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Nonlinear Dynamical Fuzzy Control Systems Design With Matching Conditions

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Abstract

In this paper, a new approach of fuzzy control for continuous nonlinear dynamical systems is developed based on the framework of Takagi-Sugeno fuzzy model and a common controller for all the local models generated by fuzzifying the nonlinear dynamical system.

Some theorems that give sufficient conditions for checking the simultaneous stabilization of fuzzy control systems via a common quadratic Lyapunov function have been presented and developed; their theoretical aspects have also been proved and discussed.

Design algorithms, illustrative examples and graphs have been presented to show effectiveness of the approach.

Introduction

In many physical and engineering systems, engineers are hindered by strong nonlinearity from successful application of linear control theory. In the past few decades, as the interest in fuzzy systems has increased, researchers have considered the stability analysis of these fuzzy using a variety of modeling and control frameworks. One of the frameworks that have attracted a great deal of attention is the so called fuzzy Takagi Sugeno (T-S) model originating from fuzzy identification (T. Takagi and M. Sugeno 1985).

The popularity of the T-S model arises not only from its simplicity, but also from the idea that local dynamics of a nonlinear plant can be represented by different fuzzy rules (linear Models) in T-S model. Recent results have also shown that T-S model can be a universal approximator of any smooth nonlinear dynamic systems (C. Fantuzzi and R. Rovatti 1996, H. Ying 1998, J. Li, H. O. Wang, D. Niemann and K. Tanaka 2000). These results differ from the commonly held view that a T-S model has only limited capability in representing nonlinear system. Moreover, it has been found out that many nonlinear systems can be represented exactly through sectorization by T-S models with only a few rules. From the point of view of system analysis, T-S model are appealing as well since the stability and performance characteristics of the system can be analyzed using a Lyapunov approach (K. Tanaka and M Sugeno 1992, K. Tanaka and M. Sano 1993, S. G. Cao and N. W. Rees 1995, S. G. Cao, N. W. Rees and G. Feng 1996, G. Kang, J. Zhao, V. Wertz and R. Gorez 1996, W. Lee and M. Sugeno 1998).

In this paper a theorem labeled theorem (1) has been developed, which describes a fuzzy controller that simultaneously quadratically stabilize the multi-input systems whose i^{th} local model is (A_i, B_i) via a common Lyapunov function $V(x) = x^T W^{-1} x$.

Also we discussed the simultaneous quadratic stabilizability of the uncertain multi-input fuzzy dynamical systems whose i^{th} local model $(A + A_i, B + B_i)$, for that a theorem labeled theorem (2) has been developed as an extension to theorem (1), which describes a fuzzy controller that simultaneously quadratically stabilize the above uncertain multi-input fuzzy dynamical systems via a common Lyapunov function $V(x) = x^T T^T W^{-1} T x$, using T-S fuzzy model and suitable coordinates transformation.

Afterwards a step by step design algorithm has been suggested. A practical example and illustration graphs for the local systems are presented.

(1) Stability Analysis of Takagi-Sugeno Fuzzy Systems:

In this section we consider the nonlinear analysis of fuzzy control systems where the plant and controller are Takagi-Sugeno fuzzy systems. The main feature of the Takagi-Sugeno fuzzy model (T-S) is the expression of each dynamic by a fuzzy implication (rule). The overall fuzzy model of the system is achieved by fuzzy interpolation of these linear systems models. We shall be concerned in the study of the continuous case of The (T-S) fuzzy model which has following form, [1].

Rule i : If $x_1(t)$ is F_1^i and $x_2(t)$ is F_2^i and $\cdots$ $x_n(t)$ is F_n^i

$$\text{Then } \dot{x}(t) = A_i x(t) + B_i u(t) \tag{1.1}$$

where $x^T(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$, $u^T(t) = [u_1(t), u_2(t), \ldots, u_m(t)]$, $i = 1, 2, \ldots, r$ and r is the number of the If-Then rules, $x_i(t)$ are some fuzzy variables, F_j^i are fuzzy sets, $\dot{x}(t) = A_i x(t) + B_i u(t)$ from the i^{th} If-Then rule "(A_i, B_i) is called the i^{th} local model".

Using Singleton fuzzification, product inference and Center-of-Gravity Defuzzification method, the expression of the (T-S) fuzzy model (1.1) takes the final state (1.2) as follows:

$$x(t) = \frac{\sum_{i=1}^{r} h_i(x)(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} h_i(x)} \qquad (1.2)$$

where $h_i(x(t)) = \Phi_i(t) / \sum_{i=1}^{r} \Phi_i(t)$,

$\Phi_i(t) = \prod_{j=1}^{n} F_j^i(x_j(t))$, and Π stands for the product operation.

Lemma (1) "Finsler's Lemma":

If $x^T A x > 0$, whenever $x^T B x = 0$ where A and B are symmetric constant matrices, then there exists a real scalar $\delta > 0$ such that $A + \delta B B^T > 0$, [2].

Definition (1)

Suppose that there exists a matrix $P > 0$ which satisfies $PA_i + A_i^T P < 0$; $i = 1, 2, \ldots, r$. Then the quadratic function $V(x) = x^T P x$ is called a common Lyapunov function for the family of asymptotically stable linear systems $\dot{x} = A_i x$, $i = 1, 2, \ldots, r$, [3].

Definition (2)

The linear system $\dot{x}(t) = Ax(t) + Bu(t)$ where $A \in \Re^{n \times n}$, $B \in \Re^{n \times m}$, $x \in \Re^n$ is the state, and $u(t) \in \Re^m$ is the control input, is said to be quadratically stabilizable if there exists a continuous feedback control $u(\cdot): \Re^n \to \Re^m$ with $u(0) = 0$, $n \times n$ positive definite symmetric matrix P, and a constant $\alpha > 0$ such that the following condition holds: the Lyapunov derivative corresponding to the closed-loop system $\dot{x} = Ax + Bu(t)$ satisfies the inequality.

$\dot{V}(x) = x^T[A^T P + PA]x + 2x^T PBu \le -\alpha \|x\|^2$ (1.3)

for all nonzero state $x \in \Re^n$ and $t \ge 0$. In this inequality, $\|\cdot\|$ standard Euclidean norm. Then it admits a Lyapunov function $V(x) = x^T P x$, [4].

Definition (3)

Consider the family of multi-input systems (A_i, B), $i = 1, 2, \ldots, r$, described by the state equation $\dot{x} = A_i x + Bu$ where $A_i \in R^{n \times n}$ and $B \in R^{n \times m}$, $x \in R^n$ is the state variables, $u \in R^m$ is the control. The set of systems is simultaneously quadratically stabilizable if there exist a matrices $W = W^T > 0$ and K such that $x^T[(A_i - BK)^T W^{-1} + W^{-1}(A_i - BK)]x < 0$ for every $x \in R^n$, $i = 1, 2, \ldots, r$ holds [5].

Remark (1):

If the condition of definition (3) is satisfied, then there exists a common quadratic Lyapunov function $V(x) = x^T W^{-1} x$, and a single state feedback $u = -Kx$ such that the derivative of $V(x)$ along the trajectory of each system is negative definite [6].

Lemma (2):

A family of plants $\dot{x} = A_i x + Bu$ is simultaneously quadratically stabilizable iff $\exists\, W = W^T > 0$ such that for each i, $x^T[A_i W + W A_i^T]x < 0$, $i = 1, 2, \ldots, r$ holds [6].

Remark (2):

If lemma (2) holds, then $V(x) = x^T W^{-1} x$ is a quadratic Lyapunov function for the closed-loop systems with control law $u = -kx = -\frac{\gamma}{2} B^T W^{-1} X$ and there exist a set of scalars $\{\gamma_i\}$ for a given matrix W such that $A_i W + W A_i^T - \gamma_i B B^T < 0, \forall i$, $\gamma \ge \max\{\gamma_i\}$ [7].

