Numerical Solution of Fuzzy Fredholm Integral Equations of the Second Kind using Bernstein Polynomials

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Abstract

In this paper, a numerical method is given for solving fuzzy Fredholm integral equations of the second kind, by using Bernstein piecewise polynomial, whose coefficients determined through solving dual fuzzy linear system. Numerical examples are presented to illustrate the proposed method, whose calculations were implemented by using the Computer software MathCadV.14.

Keywords: Fuzzy Integral Equation, Dual Fuzzy Linear System, Bernstein Polynomials.

Introduction

In recent years, the interest in fuzzy integral equation shave been rapidly growing and drawing attention by scientists, due to its importance in applications, such as, fuzzy control, approximate reasoning, fuzzy financial and economic systems, etc.

Consequently, the topic of numerical methods for solving fuzzy integral equations have been considered thoroughly, because of the difficulty of finding the analytical solution in many cases, to these equations. Some numerical methods for fuzzy integral equations illustrated by[3] using iterative method to the fuzzy function, also [5] used differential transformation method to solve fuzzy integral, and [2] gave an algorithm to solve the fuzzy integral equations by using the trapezoidal rule to compute the Riemann integrals that convert it to a linear system its unknowns are to be determined, also [8] used iterative interpolation, and [7] with finite differences and divided differences methods.

The first part of this paper, is dedicated to give some necessary theoretical background materials that lead to the understanding of the proposed method, while the second part deals with illustrating the proposed approach for solving fuzzy Fredholm integral equations of the second kind, followed by numerical examples and illustrative figures, whose calculations were implemented by using the Computer software MathCad Version 14.

1- Bernstein Polynomials [10]:

The general form of the Bernstein polynomials of the n^{th} degree over the interval $[a,b]$ is defined by:

$$B_{i,n}(t)=\binom{n}{i}\frac{1}{(b-a)^n}(t-a)^i(b-t)^{n-i}$$

where$i=0,1,2,\cdots,n$.Note that each of these $n+1$ polynomials having degree $n$satisfies the following properties:

i) $B_{i,n}(t)=0$, if $i<0$ or $i>n$.

ii) $\sum_{i=0}^{n} B_{i,n}(t)=1$.

iii) $B_{i,n}(a)=B_{i,n}(b)=0,1\leq i\leq n-1$.

Remark (1) [4]:

1. Any Bernstein polynomial of degree n may be written in terms of the power basis, as follows,
$$B_{i,n}(t)=\sum_{i=j}^{n}(-1)^{j-i}\binom{n}{j}\binom{j}{i}t^j$$
2. The 1^{st}derivative of an n^{th} degree Bernstein polynomial can be expressed as follows:
$$\frac{d}{dt}B_{i,n}(t)=n\left(B_{i-1,n-1}(t)-B_{i,n-1}(t)\right)$$
where $i=0,1,\cdots,n$.

2- Dual Fuzzy Linear System:

This section will gradually reach its purpose of finding the solution of dual fuzzy linear system through giving some necessary required definitions, and as follows:

Definition 2.1[2]:

An arbitrary fuzzy number is an ordered pair of functions $\tilde{v}=\left(\underline{v}(\alpha),\overline{v}(\alpha)\right)$, $0\leq\alpha\leq 1$, which satisfy the following requirements:

1. $\underline{v}(\alpha)$ is a bounded left continuous non decreasing function over $[0,1]$.

2. $\overline{v}(\alpha)$is a bounded left continuous non increasing function over [0,1].
3. $\underline{v}(\alpha) \leq \overline{v}(\alpha)$, $0 \leq \alpha \leq 1$.

For and arbitrary fuzzy numbers $\tilde{u} = \left(\underline{u}(\alpha), \overline{u}(\alpha)\right)$, $\tilde{v} = \left(\underline{v}(\alpha), \overline{v}(\alpha)\right)$ and $k \in \mathbb{R}$, we define the addition and scalar multiplication by k as:

$\tilde{u} + \tilde{v} = \left[\underline{u}(\alpha) + \underline{v}(\alpha), \overline{u}(\alpha) + \overline{v}(\alpha)\right]$

$k\tilde{u} = \left[k\underline{u}(\alpha), k\overline{u}(\alpha)\right]$, $k \geq 0$

and

$k\tilde{u} = \left[k\overline{u}(\alpha), k\underline{u}(\alpha)\right]$, $k < 0$

The set of all these fuzzy numbers is denoted by E.

