxies, but lenses with images at similar distances from the lens galaxy can often be modeled with other density profiles as well. Models of PG 1115+080 and B1608+656 show that steeper density profiles lead to larger predicted time delays and hence larger values for H_0 [8,9]. We analyze deviations from an isothermal profile by relaxing the condition in equation (5) is given by [3]:

$$\beta\phi = x\phi_x + y\phi_y \quad \text{.................................(9)}$$

Where β is a constant Isothermal models have β = 1. This condition can be understood by finding the general solution for the potential satisfy equation (9), then the time delay is given by [3]:

$$\Delta t(x,y) = \frac{D}{2c}(1+z_l)\left[\left(\xi^2+\eta^2\right) + 2\frac{(1-\beta)}{\beta}\left(\xi x + \eta y\right) - \frac{(2-\beta)}{\beta}\left(x^2+y^2\right)\right] \quad (10)$$

And The time delay between two images i and j is then [3]:

$$\Delta t(x,y) = \frac{D}{2c}(1+z_l)\left[\begin{array}{l}\frac{(2-\beta)}{\beta}\left(r_j^2-r_i^2\right) + \\ 2\frac{(1-\beta)}{\beta}\left\{\xi\left(x_i-x_j\right)+\eta\left(y_i-y_j\right)\right\}\end{array}\right] \quad (11)$$

Alternatively, if we substitute for the source position (ξ ,η) using the lens equation we find;

$$\Delta t(x,y) = \frac{D}{2c}(1+z_l)\left[\left(r_j^2-r_i^2\right) + 2(1-\beta)\left(\phi_j-\phi_i\right)\right] \quad (12)$$

Where $\Phi_i = \Phi(r_i, \theta_i)$.

Equations (11) and (12) shows that when the model is not isothermal (β ≠1), we cannot eliminate the model determining the source position and or the potential at each image.

The time delays for the non-isothermal elliptical potentials have been computed numerically, and we found that for small to moderate ellipticities the delays are well approximated by the isothermal time delay, i.e. Eq.(11), modified by a multiplicative factor. For opposed images $|\theta_i - \theta_j| \simeq 180$, the time delay is approximately $\Delta t_{ij} \approx [2-\beta]\,\Delta t_{ij}^{iso}$, while for orthogonal images $|\theta_i - \theta_j| \simeq 90$, the time delay is approximately $\Delta t_{ij} \approx [(2-\beta)/\beta]\,\Delta t_{ij}^{iso}$. By contrast, the isothermal time delay is not a good approximation for close image pairs $|\theta_i - \theta_j| \simeq 0$, since the images are necessarily near the lensing critical curve and hence, more sensitive to the particular lens model.

Conclusions

By assuming no knowledge of the angle between the shear and the lens galaxy major axis, the bounds would be narrower constraints on this angle. An important qualitative result is that H_0 determied from the four-image lens PG 1115+080 , figures (1) and (2), is much more strongly affected by shear than those from two-image lenses, as expected from the fact that in four-image lenses the images tend to lie at approximately the same distance from the lens galaxy. Therefore, an external shear changes the time delay by an amount proportional to the shear strength, and affects 4-image lenses more than 2-image lenses. Figures (7) and (8) shows how external shear strength affects much more strongly on four-image lenses than the 2-image lenses for $q_0 = 0.5$ model.

Furthermore, numerically, we find. that such small core radius changes the time delays negligibly ≤1%.

For the strong Lensing in the system Q0957+561, we success to minimize the value of H_0 into an acceptable range and to lie in a good fit by taking the effect of the smooth less parameter and the dark matter in care. These give us an indication that's the matter inside and around the lens galaxy are not smoothly distributed and contains in a complicated objects. Finally, there is a percentage of dark matter between 0-30 % in the way between us to this system play a main role in the strong gravitational field around the lens galaxy.

References

1. Keeton C. R., & Kochanek C. S. Seljak U., 1997, ApJ, 482, 604
2. Witt H. J., & Mao S., 1997, MNRAS, 291, 211
3. Hancock S., Rocha G., Lasenby A. N., Gutierrez C. M., 1998, "Constraints on cosmological parameters from recent measurements of cosmic microwave background anisotropy", M.N.R.A.S. 294, 1
4. Schneider P., Ehlers J., & Falco E. E. 1992, "Gravitational Lenses" (New York: Springer-Verlag)
5. Kochanek C. S. 1991, ApJ, 373, 354
6. Wallington S., & Narayan R. 1993, ApJ, 403, 517.
7. Kochanek C. S. 1996, ApJ, 466, 638
8. Impey, C. D., et al. 1998, ApJ, 509, 551

Koopmans L. V. E., & Fassnacht C. D., 1999, ApJ, 527, 513.