(2) Block controllable companion form:

A linear time-invariant system can be described by state equations in general coordinates as follows:

$$\dot{x} = Ax + Bu$$

where $x \in \Re^n$, $u \in \Re^m$, $A \in \Re^{n \times n}$, $B \in \Re^{n \times m}$. If the rank of the block controllability test matrix:

$$\rho(A, B) = [B \; AB \; A^2 B \cdots A^{n-1} B]$$

is n, then the system in (1.5) is completely block controllable and can be transformed into block controllable companion form

$$\dot{z} = A_c z + B_c u$$

where

$$A_c = TAT^{-1} = \begin{bmatrix} 0_m & I_m & 0_m & \cdots & 0_m & 0_m \\ 0_m & 0_m & I_m & \cdots & 0_m & 0_m \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_m & 0_m & 0_m & \cdots & 0_m & I_m \\ -A_n & -A_{n-1} & -A_{n-2} & \cdots & -A_2 & -A_1 \end{bmatrix}$$

$$B_c = TB = \begin{bmatrix} 0_m & 0_m & \cdots & 0_m & I_m \end{bmatrix}^T$$

where I_m is the identity matrix of order m and $z = Tx$ provided that the matrix T is nonsingular matrix. The similarity transformation matrix T is given by

$$T = [\sigma \,\vdots\, \sigma A \,\vdots\, \sigma A^2 \cdots \sigma A^{n-1}]^T$$

where $\sigma = B_c^T \rho^{-1}(A, B)$ [8].

Lemma (3):

Consider the nominal systems $\dot{x} = A_i x + Bu$ with the pair (A_i, B) are in the block companion form. Where A_i and B have the structure:

$$A_i = \begin{bmatrix} 0_m & I_m & 0_m & \cdots & 0_m & 0_m \\ 0_m & 0_m & I_m & \cdots & 0_m & 0_m \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0_m & 0_m & 0_m & \cdots & 0_m & I_m \\ -\alpha_1^i & -\alpha_2^i & -\alpha_3^i & \cdots & -\alpha_{n-1}^i & -\alpha_n^i \end{bmatrix} - \begin{bmatrix} A_{11} & A_{12} \\ A_{21}^i & A_{22}^i \end{bmatrix}$$

$$, B = \begin{bmatrix} 0_m \\ \vdots \\ I_m \end{bmatrix}$$

where $A_{11} \in \Re^{n-m \times n-m}$, $A_{12} \in \Re^{n-m \times m}$, $A_{21}^i \in \Re^{m \times n-m}$, $A_{22}^i \in \Re^{m \times m}$ and $B \in \Re^{n \times m}$. With control law $u = -\frac{\gamma}{2} B^T W^{-1} x$, then $\exists\, W = W^T > 0$ which guarantee the simultaneous quadratic stabilizability of the nominal systems $\dot{x} = A_i x + Bu$ for each i. (i.e. $V^T (A_i W + W A_i^T) V < 0$, $\forall i \in I = [1, 2, \ldots, r]$ is presumed) [Pees, 1989], [7].

Theorem (1):

Consider the nonlinear dynamical model $\dot{x}(t) = f(x(t)) + g(x(t))u(t)$ where $x(t)$ is the state vector, $u(t)$ is the control input vector, $i = 1, 2, \ldots, r$.

Using singleton fuzzification to have the (T-S) fuzzy model:

Rule i : If $x_1(t)$ is F_1^i and $x_2(t)$ is F_2^i and $\cdots$ $x_n(t)$ is F_r^i

Then $\dot{x}(t) = A_i x(t) + Bu(t)$

Using product inference and Center-of-Gravity defuzzification method we have the following model:

(1.8)

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} h_i(x)\left(A_i x(t) + B u(t)\right)}{\sum_{i=1}^{r} h_i(x)} \quad (1.11) \quad (1.9)$$

with the controllable pair (A_i, B) as the i^{th} local model, A_i is in block controllable companion form for $i = 1, 2, \ldots, r$. And linear state feedback (1.10) $u = -\frac{\gamma}{2} B^T W^{-1} x$ where $\gamma > 0$ is sufficiently large positive scalar and the matrix $W = W^T > 0$

Assume that $\sum_{i=1}^{r} h_i(x) = 1$, $h_i(x) \geq 0$, $i = 1, 2, \ldots, r$ then the model (1.11) is simultaneously quadratically stabilizable via a common Lyapunov function $V(x) = x^T W^{-1} x$ such that $A_i W + W A_i^T < 0$, $\forall i$.

Proof:

Consider the common quadratic Lyapunov function $V(x) = x^T W^{-1} x$, the Lyapunov function derivative along the trajectories of (1.11) yields:

$$\dot{V}(x) = x^T W^{-1} \dot{x} + \dot{x}^T W^{-1} x$$

$$= x^T W^{-1} \left[\frac{\sum_{i=1}^{r} h_i(x)(A_i x + Bu)}{\sum_{i=1}^{r} h_i(x)} \right] + \left[\frac{\sum_{i=1}^{r} h_i(x)(A_i x + Bu)}{\sum_{i=1}^{r} h_i(x)} \right]^T W^{-1} x \quad (1.12)$$

and since linear state feedback control is suggested to have the form $u = -\frac{\gamma}{2} B^T W^{-1} x$ for arbitrary positive constant γ, then equation (1.12) becomes:

$$\dot{V}(x) = x^T \left[\frac{\sum h_i(x) \left[W^{-1}\left(A_i - \frac{\gamma}{2} B B^T W^{-1}\right) + \left(A_i - \frac{\gamma}{2} B B^T W^{-1}\right)^T W^{-1} \right]}{\sum h_i(x)} \right] x \quad (1.13)$$

Since W is Symmetric matrix then $\left(W^{-1}\right)^T = \left(W^T\right)^{-1} = W^{-1}$ and (1.13) becomes:

$$\dot{V}(x) = x^T \left[\frac{\sum_{i=1}^{r} h_i(x) \left[W^{-1} \left(A_i W + W A_i^T - \gamma B B^T\right) W^{-1} \right]}{\sum_{i=1}^{r} h_i(x)} \right] x$$

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Since by assumption $\sum_{i=1}^{r} h_i(x) = 1$, then
$$\dot{V}(x) = x^T\left[\sum_{i=1}^{r} h_i(x)\left[W^{-1}\left(A_iW + WA_i^T - \gamma BB^T\right)W^{-1}\right]\right]x$$
.

and we have $A_iW + WA_i^T < 0$, $\forall i$, then by remark (2), one can always find $\gamma \geq 0$ such that
$A_iW + WA_i^T - \gamma BB^T < 0$ Let
$A_iW + WA_i^T - \gamma BB^T = -Q$, then
$$\dot{V}(x) = x^T\left(\sum_{i=1}^{r} h_i(x)W^{-1}(-Q)W^{-1}\right)x$$
$$\leq x^T M x < 0$$
where $M = W^{-1}(-Q)W^{-1}$.

Which completes the proof of the theorem.

(3) Design Algorithm:

The following is a suggested algorithm for the fuzzy system (1.11) using theorem (1) has been discussed in order to find such a stabilizing controller. This algorithm is divided into two categories, the first is to compute the linear state feedback stabilizing controller. While the second is to compute the common Lyapunov function that will stabilize the fuzzy model:

1. Approximating the nonlinear model by linear local fuzzy models (In our work we shall use singleton fuzzification):

Rule i: If $x_1(t)$ is F_1^i and $x_2(t)$ is F_2^i and ... $x_n(t)$ is F_n^i (1.14)

Then $\dot{x}(t) = A_i x(t) + Bu(t)$

   where $i = 1, 2, \dots, r$, (A_i, B) is the i^{th} local model, $A_i \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, and the membership functions of the fuzzy sets F_j^i, $i = 1, 2, \dots, r$, are defined suitably according to the decision maker depending on the system.