Definition 2.2 [9]:

The fuzzy system $A\tilde{x} = B\tilde{x} + \tilde{y}$ where the coefficients matrix $A = (a_{ij})$, and $B = (b_{ij})$, $1 \leq i \leq m$, $1 \leq j \leq n$ are crisp $m \times n$ matrices, and $m \leq n$, $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^T$ and $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \cdots, \tilde{y}_m)^T$ are fuzzy number vectors. The above system is called the dual fuzzy linear system.

Remark (2) [9]:

Usually, there is no inverse element for an arbitrary fuzzy number $\tilde{a} \in E$, i.e. there exists no element $\tilde{b} \in E$ such that $\tilde{a} + \tilde{b} = \tilde{0}$. Actually, for all non-crisp fuzzy number $\tilde{a} \in E$ we have $\tilde{a} + (-\tilde{a}) \neq \tilde{0}$. So the above system cannot be equivalently replaced by the fuzzy system,$(A - B)\tilde{x} = \tilde{y}$.

Definition 2.3[6]:

A fuzzy number vector $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^T$is given by:

$\tilde{x}_i = \left(\underline{x_i}(\alpha), \overline{x}_i(\alpha)\right)$, $1 \leq i \leq n$, $0 \leq \alpha \leq 1$

is called a solution of the system in Definition 2.2 if

$$\underline{\sum_{j=1}^{n} a_{ij} x_j} = \sum_{j=1}^{n} \underline{a_{ij} x_j} = \underline{y_i} = \underline{\sum_{j=1}^{n} b_{ij} x_j} = \sum_{j=1}^{n} \underline{b_{ij} x_j}$$

$$\overline{\sum_{j=1}^{n} a_{ij} x_j} = \sum_{j=1}^{n} \overline{a_{ij} x_j} = \overline{y}_i = \overline{\sum_{j=1}^{n} b_{ij} x_j} = \sum_{j=1}^{n} \overline{b_{ij} x_j}$$

for a particular i, $a_{ij} > 0$ and $b_{ij} > 0$, $1 \leq j \leq n$, we simply get

$$\sum_{j=1}^{n} a_{ij} \underline{x}_j = \underline{y}_i + \sum_{j=1}^{n} b_{ij} \underline{x}_j$$

$$\sum_{j=1}^{n} a_{ij} \overline{x}_j = \overline{y}_i + \sum_{j=1}^{n} b_{ij} \overline{x}_j$$

Remark (3):

Follow the work in [1], [6] and consider the dual fuzzy linear system, $A\tilde{x} = B\tilde{x} + \tilde{y}$ and transform its $n \times n$ coefficient matrices A, and B into $2n \times 2n$ crisp linear system:

$SX = TX + Y \Longrightarrow (S - T)X = Y$

$\Longrightarrow X = (S - T)^{-1}Y$

where the coefficients matrix $S = (s_{ij})$, and $T = (t_{ij})$,$1 \leq i, j \leq 2n$. The elements s_{ij} and t_{ij}, are determined as follows:

- If $a_{ij} \geq 0 \Longrightarrow s_{ij} = a_{ij}$ and $s_{i+n\, j+n} = a_{ij}$.
- If $a_{ij} < 0 \Longrightarrow s_{i\, j+n} = -a_{ij}$ and $s_{i+n\, j} = -a_{ij}$.
- If $b_{ij} \geq 0 \Longrightarrow t_{ij} = b_{ij}$ and $t_{i+n\, j+n} = b_{ij}$.
- If $b_{ij} < 0 \Longrightarrow t_{i\, j+n} = -b_{i,j}$ and $t_{i+n\, j} = -b_{ij}$.

and any element s_{ij} and t_{ij} which has no assigned value from the coefficient matrices A, and B is set as zero.Also, the variables vectors are:

$X = [\underline{x}_1 \quad \underline{x}_2 \quad \cdots \quad \underline{x}_n \quad -\overline{x}_1 \quad -\overline{x}_2 \quad \cdots \quad -\overline{x}_n]^T$

and

$Y = [\underline{y}_1 \quad \underline{y}_2 \quad \cdots \quad \underline{y}_n \quad -\overline{y}_1 \quad -\overline{y}_2 \quad \cdots \quad -\overline{y}_n]^T$

3- Fuzzy Fredholm Integral Equations [7]:

