Fixed Point Theorem on (F, ρ) Fuzzy Metric Space

Fadhel Subhi Fadhel and Rasha Naser Majeed
College of Science, Al-Nahrain University, Baghdad- Iraq.
Department of Mathematics, College of Education – Ibn Al- Haitham, Baghdad University, Baghdad-Iraq.

Abstract

In this paper we prove that the fuzzy metric space (F, ρ) is a fuzzy complete metric space, (where F is the family of all fuzzy sets in general fuzzy numbers F_G and fuzzy points F_P and any numbers of intersection, union of fuzzy sets in F_G or F_P), then the proof of the fixed point theorem in the fuzzy metric space (F, ρ) is given as a main result.

1. Introduction

The concept of fuzzy set was first introduced by L. Zadeh [1] in 1965, several researches were conducted on the generalization of the concept of fuzzy set. In the next decade I. Kramosil and J. Michalek [5] in 1975 introduced the concept of fuzzy metric spaces, which opens an avenue for further development of analysis in such spaces. Consequently, in due course of time some metric fixed point results were generalized to fuzzy metric spaces by A. George and P. Veeramani [6] in 1994 and M. Grabiec [7] in 1998 and others.

Several approaches are proposed to study fuzzy metric spaces depending on the definition of the distance function either using the α-level sets, or using the membership function, or using fuzzy numbers, etc.

Therefore, the study of some well know results on fuzzy metric spaces will depend on the structure of the fuzziness, such as the completeness of such spaces, fixed point theorem, etc.

In this paper, the distance on F is based on the distance defined and given by Kaufmann and Gupta [8] in 1991 (where F is the families of fuzzy sets which are any intersection union of fuzzy sets are fuzzy numbers or fuzzy point). M. Aziz [9] in 2006 studied fuzzy numbers and fuzzy points again in different approachs and given the construction of fuzzy metric space, as well as, a modified distance function between fuzzy sets.

Also, in this paper we prove this fuzzy metric space is complete and the fixed point theorem is valued.

2. Preliminaries

In this section, some fundamental and primitive concepts related to fuzzy set theory, in general, and fuzzy metric space, in particular are given.

Definition 2.1 [2]:

If X is a collection of objects with generic element x, then a fuzzy subset $\tilde{A}$ in X is characterized by a membership function; $\mu_{\tilde{A}}:X\to I$, where I=[0,1], then we write a fuzzy set $\tilde{A}$ by the set of points:
$\tilde{A}=\{(x, \mu_{\tilde{A}}(x)) \mid x\in X,\ 0\le \mu_{\tilde{A}}(x) \le 1\}$.

Remark 2.2 [2]:

The set of all fuzzy subsets of a set X is denoted by I^X that is
$I^X = \{\tilde{A}: \tilde{A} \text{ is fuzzy subset of } X\}$.

Definition 2.3 [3], [4]:

A fuzzy point $\tilde{p}$ in a set X is a fuzzy set with membership function:

$$\mu_{\tilde{p}}(x) = \begin{cases} r & \text{for } x = y \\ 0 & \text{for } x \neq y \end{cases}$$

where $x\in X$ and $0 < r \le 1$, y is called the support of $\tilde{p}$ and r the value of $\tilde{p}$.

We denote this fuzzy point by y_r or p̃.tow fuzzy points x_r and y_s are said to be distinct if and only if $x \neq y$.

Definition 2.4 [3], [4]:

A fuzzy point x_r is said to be belong to a fuzzy subset Ã in X, denoted by $x_r \in \tilde{A}$ if and only if $r \leq \mu_{\tilde{A}}(x)$.

Definition 2.5 [10]:

Let Ã be a fuzzy subset of X, for any $\alpha \in (0,1]$, the set of all elements x such that $\mu_{\tilde{A}}(x) \geq \alpha$ is called the α-level (or α-cut) set of Ã and is denoted by:

$$\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha\}.$$

The next definition depends on the extension principle:

Definition 2.6 [9]:

Let f be a function from the universal set X to the universal set Y. Let Ã be a fuzzy subset in X with the membership function $\mu_{\tilde{A}}(x)$. The image of Ã, written as f(Ã), is a fuzzy subset in Y whose the membership function is given by:

$$\mu_{f(\tilde{A})}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\mu_{\tilde{A}}(x)\}, & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

Now, before introducing the definition of fuzzy number, we introduce the definition of metric space based on real numbers.

Definition 2.7 [9] :

Let ℝ be the set of real numbers, let $\rho' : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ such that $\rho'(x,y) = |x-y| \ \forall x, y \in \mathbb{R}$, then $(\mathbb{R}, \rho')$ is a metric space.

Definition 2.8 [11]:

If a fuzzy set is convex and normalized, and its membership function is defined in ℝ and piecewise continuous, then it is called a fuzzy number.

Remark 2.9 [11]:

Fuzzy number is expressed as a fuzzy interval in the real line ℝ represented by two end points a and c and peak point b as [a, b, c], such that the highest of the fuzzy number at b is equal to 1.

Now, we can put (Remark 2.9) as the following to make the definition of fuzzy number more clarify.

Definition 2.10 [4],[8]:

A fuzzy number $\widetilde{M}$ with the membership function is given by:

$$\mu_{\widetilde{M}}(x) = \begin{cases} f_L(x); & \text{if } a \leq x \leq b \\ f_R(x); & \text{if } b \leq x \leq c \\ 0 & \text{other wise} \end{cases}$$

Where $f_L(x)$ is a continuous increasing function in [a, b], $f_R(x)$ is a continuous decreasing function in [b, c] and $f_L(a) = f_R(c) = 0, f_L(b) = f_R(b) = 1$, is called general fuzzy number.