2. Check the controllability of the pairs (A_i, B) and whether A_i is in the companion form $\forall i$, if not one can always transform it (see section 2). And let it be partitioned according to lemma (3).
3. Using Center-Of-Gravity Defuzzification method, we connect all local models produced by fuzzification, into one global model (see section 1).
4. Calculate the matrix M such that the Matrix $A_{11} + A_{12}M$ is stable. We shall suggest using the Pole-Placement method to calculate M.
5. Calculate the positive definite symmetric matrix W defined by:

$$W = \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \quad (1.15)$$

where $W_1 \in \Re^{(n-m)\times(n-m)}$ is $W_1 = W_1^T > 0$ computed by solving the Lyapunov equation:

$$(A_{11} + A_{12}M)W_1 + W_1(A_{11} + A_{12}M)^T = Q_o \quad (1.16)$$

and $Q_o \in \Re^{(n-m)\times(n-m)}$ is any negative definite symmetric matrix, $W_2 \in \Re^{(n-m)\times m}$ is computed from $W_2 = (MW_1)^T$, and $W_3 \in \Re^{m\times m}$ is computed from $W_3 = N + W_2^T W_1^{-1} W_2$ where N is any positive definite symmetric matrix.

6. Calculate the scalar γ which satisfies $\gamma \geq \max(\gamma_i)$. Define

$$Q^i = A_iW + WA_i^T - \gamma_i BB^T = \begin{pmatrix} Q_o & q_1 \\ q_1^T & q_{2i} - \gamma_i I_m \end{pmatrix} \quad (1.17)$$

   Where Q_o is any negative definite matrix (defined in (1.16)), γ_i are calculated so as to satisfy $q_{2i} - \gamma_i I_m - q_1^T Q_o^{-1} q_1 < 0$, and q_1, q_2 are calculated using (1.18), (1.19):

$$q_1 = A_{11}W_2 + A_{12}W_3 + W_1(A_{21})^T + W_2(A_{22})^T \quad (1.18)$$

$$q_{2i} = A_{21}W_2 + A_{22}W_3 + W_2^T(A_{21})^T + W_3(A_{22})^T \quad (1.19)$$

7. Calculate the linear state feedback $u = -\frac{\gamma}{2}B^T W^{-1}x$ and substitute it in original model and in the global model in (equation 1.11).
8. Calculate the common Lyapunov function $V(x) = x^T W^{-1} x$.

Example (2.1): Consider the following nonlinear dynamical model

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \text{[illegible]}$$

1. Notice that the sine function on the two operating points of the systems has the property that:

$$\sin(x) \approx x \quad \text{when } x \approx 0 \quad (1)$$

$$\sin(x) = \frac{2}{\pi}x = 1 \quad \text{when } x = \frac{\pi}{2} \quad (2)$$

   Then the state equation of the nonlinear model is simplified by the following linear systems, From (1) we have:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix}=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&17.294117&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}+\begin{pmatrix}0\\0\\0\\-0.17647\end{pmatrix}u \quad (3)$$

From (2) we have:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix}=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&17.294117\frac{2}{\pi}&0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}+\begin{pmatrix}0\\0\\0\\-0.17647\end{pmatrix}u \quad (4)$$

Now, from (3) and (4), we have the following linear local fuzzy models:

Rule1: if $(x_3 \text{ is about } 0)$ then $\dot{x}=A_1x+Bu$

Rule2: if $(x_3 \text{ is about } \pi/2)$ then $\dot{x}=A_2x+Bu$

where the membership functions for the fuzzy sets

$F_1^1=(x_3 \text{ is about } 0)$,

$F_1^2=(x_3 \text{ is about } \pi/2)$, are chosen respectively,

$F_1^1(x_3(t))=\exp(-\frac{\pi}{2}x_3^2(t))$,

$F_1^2(x_3(t))=1-\exp(-\frac{\pi}{2}x_3^2(t))$. And:

$$A_1=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&17.294117&0\end{pmatrix},$$

$$A_2=\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&11.005347&0\end{pmatrix},$$

$$B=\begin{pmatrix}0\\0\\0\\-0.17647\end{pmatrix}.$$

2. Using the condition in (1.6), then the pairs (A_1,B) and (A_2,B) are controllable. Comparing with equation (1.8), one can see that A_1 and A_2 are in the block companion form. So the Partitioning will be:

$$A_{11}=\begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix},\ A_{12}=\begin{pmatrix}0\\0\\1\end{pmatrix},\ A_{21}^1=\begin{pmatrix}0&0&17.294117\end{pmatrix},$$

$A_{21}^2=\begin{pmatrix}0&0&11.005347\end{pmatrix}$, $A_{22}=(0)$.

3. Defuzzification to global model:

$$\dot{x}=\frac{\sum_1^2 h_i(t)(A_ix+Bu)}{\sum_1^2 h_i(t)},$$

$M_1(t)=F_1^1(x_3(t))=\exp\left(-\frac{\pi}{2}x_3^2(t)\right)$,

$h_1(x_3(t))=\exp\left(-\frac{\pi}{2}x_3^2(t)\right)$,

$M_2(t)=F_1^2(x_3(t))=1-\exp\left(-\frac{\pi}{2}x_3^2(t)\right)$

, $h_2(x_3(t))=1-\exp\left(-\frac{\pi}{2}x_3^2(t)\right)$,

then we have the following dynamical system:

$$\dot{x}=\exp\left(-\frac{\pi}{2}x_3^2(t)\right)(A_1x+Bu)+\left[1-\exp\left(-\frac{\pi}{2}x_3^2(t)\right)\right](A_2x+Bu)$$

4. Calculating the matrix M (Poles-Placement): $M=\begin{pmatrix}-24&-26&-9\end{pmatrix}$.

5. First we shall proceed in the procedure of finding the matrix W where $W_1\in\Re^{3\times3}$, $W_2\in\Re^{3\times1}$, $W_3\in\Re^{1\times1}$. Q_o is chosen to be:

$$Q_o=\begin{pmatrix}-0.001&0.003&0\\0.003&-0.01&0.001\\0&0.001&-0.004\end{pmatrix},$$

hence

$$W_1=\begin{pmatrix}0.0002155&-0.0005&0.0003143\\-0.0005&0.0026857&-0.005\\0.0003143&-0.005&0.0138286\end{pmatrix},$$

and $W_2=\begin{pmatrix}0.0049993\\-0.0128282\\-0.0020006\end{pmatrix}$, Chose

$N=(1)$ then $W_3=(1.2315551)$. Finally,

$$W=\begin{pmatrix}0.0002155&-0.0005&0.0003143&0.0049993\\-0.0005&0.0026857&-0.005&-0.0128282\\0.0003143&-0.005&0.0138286&-0.0020006\\0.0049993&-0.0128282&-0.0020006&1.2315551\end{pmatrix}$$

6. The scalar γ_1 for A_1:

$$q=\begin{pmatrix}-0.00739\\-0.08847\\1.47071\end{pmatrix},$$

$q_{21}=(-0.0692)$, $\gamma_1=628.81612$

For A_2:

$$q_1 = \begin{pmatrix} -0.00937 \\ 0.05703 \\ 1.38374 \end{pmatrix}, \; q_{22} = (-0.04403),$$