In this section, the definition of Fuzzy Fredholm integral equation of the second kind, will be studied, and as follows:

$$\tilde{y}(x;\alpha) = \tilde{f}(x;\alpha) + \lambda \int_a^b k(x,t)\tilde{y}(t;\alpha)dt \quad (1)$$

where $\lambda > 0$, and $k(x,t)$ is an arbitrary function called the kernel over the rectangle $a \leq x, t \leq b$, $\tilde{f}(x;\alpha) = \left[\underline{f}(x;\alpha),\ \overline{f}(x;\alpha)\right]$ is a fuzzy functions on the interval $[a, b]$, where $0 \leq \alpha \leq 1$, and $\tilde{y}(x;\alpha) = \left[\underline{y}(x;\alpha), \overline{y}(x;\alpha)\right]$ hence equation (1) can be replaced by two equations,

$\underline{y}(x;\alpha) = \underline{f}(x;\alpha) + \int_a^b \underline{U}(t;\alpha)dt$,

and

$$\overline{y}(x;\alpha) = \overline{f}(x;\alpha) + \int_a^b \overline{U}(t;\alpha)dt$$

where

$$\underline{U}(t;\alpha)=\begin{cases}k(s,t)\underline{y}(t;\alpha) & k(s,t)\geq 0\\ k(s,t)\overline{y}(t;\alpha) & k(s,t)<0\end{cases}$$

and

$$\overline{U}(t;\alpha)=\begin{cases}k(s,t)\overline{y}(t;\alpha) & k(s,t)\geq 0\\ k(s,t)\underline{y}(t;\alpha) & k(s,t)<0\end{cases}$$

Sufficient conditions for the existence of a unique solution to the fuzzy Fredholm integral equations of the second kind, have been given in [3].

4- Main Results:

In this section, the solution of equation (1) by substituting the Bernstein polynomials in $\tilde{y}(x;\alpha)$ to give $\tilde{y}(x;\alpha)=\sum_{i=1}^{n}\tilde{a}_iB_{i,n}(x)$, and hence:

$$\sum_{i=0}^{n}\tilde{a}_iB_{i,n}(x)=\tilde{f}(x;\alpha)+\lambda\int_a^b k(x,t)\sum_{i=1}^{n}\tilde{a}_iB_{i,n}(t)\,dt$$

or equivalently,

$$\sum_{i=0}^{n}\tilde{a}_iB_{i,n}(x)=\tilde{f}(x;\alpha)+\lambda\sum_{i=1}^{n}\tilde{a}_i\int_a^b k(x,t)B_{i,n}(t)dt \quad (2)$$

Now, in order to fined $\tilde{a}_i$, choose $x_i\in[a,b], i=0,1,\cdots,n$, and substitute them into equation (2) to obtain the dual fuzzy linear system of the form:

$$A\tilde{a}=\tilde{f}+B\tilde{a}$$

where $A=[a_{i,j}]$, $B=[b_{i,j}]$, $0\leq i,j\leq n$, $a_{i,j}=B_{j,n}(x_i)$, $b_{i,j}=\lambda\int_a^b k(x_i,t)B_{j,n}(t)dt$ and $\tilde{f}=[\tilde{f}(x_0)\ \cdots\ \tilde{f}(x_n)]^T$ is an arbitrary fuzzy number vector.

Now, transform the $n\times n$ coefficient matrices A and B into $2n\times 2n$ crisp linear matrices S and T respectively as mentioned in section (2) to obtain the values of $\tilde{a}$, as follows:

$$S\tilde{a}=T\tilde{a}+F$$

and hence

$$(S-T)\tilde{a}=F\Longrightarrow\tilde{a}=(S-T)^{-1}F$$

Where

$$F=\left[\underline{f}_1\ \ \underline{f}_2\ \cdots\ \underline{f}_n\ \ -\overline{f}_1\ \ -\overline{f}_2\ \cdots\ -\overline{f}_n\right]^T$$

are functions whose values must be determined at $x_i\in[a,b], i=0,1,\cdots,n$, and the fuzzy approximate solution of equation (1) will be given by:

$$\tilde{y}(x;\alpha)=\left[\underline{y}(x;\alpha),\overline{y}(x;\alpha)\right]$$

where the lower solution $\underline{y}(x;\alpha)=\sum_{i=1}^{n}\underline{a}_iB_{i,n}(x)$ and the upper solution $\overline{y}(x;\alpha)=\sum_{i=1}^{n}\overline{a}_iB_{i,n}(x)$.

5- Numerical Examples:

Now, two examples will be presented and solved,alongside illustrating graphs to compare with the exact solution to illustrate the proposed method.

Example (1):

Consider the fuzzy Fredholm integral equation of the second kind, with $\lambda=1$, $a=0$, and $b=1$, such that:

$$\tilde{y}(x;\alpha)=\tilde{f}(x;\alpha)+\int_0^1 k(x,t)\tilde{y}(t;\alpha)dt \quad (3)$$

where

$k(x,t)=x+t, 0\leq x,t\leq 1$

$\underline{f}(x;\alpha)=\alpha\left(\frac{1}{2}x-\frac{1}{3}\right)$,

$\overline{f}(x;\alpha)=(2-\alpha)\left(\frac{1}{2}x-\frac{1}{3}\right)$

where $\underline{y}(x;\alpha)=\alpha x$ is the exact lower solution, and $\overline{y}(x;\alpha)=(2-\alpha)x$ is the exact upper solution [2].

Now, by substituting $\tilde{y}(x;\alpha)=\sum_{i=0}^{3}\tilde{a}_iB_{i,3}(x)$, into equation (3) we get:

$$\sum_{i=0}^{3}\tilde{a}_iB_{i,3}(x)=\tilde{f}(x;\alpha)+\int_0^1 k(x,t)\sum_{i=0}^{3}\tilde{a}_iB_{i,3}(t)\,dt$$

or equivalently:

$$\sum_{i=0}^{3}\tilde{a}_iB_{i,3}(x)=\tilde{f}(x;\alpha)+\sum_{i=0}^{3}\tilde{a}_i\int_0^1 k(x,t)B_{i,3}(t)dt \quad (4)$$

and in order to fined $\tilde{a}_i$, $i=0,1,2,3$, let us take $x_0=\frac{1}{4}$, $x_1=\frac{1}{2}$, $x_2=\frac{3}{4}$, and $x_3=1$then substitute them into equation (4) to obtain the dual fuzzy linear system:

$$A\tilde{a}=\tilde{f}+B\tilde{a}$$

where

$$A=\begin{bmatrix}0.422 & 0.422 & 0.141 & 0.016\\ 0.125 & 0.375 & 0.375 & 0.125\\ 0.016 & 0.141 & 0.422 & 0.422\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$\tilde{a}=\begin{bmatrix}\tilde{a}_0\\ \tilde{a}_1\\ \tilde{a}_2\\ \tilde{a}_3\end{bmatrix}, \tilde{f}=\begin{bmatrix}\tilde{f}(x_0)\\ \tilde{f}(x_1)\\ \tilde{f}(x_2)\\ \tilde{f}(x_3)\end{bmatrix}$$

and

$$B=\begin{bmatrix}0.113 & 0.163 & 0.213 & 0.263\\ 0.175 & 0.225 & 0.275 & 0.325\\ 0.238 & 0.288 & 0.338 & 0.388\\ 0.3 & 0.35 & 0.4 & 0.45\end{bmatrix}$$

Now, transform the above 4×4 coefficient matrices A and B into 8×8 crisp linear Matrices S, T respectively as mentioned in section (2), to get:

$$S=\begin{bmatrix}0.422 & 0.422 & 0.141 & 0.016 & 0 & 0 & 0 & 0\\ 0.125 & 0.375 & 0.375 & 0.125 & 0 & 0 & 0 & 0\\ 0.016 & 0.141 & 0.422 & 0.422 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.422 & 0.422 & 0.141 & 0.016\\ 0 & 0 & 0 & 0 & 0.125 & 0.375 & 0.375 & 0.125\\ 0 & 0 & 0 & 0 & 0.016 & 0.141 & 0.422 & 0.422\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}$$

and

$$T=\begin{bmatrix}0.113 & 0.163 & 0.213 & 0.263 & 0 & 0 & 0 & 0\\ 0.175 & 0.225 & 0.275 & 0.325 & 0 & 0 & 0 & 0\\ 0.238 & 0.288 & 0.338 & 0.388 & 0 & 0 & 0 & 0\\ 0.3 & 0.35 & 0.4 & 0.45 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0.113 & 0.163 & 0.213 & 0.263\\ 0 & 0 & 0 & 0 & 0.175 & 0.225 & 0.275 & 0.325\\ 0 & 0 & 0 & 0 & 0.238 & 0.288 & 0.338 & 0.388\\ 0 & 0 & 0 & 0 & 0.3 & 0.35 & 0.4 & 0.45\end{bmatrix}$$

to obtain the values of $\tilde{a}$, as follows:

$$\begin{bmatrix}\underline{a}_0\\ \underline{a}_1\\ \underline{a}_2\\ \underline{a}_3\\ -\overline{a}_0\\ -\overline{a}_1\\ -\overline{a}_2\\ -\overline{a}_3\end{bmatrix}=(S-T)^{-1}F=\begin{bmatrix}0\\ 0.333\alpha\\ 0.667\alpha\\ 0.999\alpha\\ 0\\ 0.333\alpha-0.667\\ 0.667\alpha-1.333\\ \alpha-2\end{bmatrix}$$