$\gamma_2 = 569.42011$

hence $\gamma = 628.81612$

7. The calculation of the linear state feedback

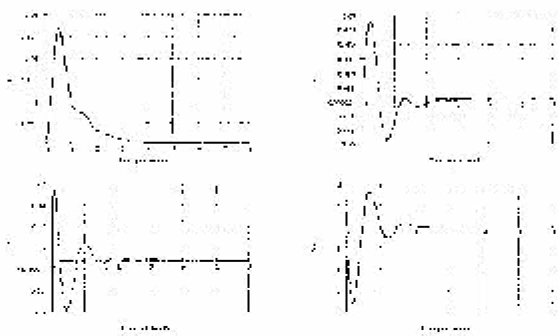
$$u = -\frac{\gamma}{2} B^T W^{-1} x ;$$

$u = (1331.6061684 \quad 1442.573349 \quad -497.5523131 \quad 33.4832903)x$

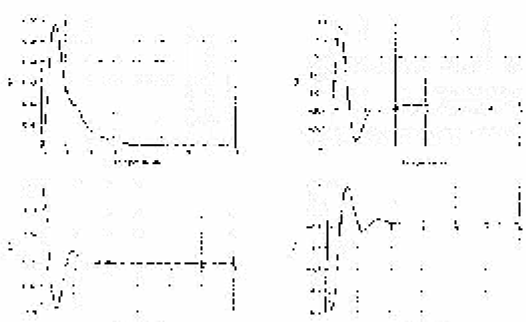
8. The common Lyapunov function

$$V(x) = x^T W^{-1} x ;$$

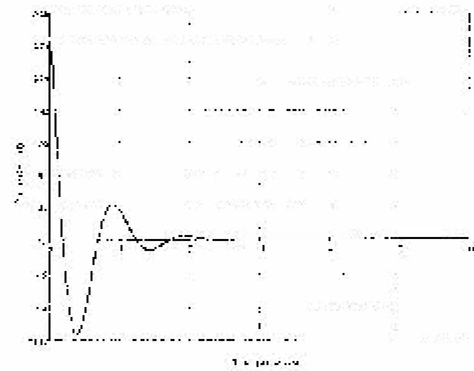
$V(x) = 23698.7816x_1^2 + 24222.8942x_1x_2 - 7552.54\ldots x_1x_3 - 48x_1x_4 +$ $+ 6576.55061x_2^2 - 4425.60935x_2x_3 + 52x_2x_4 + 786.67510x_3^2 - [illegible]$



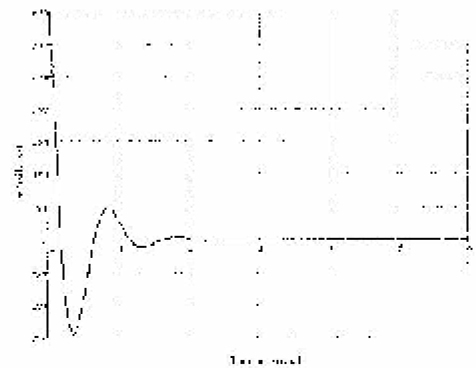
Graph (1): The state-space components versus time of the first local system with initial condition $(0, 0, \frac{7\pi}{36}, 0)$.



Graph (2): The state-space components versus time of the second local system with initial condition $(0, 0, \frac{15\pi}{36}, 0)$.



(a)



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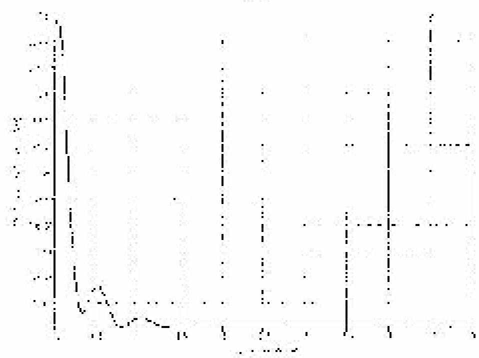
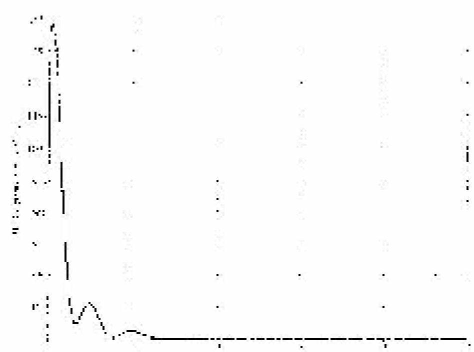
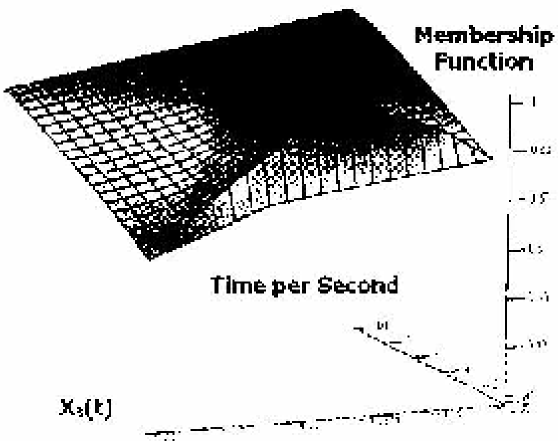


(c)

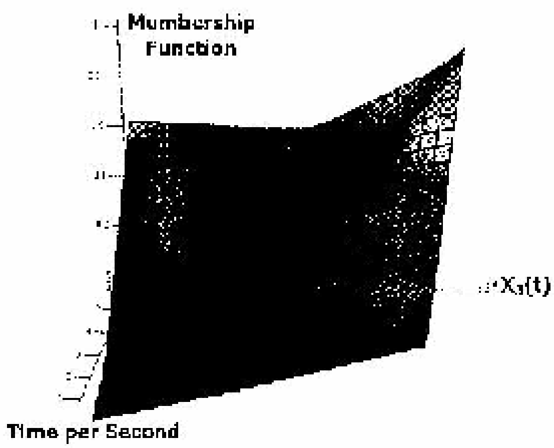


(d)

Graph (3):(a) The controller at the 1st local system. (b) The controller at the 2nd local system. (c) The common Lyapunov function at the 1st local system. (d) The common Lyapunov function at the 2nd local system.



Graph (4): The membership function of the fuzzy inferred set (x_3 is about 0) versus x_3 and the time.



Graph (5): The membership function of the fuzzy inferred set (x_3 is about $\pi/2$) versus x_3 and the time.

(4) Matching Conditions:

The matching conditions are preconditions, which constrain the manner in which the uncertainty is permitted to enter into the dynamics. The satisfaction of matching conditions is sufficient for stabilizability. Consider the systems of the following form $\dot{x} = (A + A_i)x(t) + (B + B_i)u(t)$, now A_i, B_i are said to satisfy matching conditions if there exists matrices E_i, D_i such that $A_i = BE_i$, $B_i = BD_i$ where $i \in I = \{1, 2, \ldots, r\}$ [9].

Lemma (4):

If A is an $n \times n$ positive definite nonsingular real symmetric matrix, then there exist a nonsingular matrix S such that $A = SS^T$ [10].

The following theorem has been developed to design a suitable controller for some nonlinear fuzzy dynamical systems.