Hence

$\tilde{a}_0=(0,0)$, $\tilde{a}_1=(0.333\alpha, 0.667-0.333\alpha)$,

$\tilde{a}_2=(0.667\alpha, 1.333-0.667\alpha)$,

$\tilde{a}_3=(0.999\alpha, 2-\alpha)$,

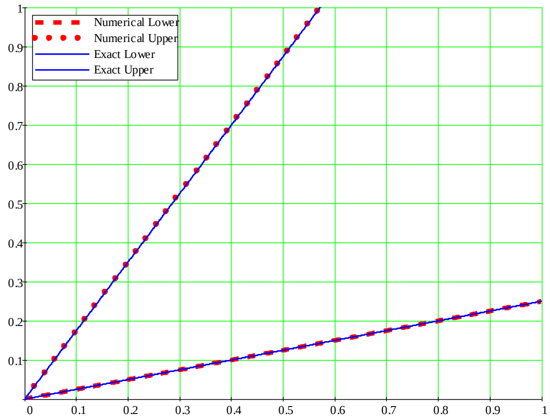
therefore the fuzzy approximate solution $\tilde{y}(x;\alpha)=\left[\underline{y}(x;\alpha),\overline{y}(x;\alpha)\right]$, will be given as:

$$\underline{y}(x;\alpha)=\underline{a}_0(1-x)^3+\underline{a}_1 3x(1-x)^2+\underline{a}_2 3x^2(1-x)+\underline{a}_3 x^3,$$

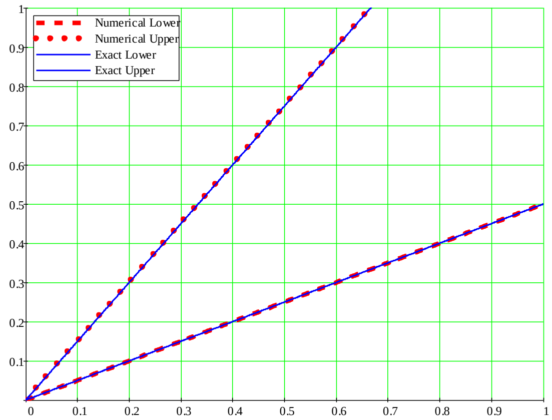
and

$$\overline{y}(x;\alpha)=\overline{a}_0(1-x)^3+\overline{a}_1 3x(1-x)^2+\overline{a}_2 3x^2(1-x)+\overline{a}_3 x^3$$

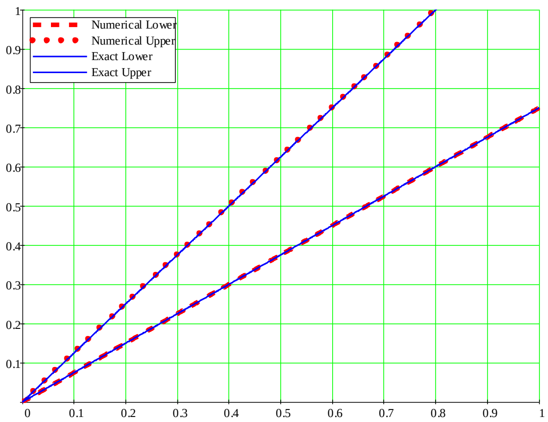
Figs. (1.a), (1.b), (1.c), and (1.d) gives a comparison between the exact and approximate solutions for $\alpha=0.25$, 0.5, 0.75, and1 respectively.



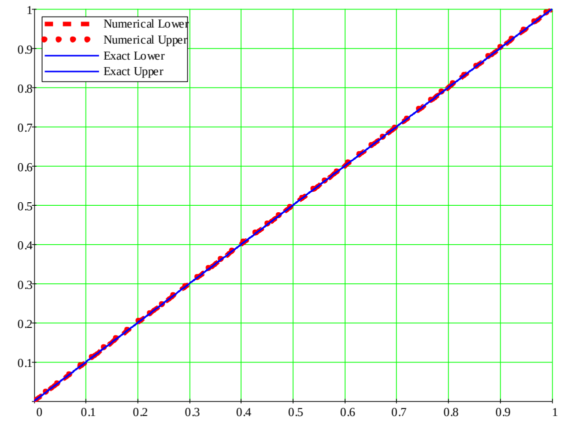
(1.a) $\alpha=0.25$



(1.b) $\alpha=0.5$



(1.c) $\alpha=0.75$



(1.d) $\alpha = 1$

Fig. (1) The exact solution alongside the numerical solution of example (1) for different values of α.

<u>Example</u> (2):

Consider the fuzzy Fredholm integral equation of the second kindwith $\lambda = 1$, $a = 0$, and $b = 2\pi$, such that:

$$\tilde{y}(x;\alpha) = \tilde{f}(x;\alpha) + \int_0^{2\pi} k(x,t)\tilde{y}(t;\alpha)dt \quad (5)$$

where

$$k(x,t) = 0.1\sin(t)\sin(\tfrac{1}{2}x),\ 0 \le x,t \le 2\pi$$

$$\underline{f}(x;\alpha) = \left[\frac{13}{15}(\alpha^2+\alpha) + \frac{2}{15}(4-\alpha-\alpha^3)\right]\sin(\tfrac{1}{2}x)$$

$$\overline{f}(x;\alpha) = \left[\frac{2}{15}(\alpha^2+\alpha) + \frac{13}{15}(4-\alpha-\alpha^3)\right]\sin(\tfrac{1}{2}x)$$

Where the exact lower solution is $\underline{y}(x;\alpha) = (\alpha^2+\alpha)\sin(\frac{1}{2}x)$, and the exact upper solution is $\overline{y}(x;\alpha) = (4-\alpha-\alpha^3)\sin(\frac{1}{2}x)$[2].

Now, by substituting $\tilde{y}(x;\alpha) = \sum_{i=0}^{3}\tilde{a}_i B_{i,3}(x)$, into equation (5) we get:

$$\sum_{i=0}^{3}\tilde{a}_i B_{i,3}(x) = \tilde{f}(x;\alpha) + \int_0^{2\pi} k(x,t)\sum_{i=0}^{3}\tilde{a}_i B_{i,3}(t)\,dt$$

or equivalently,

$$\sum_{i=0}^{3}\tilde{a}_i B_{i,3}(x) = \tilde{f}(x;\alpha) + \sum_{i=0}^{3}\tilde{a}_i \int_0^{2\pi} k(x,t)B_{i,3}(t)dt \quad (6)$$

Now, in order to fined $\tilde{a}_i$, $i = 0,1,2,3$, let us take $x_0 = \frac{\pi}{4}, x_1 = \frac{3\pi}{4}, x_2 = \frac{5\pi}{4}$, and$x_3 = \frac{7\pi}{4}$, then substitute them into equation (6) to obtain the dual fuzzy linear system:

$$A\tilde{a} = \tilde{f} + B\tilde{a}$$

where

$$A = \begin{bmatrix} 0.67 & 0.287 & 0.041 & 0.002 \\ 0.244 & 0.439 & 0.264 & 0.053 \\ 0.053 & 0.264 & 0.439 & 0.244 \\ 0.002 & 0.041 & 0.287 & 0.67 \end{bmatrix}, \qquad \tilde{a} = \begin{bmatrix} \tilde{a}_0 \\ \tilde{a}_1 \\ \tilde{a}_2 \\ \tilde{a}_3 \end{bmatrix},$$

$$\tilde{f} = \begin{bmatrix} \tilde{f}(x_0) \\ \tilde{f}(x_1) \\ \tilde{f}(x_2) \\ \tilde{f}(x_3) \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0.032 & 0.017 & -0.017 & -0.032 \\ 0.078 & 0.042 & -0.042 & -0.078 \\ 0.078 & 0.042 & -0.042 & -0.078 \\ 0.032 & 0.017 & -0.017 & -0.032 \end{bmatrix}$$