Theorem (2):

Consider the nonlinear dynamical model:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t)$$

where $x(t)$ is the state vector, $u(t)$ is the control input vector. Using singleton fuzzification to have the (T-S) fuzzy model:

$$\text{Rule } i: \text{If } x_1(t) \text{ is } F_1^i \text{ and } x_2(t) \text{ is } F_2^i \text{ and} \cdots x_p(t) \text{ is } F_p^i$$
$$\text{Then } \dot{x}(t) = (A + A_i)x(t) + (B + B_i)u(t)$$

where $i = 1, 2, \ldots, r$ is the number of the rules, $j = 1, 2, \ldots, p$ is the number of the inferred fuzzy sets. Let $F_j^i(x_j(t))$ be the membership function of the inferred fuzzy sets F_j^i and $h_i(x)$ to be find as shown in section (1). Using product inference and Center-of-Gravity defuzzification method on (1.21), we have the following model:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} h_i(x)\big((A + A_i)x(t) + (B + B_i)u(t)\big)}{\sum_{i=1}^{r} h_i(x)}$$

with controllable nominal system (A, B) and $(A + A_i, B + B_i)$ as the controllable i^{th} local model, $i = 1, 2, \ldots, r$. Let $A_i = BE_i$, $B_i = BD_i$ $\forall i \in I$ and $D_i > 0$ be the matching conditions. Define linear state feedback $u(t) = u_o(t) + u_1(t) = (K_o + K_1)x(t)$ where $u_o(t)$ is selected such that the nominal model $\dot{x}(t) = Ax(t) + Bu(t)$ is asymptotically stable and $u_1(t) = K_1x(t) = -\frac{\gamma}{2}(TB)^T W^{-1}Tx(t)$ where $\gamma > 0$ is sufficiently large positive scalar, T is any transformation matrix selected as described in section (2), and $W = W^T > 0$ such that $\bar{M}_iW + W\bar{M}_i^T < 0$ $\forall i \in I$ where $\bar{A} = A + BK_o$ and $\bar{M}_i = T\bar{A}T^{-1} + T[A_i + B_iK_o]T^{-1}$. Assume that $\sum_{i=1}^{r} h_i(x) = 1$, $h_i(x) \geq 0$ then the model (1.22) is simultaneously quadratically stabilizable via a common Lyapunov function $V(x) = z^TW^{-1}z$.

Proof:

Define the coordinates transformation for the model (1.22) to be $z(t) = Tx(t)$ and substitute it in model (1.22) yields:

$$\dot{z}(t) = (Tx(t)) = T\dot{x}(t)$$

$$= T\frac{\sum_{i=1}^{r} h_i(x(t))\left[(A + A_i)x(t) + (B + B_i)u(t)\right]}{\sum_{i=1}^{r} h_i(x(t))}$$

Now, consider the linear state feedback $u(t) = (K_o + K_1)x(t)$ then (1.23) becomes:

$$\dot{z}(t) = T\frac{\sum_{i=1}^{r} h_i(x(t))\left[(A + A_i)x(t) + (B + B_i)(K_o + K_1)x(t)\right]}{\sum_{i=1}^{r} h_i(x(t))}$$

Since $x(t) = T^{-1}z(t)$ and the matching conditions $A_i = BF_i$, $B_i = BD_i$, $\forall i \in I$, then

$$\dot{z}(t) = \frac{\sum_{i=1}^{r} h_i(z(t))\left[T\overline{A}T^{-1}z + T(A_i + B_iK_o)T^{-1}z + TBK_1T^{-1}z + TBD_iK_1T^{-1}z\right]}{\sum_{i=1}^{r} h_i(z(t))}$$

(1.24)

Let $\overline{F} = T\overline{A}T^{-1}$ and let it be partitioned as follows:

$$\overline{F} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}$$

where $F_{11} \in R^{n-m \times n-m}$, $F_{12} \in R^{n-m \times m}$, $F_{21} \in R^{m \times n-m}$, $F_{22} \in R^{m \times m}$. Let $F^i = T(A_i + B_iK_o)T^{-1}$, and define $\overline{M}_i = (\overline{F} + F^i)$ and let it be partitioned as follows:

$$\overline{M}_i = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} + F_{21}^i & F_{22} + F_{22}^i \end{bmatrix},$$

Let $G = TB$, $G_i = TBD_i$ and $u^*(t) = K_1T^{-1}z$ then

$$(G + G_i)u^*(t) =$$

$$(TB + TBD_i)K_1T^{-1}z =$$

$$TB(I_m + D_i)K_1T^{-1}z = TBC_iK_1T^{-1}z,$$

where $C_i = I_m + D_i$, since $D_i > 0$, $I_m > 0$ implies $I_m + D_i > 0 \Rightarrow C_i > 0$, then (1.24) will become:

$$\dot{z}(t) = \frac{\sum_{i=1}^{r} h_i(z(t))\left[\overline{M}_i z(t) + GC_iu^*(t)\right]}{\sum_{i=1}^{r} h_i(z(t))}$$

Now, consider the Lyapunov function $v(z(t)) = z^TW^{-1}z$ its differentiation yields.

$$\dot{v}(z(t)) = z^TW^{-1}\dot{z} + \dot{z}^TW^{-1}z$$

$$= z^TW^{-1}\left[\frac{\sum_{i=1}^{r} h_i(z)(\overline{M}_i z + GC_iu^*)}{\sum_{i=1}^{r} h_i(z)}\right] + \left[\frac{\sum_{i=1}^{r} h_i(z)(\overline{M}_i z + GC_iu^*)}{\sum_{i=1}^{r} h_i(z)}\right]^T W^{-1}z$$

(1.26)

Since the linear state feedback (1.25) $u^* = -\frac{\gamma}{2}G^TW^{-1}z$ where $\gamma > 0$ is sufficiently large positive scalar, then (1.26) becomes:

$$\dot{v} = z^T\left[\frac{\sum_{i=1}^{r} h_i(z)\left[W^{-1}\left(\overline{M}_i - \frac{\gamma}{2}GC_iG^TW^{-1}\right) + \left(\overline{M}_i - \frac{\gamma}{2}GC_iG^TW^{-1}\right)^T W^{-1}\right]}{\sum_{i=1}^{r} h_i(z)}\right]z$$

Since $\left(W^{-1}\right)^T = \left(W^T\right)^{-1} = W^{-1}$ "Symmetric",

$$\dot{v} = z^T\left[\frac{\sum_{i=1}^{r} h_i(z)W^{-1}\left[\overline{M}_iW - \frac{\gamma}{2}GC_iG^T + W\overline{M}_i^T - \frac{\gamma}{2}GC_i^TG^T\right]W^{-1}}{\sum_{i=1}^{r} h_i(z)}\right]z$$

$C_i > 0$ then by lemma(4) $C_i = L_iL_i^T > 0\ \forall i$, and the Lyapunov function derivative becomes:

$$\dot{v} = z^T\left[\frac{\sum_{i=1}^{r} h_i(z)W^{-1}\left(\overline{M}_iW + W\overline{M}_i^T - \gamma GL_iL_i^TG^T\right)W^{-1}}{\sum_{i=1}^{r} h_i(z)}\right]z$$

Since by assumption $\overline{M}_iW + W\overline{M}_i^T < 0$, $W > 0$ and $\gamma > \max(\gamma_i)$ then using Finsler's Lemma (Lemma 1), we get

$$\overline{M}_iW + W\overline{M}_i^T - \gamma GL_iL_i^TG^T$$

$$< \overline{M}_iW + W\overline{M}_i^T - \gamma_i GL_iL_i^TG^T < 0 \quad \forall i \in I$$

Now, let $\overline{M}_iW + W\overline{M}_i^T - \gamma GL_iL_i^TG^T = -Q$, then

$$\dot{v} = z^T\left[\frac{\sum_{i=1}^{r} h_i(z)\left(W^{-1}(-Q)W^{-1}\right)}{\sum_{i=1}^{r} h_i(z)}\right]z,$$

and $M = W^{-1}(-Q)W^{-1}$, then

$$\dot{v} = z^T\sum_{i=1}^{r} h_i(z)\left(W^{-1}(-Q)W^{-1}\right)z < -z^TMz < 0$$

And this completes the proof of the theorem.

(5) Design Algorithm:

Using what have been presented in the first and second chapter, above, and more from the literature, we shall suggest a step by step design algorithm for the system under discussion to achieve the stabilizability of model (1.22).

1. Approximating the nonlinear model by linear local fuzzy models (In our example we shall use singleton fuzzification):

Rule i : If $x_1(t)$ is F_1^i and $x_2(t)$ is F_2^i and $\cdots x_n(t)$ is F_n^i

Then $\dot{x} = (A + A_i)x(t) + (B + B_i)u(t)$

where $(A + A_i, B + B_i)$ is the i^{th} local model where $A, A_i \in \Re^{n\times n}$, $B, B_i \in \Re^{n\times m}$.

2. Using Center-Of-Gravity Defuzzification method we connect all local models produced by fuzzification into one global model:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} h_i(x)\left((A + A_i)x(t) + (B + B_i)u(t)\right)}{\sum_{i=1}^{r} h_i(x)} \qquad (1.28)$$

where $h_i(x(t)) = \frac{\Phi_i(t)}{\sum_{i=1}^{r}\Phi_i(t)}$, $\Phi_i(t) = \prod_{j=1}^{n} F_j^i(x_j(t))$, and $F_j^i(x_j(t))$ is the membership function of the fuzzy set F_j^i.

3. Check for matching conditions $A_i = BE_i$ and $B_i = BD_i$, where E_i are $(m_i \times n_i)$ constant matrices and D_i are $(m_i \times m_i)$ constant matrices.

4. Construct a matrix K_o such that the nominal system $(A + BK_o)x(t) = \overline{A}x(t)$ is asymptotically stable "Use pole-placement".

5. Choose an $n\times n$ invertible transformation matrix T selected as described in section (2).

6. Form the matrices $\overline{F} = T\overline{A}T^{-1}$, $G = TB$, and $F^i = TA_iT^{-1}$, $i \in I$.

7. Construct the matrices $M_i = \overline{F} + F^i$ and partition them as follows:

$$M_i = \begin{bmatrix} M_{11} & M_{12} \\ M_{21}^i & M_{22}^i \end{bmatrix} \qquad (1.29)$$

where $M_{11} \in \Re^{n-m\times n-m}$, $M_{12} \in \Re^{n-m\times m}$, $M_{21}^i \in \Re^{m\times n-m}$, and $M_{22}^i \in \Re^{m\times m}$, $i \in I$.

8. Calculate matrix N such that the Matrix $M_{11} + M_{12}N$ is stable. "Use pole-placement".

9. Calculate the positive definite symmetric matrix W defined by:

$$W = \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix}$$

where $W_1 \in \Re^{n-m\times n-m}$ is $W_1 = W_1^T > 0$ computed by solving the Lyapunov equation that has the form (1.27)

$$(M_{11} + M_{12}N)W_1 + W_1(M_{11} + M_{12}N)^T = Q_o$$

, where $Q_o \in \Re^{n-m\times n-m}$ is any negative definite symmetric matrix, $W_2 \in \Re^{n-m\times m}$ is computed from $W_2 = (NW_1)^T$ and $W_3 \in \Re^{m\times m}$ is computed from $W_3 = P + W_2^T W_1^{-1} W_2$ where $P = P^T > 0$.

10. Calculate the scalar γ which satisfies $\gamma > \max(\gamma_i)$ Define:

$$Q^i = M_i W + W M_i^T - \gamma_i G G^T = \begin{pmatrix} Q_o & q_i \\ q_i^T & q_{2i} - \gamma_i I_m \end{pmatrix}$$

Where Q_o is any negative definite matrix (defined in step(9)), γ_i are calculated so as to satisfy $q_{2i} - \gamma_i I_m - q_i^T Q_o^{-1} q_i < 0$, where

$$q_i = M_{11}W_2 + M_{12}W_3 + W_1(M_{21}^i)^T + W_2 M_{22}^i$$

and

$$q_{2i} = M_{21}^i W_2 + M_{22}^i W_3 + W_2^T (M_{21}^i)^T + W_3 (M_{22}^i)^T$$

.

11. Calculate the linear state feedback $u(t) = u_o(t) + u_1(t) = (K_o + K_1)x(t)$ where K_o as in step (3) and

$$K_1 = -\frac{\gamma}{2}(TB)^T W^{-1} T.$$

12. Calculate the common Lyapunov function $v(x) = (Tx)^T W^{-1} (Tx)$.

Example (2): Consider the dynamical system described by the following equation:

$$\dot{x} = f(x) + g(x)u \qquad (1.32)$$

where

$$f(x)=\begin{bmatrix} x_2 \\ x_1+0.31831x_3+\pi x_1\frac{\sin(x_3)}{x_3}+\frac{1}{3}\pi\tan(x_3) \\ x_4 \\ 0.63662x_1+0.95493x_3+\frac{3}{2}\pi x_1\frac{\sin(x_3)}{x_3}+\frac{1}{2}\pi\tan(x_3) \end{bmatrix}$$

$$, g(x)=\begin{bmatrix} 0 \\ 0.30329\pi e^{x_3} \\ 0 \\ 0.45493\pi e^{x_3} \end{bmatrix}.$$

1. Fuzzifying the system along the desired operating points yields (recall the properties of the sine function in example (1)):

$$\text{Rule1: if } (x_3 \text{ is near } 0) \text{ then } \dot{x}=(A+A_1)x+(B+B_1)u$$

$$\text{Rule2: if } (x_3 \text{ is near } \tfrac{\pi}{2}) \text{ then } \dot{x}=(A+A_2)x+(B+B_2)u$$

where

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0.31831 & 0 \\ 0 & 0 & 0 & 1 \\ 0.63662 & 0 & 0.95493 & 0 \end{bmatrix},$$

$$A_1=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 3.14159 & 0 & 1.0472 & 0 \\ 0 & 0 & 0 & 0 \\ 4.71239 & 0 & 1.5708 & 0 \end{bmatrix},$$

$$A_2=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 0 & 0.66667 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{bmatrix},$$

$$B=\begin{bmatrix} 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \quad B_1=\begin{bmatrix} 0 \\ 1.0472 \\ 0 \\ 1.5708 \end{bmatrix},$$

$$B_2=\begin{bmatrix} 0 \\ 2.58342 \\ 0 \\ 3.87513 \end{bmatrix}.$$

2. Defuzzificating to global model; The Fuzzy sets are: $F_1^1=(x_3 \text{ is near } 0)$, $F_1^2=(x_3 \text{ is near } \frac{\pi}{2})$, the membership functions respectively are:

$$F_1^1(x_3(t))=\frac{0.1}{0.3x^4+0.1},$$

$$F_1^2(x_3(t))=1-\frac{0.1}{0.3x^4+0.1}$$

The functions Φ_i, h_i and the global model are:

$$\Phi_1(t)=\frac{0.1}{0.3x^4+0.1},$$

$$\Phi_2(t)=1-\frac{0.1}{0.3x^4+0.1}, \; h_1=\Phi_1, \; h_2=\Phi_2,$$

$$\dot{x}(t)=\sum_{i=1}^{r} h_i(x)\big((A+A_i)x(t)+(B+B_i)u(t)\big)$$

The constant matrices E_i are $E_1=(1.5708 \quad 0 \quad 0.5236 \quad 0)$, $E_2=(1 \quad 0 \quad 0.33333 \quad 0)$ and D_i are $D_1=-\frac{\pi}{6}$, $D_2=1.29171$.