Now, transform the above 4×4 coefficient matrices A and B into 8×8 crisp linear Matrices S, T respectively as mentioned in section (2), to get:

$$S = \begin{bmatrix} 0.67 & 0.287 & 0.041 & 0.002 & 0 & 0 & 0 & 0 \\ 0.244 & 0.439 & 0.264 & 0.053 & 0 & 0 & 0 & 0 \\ 0.053 & 0.264 & 0.439 & 0.244 & 0 & 0 & 0 & 0 \\ 0.002 & 0.041 & 0.287 & 0.67 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.67 & 0.287 & 0.041 & 0.002 \\ 0 & 0 & 0 & 0 & 0.244 & 0.439 & 0.264 & 0.053 \\ 0 & 0 & 0 & 0 & 0.053 & 0.264 & 0.439 & 0.244 \\ 0 & 0 & 0 & 0 & 0.002 & 0.041 & 0.287 & 0.67 \end{bmatrix},$$

and

$$T = \begin{bmatrix} 0.032 & 0.017 & 0 & 0 & 0 & 0 & 0.017 & 0.032 \\ 0.078 & 0.042 & 0 & 0 & 0 & 0 & 0.042 & 0.078 \\ 0.078 & 0.042 & 0 & 0 & 0 & 0 & 0.042 & 0.078 \\ 0.032 & 0.017 & 0 & 0 & 0 & 0 & 0.017 & 0.032 \\ 0 & 0 & 0.017 & 0.032 & 0.032 & 0.017 & 0 & 0 \\ 0 & 0 & 0.042 & 0.078 & 0.078 & 0.042 & 0 & 0 \\ 0 & 0 & 0.042 & 0.078 & 0.078 & 0.042 & 0 & 0 \\ 0 & 0 & 0.017 & 0.032 & 0.032 & 0.017 & 0 & 0 \end{bmatrix}$$

to obtain the values of $\tilde{a}$, as follows:

$$\begin{bmatrix} \underline{a}_0 \\ \underline{a}_1 \\ \underline{a}_2 \\ \underline{a}_3 \\ -\overline{a}_0 \\ -\overline{a}_1 \\ -\overline{a}_2 \\ -\overline{a}_3 \end{bmatrix} = (S-T)^{-1}F = \begin{bmatrix} -0.032 - 0.075\alpha - 0.083\alpha^2 + 0.008\alpha^3 \\ 0.481 + 1.112\alpha + 1.232\alpha^2 - 0.12\alpha^3 \\ 0.481 + 1.112\alpha + 1.232\alpha^2 - 0.12\alpha^3 \\ -0.032 - 0.075\alpha - 0.083\alpha^2 + 0.008\alpha^3 \\ 0.331 - 0.075\alpha + 0.008\alpha^2 - 0.083\alpha^3 \\ -4.928 + 1.112\alpha - 0.12\alpha^2 + 1.232\alpha^3 \\ -4.928 + 1.112\alpha - 0.12\alpha^2 + 1.232\alpha^3 \\ 0.331 - 0.075\alpha + 0.008\alpha^2 - 0.083\alpha^3 \end{bmatrix}$$

Hence:

$\tilde{a}_0 = (-0.032 - 0.075\alpha - 0.083\alpha^2 + 0.008\alpha^3, -0.331 + 0.075\alpha - 0.008\alpha^2 + 0.083\alpha^3)$,

$\tilde{a}_1 = (0.481 + 1.112\alpha + 1.232\alpha^2 - 0.12\alpha^3, 4.928 - 1.112\alpha + 0.12\alpha^2 - 1.232\alpha^3)$,

$\tilde{a}_2 = (0.481 + 1.112\alpha + 1.232\alpha^2 - 0.12\alpha^3, 4.928 - 1.112\alpha + 0.12\alpha^2 - 1.232\alpha^3)$,

$\tilde{a}_3 = (-0.032 - 0.075\alpha - 0.083\alpha^2 + 0.008\alpha^3, -0.331 + 0.075\alpha - 0.008\alpha^2 + 0.083\alpha^3)$,

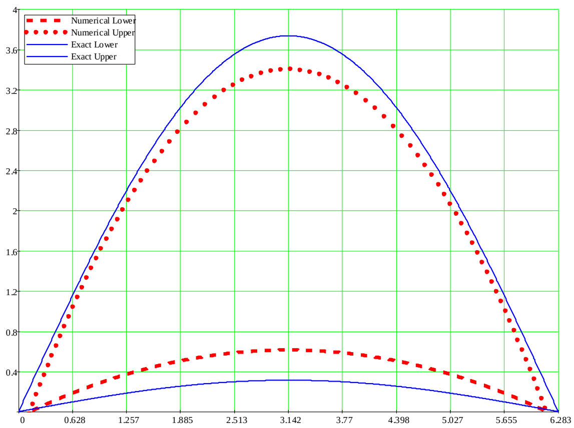
Therefore the fuzzy approximate solution is given by $\tilde{y}(x;\alpha) = \left[\underline{y}(x;\alpha), \overline{y}(x;\alpha)\right]$, where

$$\underline{y}(x;\alpha) = \underline{a}_0 \frac{1}{(2\pi)^3}(2\pi - x)^3 + \underline{a}_1 \frac{3}{(2\pi)^3} x(2\pi - x)^2 + \underline{a}_2 \frac{3}{(2\pi)^3} x^2(2\pi - x) + \underline{a}_3 \frac{1}{(2\pi)^3} x^3,$$