4. Construct the matrix K, (Poles Placement)

$K=($ [illegible] [illegible] [illegible] [illegible] $)$

5. The transformation matrix T is chosen to be:

$$T_1=\begin{bmatrix} 1.27324 & 0 & 0 & 0 \\ 0 & 0 & 2.22816 & 0 \\ 3.05577 & 3.81971 & 4.45633 & 2.5464 \\ 0.4 & 0.5 & -0.58333 & 0 \end{bmatrix}$$

$$, T_2=\begin{bmatrix} 0.7854 & 0 & 0 & 0 \\ 0.62832 & 0.5236 & 0 & 2 \\ 0 & 0.4488 & 0 & 0 \\ 0 & 0 & 0.3927 & 3 \end{bmatrix}.$$

6. The matrix G is $G=(0 \quad 0 \quad 0 \quad 1)^T$.

the matrix F is:

$$F=\begin{bmatrix} [illegible] & [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] & [illegible] \end{bmatrix}$$

The matrices F^i are:

$$F^1=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ [illegible] & [illegible] & 0 & 0 \\ [illegible] & [illegible] & 0 & 0 \end{bmatrix}$$

$$B^2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.000007854 & -0.00000726 & 0 & 0 \\ 0.735797885 & 0.14560616 & 0 & 0 \end{pmatrix}$$

7. The matrices M_i are:

$$M_1=\begin{pmatrix} 0.8 & 0.66667 & 0 & 2.54648 \\ 0 & -0.00001 & 0.875 & 6.68119 \\ 0.25182 & 2.14889 & 1.74135 & 19.48077 \\ -5857.52647 & -324.67147 & 864.91428 & -24.54134 \end{pmatrix}$$

$$M_2=\begin{pmatrix} 0.8 & 0.66667 & 0 & 2.54648 \\ 0 & -0.00001 & 0.875 & 6.69449 \\ 0.25182 & 2.14889 & 1.74135 & 19.48077 \\ -5857.97477 & -324.75686 & 864.91428 & -24.54134 \end{pmatrix}$$

And the partitions are as follows:

$$M_{11}=\begin{pmatrix} 0.8 & 0.66667 & 0 \\ 0 & -0.00001 & 0.875 \\ 0.25182 & 2.14889 & 1.74135 \end{pmatrix}$$

$M_{12}=\begin{pmatrix} 2.54648 \\ 6.68449 \\ 19.48077 \end{pmatrix}$, $M_{22}=-24.54134$,

$M_{21}^1=(-5857.52647 \quad -324.67147 \quad 864.91428)$

$M_{21}^2=(-5857.97477 \quad -324.75686 \quad 864.91428)$

8. Calculating the matrix **N** (Poles-Placement):

$N=(-186499171 \quad 33668943 \quad 19567623)$

9. Now we shall proceed in the procedure of finding the matrix W:

$$W=\begin{pmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{pmatrix}$$

where $W_1 \in \Re^{3\times 3}$, $W_2 \in \Re^{3\times 1}$, $W_3 \in \Re^{1\times 1}$. The Lyapunov equation $(M_{11}+M_{12}N)W_1 + W_1(M_{11}+M_{12}N)^T = Q_c$, where Q_c is chosen to be:

$$Q_c=\begin{pmatrix} -1 & 2 & 0 \\ 2 & -10 & 3 \\ 0 & 3 & -4 \end{pmatrix}.$$ Hence

$$W_1=\begin{pmatrix} 6046.1477426 & 15949.608694 & 46453.252038 \\ 15949.608694 & 42080.707078 & 122538.43585 \\ 46453.252038 & 122538.43585 & 356912.026115 \end{pmatrix}$$

$$W_2=(N\,W_1)^T = \begin{pmatrix} -6075.2885118 \\ -16041.029944 \\ -46021.464984 \end{pmatrix},$$

$W_3 = P + W_2^T W_1^{-1} W_2$ where

$P=(583.2738711)$ then

$W_3=(133000)$. Finally,

$$W=\begin{pmatrix} 6046.1477426 & 15949.608694 & 46453.252038 & -6075.2885118 \\ 15949.608694 & 42080.707078 & 122538.43585 & -16041.029944 \\ 46453.252038 & 122538.43585 & 356912.026115 & -46021.464984 \\ -6075.2885118 & -16041.029944 & -46021.464984 & 133000 \end{pmatrix},$$

10. The calculations of the scalar γ:

$$q_1=\begin{pmatrix} 56450.9529435 \\ 140019.793719 \\ 416651.625895 \end{pmatrix},$$

$q_{21}=(454878.8565141)$,

$\gamma_1 = 131832190162.193$

$$q_2=\begin{pmatrix} 52378.5278241 \\ 129276.312564 \\ 385363.0759697 \end{pmatrix},$$

$q_{22}=(-4540598.2657406)$,

$\gamma_2 = 112821741818.447$

hence $\gamma = 131832190162.193$.

11. The feed back gain matrix:

$K_1=(-1807312.3 \quad -3315.59241 \quad 3144.55645 \quad 2203.06161)$,

$K_2=(-2010353402.9999 \quad -345908184.0883 \quad$ [illegible] $\quad$ [illegible]$)$,

$K=($[illegible]$)$,

$u=$ [illegible],

Substitute u in (1.32) yields:

$$\dot{x}=Ax+f(x)$$

where

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0.31831 & 0 \\ 0 & 0 & 0 & 1 \\ 0.63662 & 0 & 0.95493 & 0 \end{bmatrix},$$

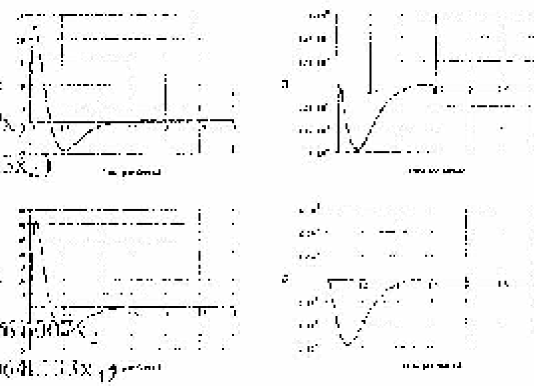
$$f(x)=\begin{bmatrix} 0 \\ g_1(x)-g_2(x) \\ 0 \\ g_3(x)+g_4(x) \end{bmatrix};$$

$$g_1(x) = \frac{\pi x_1 \sin(x_3)}{x_3} - \frac{\pi \sin(x_3)}{3},$$

$$g_2(x) = 0.30329\pi e^{x_3}(-2010353449\ldots331x_1 - 8465052146.6007x_2 - 13322980449.633x_3 + 5639588064.133x_4)$$

$$g_3(x) = -\frac{3\pi x_1 \sin(x_3)}{2x_3} - \frac{\pi \sin(x_3)}{2}$$

$$g_4(x) = 0.45493\pi e^{x_3}(-20103534\ldots7331x_1 - 8465052145.6007x_2 - 13322980449.633x_3 + 5639588064.133x_4)$$



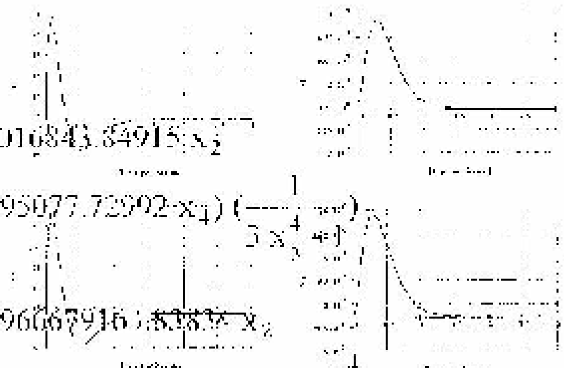
Graph (9): The state-space components versus time of the first local system with initial condition (0, 0, π/12, 0).