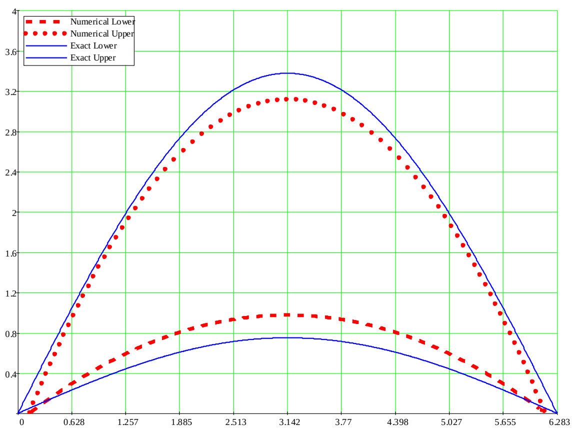
and

$$\overline{y}(x;\alpha) = \overline{a}_0 \frac{1}{(2\pi)^3}(2\pi - x)^3 + \overline{a}_1 \frac{3}{(2\pi)^3} x(2\pi - x)^2 + \overline{a}_2 \frac{3}{(2\pi)^3} x^2(2\pi - x) + \overline{a}_3 \frac{1}{(2\pi)^3} x^3,$$

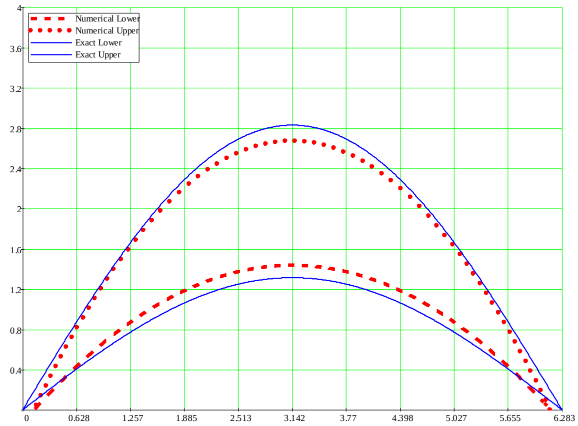
Figs. (2.a), (2.b), (2.c), and (2.d) gives a comparison between the exact and approximate solutions for $\alpha = 0.25, 0.5, 0.75,$ and 1 respectively.



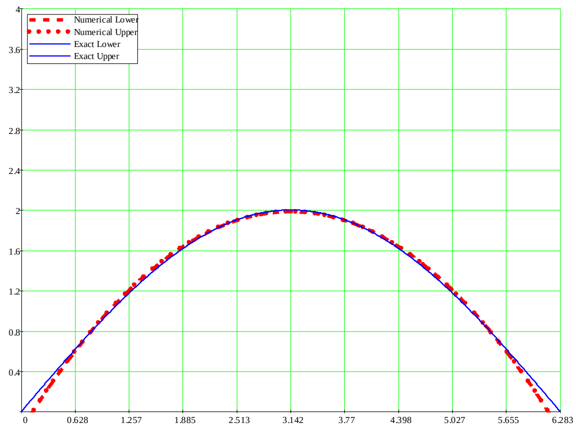
(2.a) $\alpha = 0.25$



(2.b) $\alpha = 0.5$



(2.c) $\alpha = 0.75$



(2.d) $\alpha = 1$

Fig.(2) The exact solution alongside the numerical solution of example (2) for different values of α.

Conclusions

In this paper a very simple and straight forward method for approximating the solution of the given fuzzy integral equation using Bernstein polynomial basis and depending on solving a dual fuzzy linear system of equations is presented, and the results shown in example (1) and (2) are very good if compared with the exact solution.

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الخلاصة

في هذا البحث، تم أقتراح طريقة عددية لتقريب حل معادلات فريدهولم التكاملية الضبابية من النوع الثاني بأستخدام متعددات حدود برنشتاين، والتي تم تحديد معاملاتها من خلال حل نظام ضبابي خطي مزدوج، ومن ثم طرح أمثلة عددية لتوضيح الطريقة المقترحة، والتي قد تم عمل حساباتها باستخدام برنامج الكومبيوتر MathCad V.14.