Substitute u in (1.33) yields:

$$\dot{x} = \begin{bmatrix} x_2 \\ f_1(x) - f_2(x) \\ x_4 \\ f_3(x) + f_4(x) \end{bmatrix} \qquad (1.35)$$

where

$$f_1(x) = (-1915464766652.12853\, x_1 - 806550168\ldots3.84915\, x_2 + 1269413577\ldots28.61583\, x_3 + 5373995077.72992\, x_4)\left(\frac{1}{3x_3^4 + 1}\right)$$

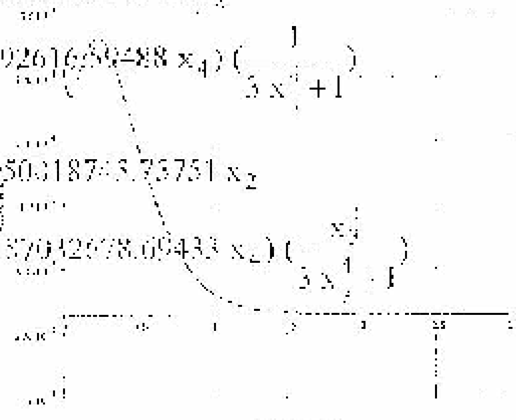


$$f_2(x) = (-2764288267068.70995\, x_1 - 1163960679\ldots16.83837\, x_2 + 1831944451622.07859\, x_3 + 775458021785.79632\, x_4)\left(\frac{x_3^4}{3x_3^4 + 1}\right)$$

Graph (10): The state-space components versus time of the second local system with initial condition (0, 0, 5π/12, 0).

$$f_3(x) = (-287319714979.03587\, x_1 - 120982\ldots\, x_2 + 190412036593.40121\, x_3 + 80660992616.59488\, x_4)\left(\frac{1}{3x_3^4 + 1}\right)$$

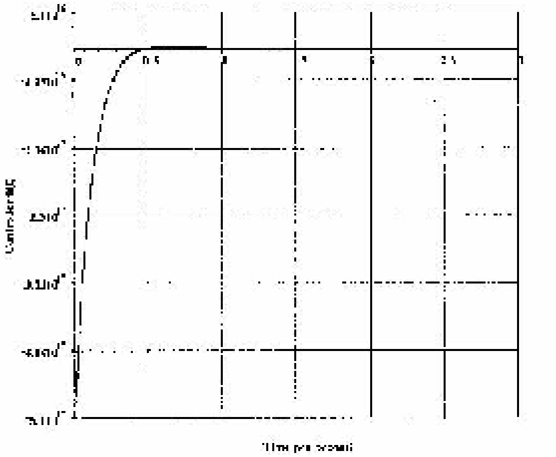
$$f_4(x) = (-4146432393105.65506\, x_1 - 17\ldots5950318743.73751\, x_2 + 2747916677\ldots3\ldots.55027\, x_3 + 116318703\ldots2698.09433\, x_4)\left(\frac{x_3^4}{3x_3^4 + 1}\right)$$



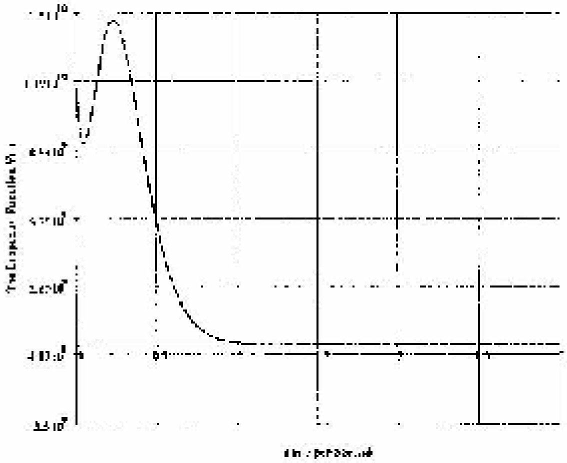
(a)

(2) The common Lyapunov function $V(x) = (Tx)^T W^{-1}(Tx)$, where

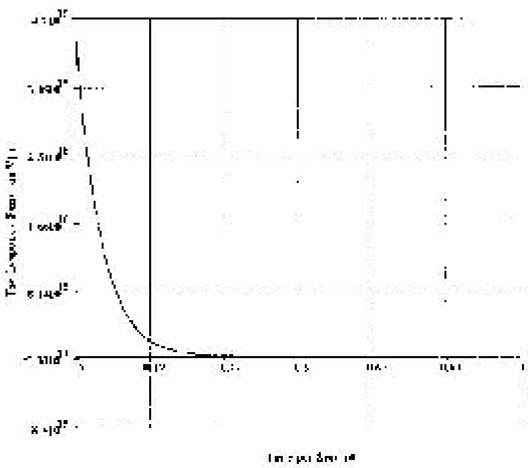
$V(x) = 5411.93390\, x_1^2 - 1571.96540\, x_1 x_2 - 7962.07.577\, x_1 x_3 - 3447.31992\, x_1 x_4 + [illegible]\, x_2^2 - [illegible]\, x_2 x_3 - 1231.5291\ldots\, x_2 x_4 - 2335.3\ldots\, x_3^2 + [illegible]\, x_3 x_4 + [illegible]\, x_4^2$



(b)

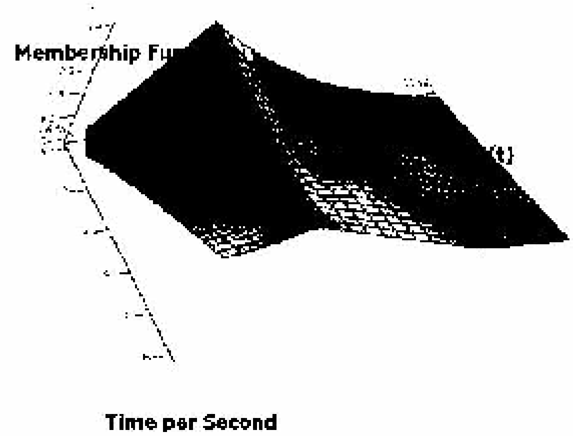


(c)

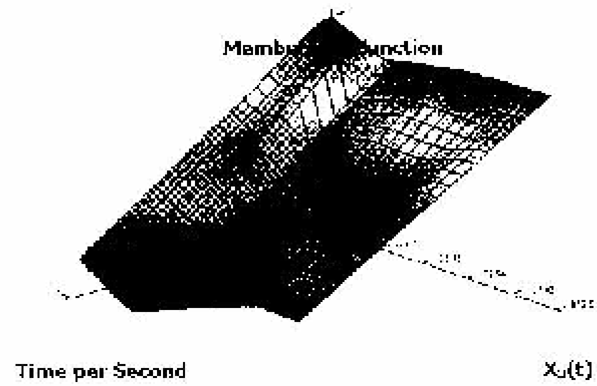


(d)

Graph (11): (a) The controller at the 1st local system. (b) The controller at the 2nd local system. (c) The common Lyapunov function at the 1st local system. (d)The common Lyapunov function at the 2nd local system.



Graph (12): The membership function of the fuzzy inferred set (x_3 is near 0) versus x_3 and the time.



Graph (13): the membership function of the fuzzy inferred set (x_3 is near $\pi/2$) versus x_3 and the time.

Conclusions and Future Studies.

From the calculations we can see the flexibility in the choice of the operating points, the poles in the pole-placement method, and the matrix Q_i that enables us to manipulate the plant to give results with desired characteristics, and that this approach is effective.

A future studies can be studying the effect of the no satisfaction of the matching condition and how to deal with the system in that case, applying the ideas and theorems to discrete time nonlinear dynamical systems or to chaotic dynamical systems.

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الخلاصة

يتضمن هذا البحث طريقة جديدة في دراسة ايجاد "التثبيت التربيعي" لأنظمة الآنية المتغيرة عبر الزمن "المتغيرة" مبنية على اساس دالة ليابونوف (كلاسيكي-بورجيتو) "التسلسل و سيطرة حل مجموع التلاؤم لدالة المولدة من تحسين" لأنظمة شبه خطية [illegible] الخطية [illegible] "الطريقة التي تعطي شروط كافية لأختبار "السيطرة الآنية لأنظمة" "المتوازية المتغيرة بواسطة [illegible] [illegible] [illegible] أمثلة توضيحية ورسوم لإيضاح فعالية هذه الطريقة.