We denote the family of this kind of fuzzy numbers as $F_G = \{[a, b, c]: \forall a \leq b \leq c \ ; a, b, c \in \mathbb{R}\}$.

Definition 2.11 [4],[8]:

Let $\widetilde{M}$ be any general fuzzy number, then if $f_L(x) = (x-a)/(b-a)$, $a \leq x \leq b$ and $f_R(x) = (c-x)/(c-b)$, $b \leq x \leq c$, then we call this fuzzy number as triangular fuzzy number which is denoted by $\widetilde{M} = (a, b, c)$ with $a < b < c$.

The family of all triangular fuzzy numbers is denoted by F_T, where:

$$F_T = \{(a,b,c): \forall a < b < c, \ a, b, c \in \mathbb{R}\}.$$

Remark 2.12 [4],[8]:

For a level $\lambda \in (0, 1]$ we can define a fuzzy point a_λ with the membership function:

$$\mu_{a_\lambda}(x) = \begin{cases} \lambda & , \ x = a \\ 0 & , \ x \neq a \end{cases}$$

All λ-levels fuzzy points a_λ, form a family given by:

$$F_P(\lambda) = \{a_\lambda : a \in \mathbb{R}\}$$

Especially: if λ=1, then this degenerates a fuzzy number (a, b, c), with a=b=c, which is denoted by $\tilde{a} = (a, a, a)$.

i.e. $F_P(1) = \{a_1 : a \in \mathbb{R}\} = \{\tilde{a} = (a, a, a) \mid a \in \mathbb{R}\}$ and let $F_P = \bigcup_{0 < \lambda \leq 1} F_P(\lambda)$.

Remark 2.13 [4], [8]:

Let F be the family of all fuzzy sets in general fuzzy numbers F_G and fuzzy points F_P such that $F_G \cap F_P \subset F$ and $F_G \cup F_P \subset F$.

Notation 2.14 [9]:

For each λ, $0 < \lambda \le 1$, there is a one-one and onto mapping between $F_P(\lambda)$ and $\mathbb{R}$, which maps $a_\lambda \in F$ onto $a \in \mathbb{R}$.

The next definition is a modified approach for the distance function between two fuzzy numbers, which will be used later in defining the fuzzy metric space.

Definition 2.15 [9]:

Let $\widetilde{A}, \widetilde{B} \in F$ where $\widetilde{A} = [a_1, a_2, a_3], \widetilde{B} = [b_1, b_2, b_3]$, then the distance between $\widetilde{A}$ and $\widetilde{B}$ is defined as:

$$\rho(\widetilde{A}, \widetilde{B}) = \frac{1}{2}\int_0^1 |x_{\widetilde{A}L}(\alpha) - x_{\widetilde{B}L}(\alpha)| d\alpha + \frac{1}{2}\int_0^1 |x_{\widetilde{A}R}(\alpha) - x_{\widetilde{B}R}(\alpha)| d\alpha$$

where $\alpha \in (0,1]$, and

$x_{\widetilde{A}L}(\alpha) = a_1 + (a_2 - a_1)\alpha$ is α-cuts of $\widetilde{A}$ from the left-hand side.

$x_{\widetilde{B}L}(\alpha) = b_1 + (b_2 - b_1)\alpha$ is α-cuts of $\widetilde{B}$ from the left-hand side.

$x_{\widetilde{A}R}(\alpha) = a_3 - (a_3 - a_2)\alpha$ is α-cuts of $\widetilde{A}$ from the right-hand side.

$x_{\widetilde{B}R}(\alpha) = b_3 - (b_3 - b_2)\alpha$ is α-cuts of $\widetilde{B}$ from the right-hand side.

Properties 2.16 [4],[8]:

1- If $a_\lambda, b_\beta \in F_P, 0 < \lambda < \beta \le 1$, then

$$\rho(a_\lambda, b_\beta) = \int_0^\lambda |a - b| d\alpha + \int_\lambda^\beta |b| d\alpha = \lambda|a - b| + (\beta - \lambda)|b|.$$

2- If $\tilde{a} = (a, a, a), \tilde{b} = (b, b, b) \in F_P(1)$, then $\rho(\tilde{a}, \tilde{b}) = |b - a|$.

As an illustration, consider the following example:

Example 2.17 :

Let $\widetilde{A}, \widetilde{B} \in F$ be any two fuzzy numbers, where $\widetilde{A} = (1,3,5)$, $\widetilde{B} = (2,4,6)$ membership functions:

$$\mu_{\widetilde{A}}(x) = \begin{cases} f_L(x) = \dfrac{x-1}{2}; & \text{if } 1 \le x \le 3 \\ f_R(x) = \dfrac{5-x}{2}; & \text{if } 3 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{\widetilde{B}}(x) = \begin{cases} f_L(x) = \dfrac{x-2}{2}; & \text{if } 2 \le x \le 4 \\ f_R(x) = \dfrac{6-x}{2}; & \text{if } 4 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

then in order to find the distance between $\widetilde{A}$ and $\widetilde{B}$, we must find α-cuts of $\widetilde{A}$ and $\widetilde{B}$, where $\alpha \in (0,1]$, as follows:

For $\widetilde{A}$ the left-hand side is $x_{\widetilde{A}L}(\alpha) = a_1 + (a_2 - a_1)\alpha = 1 + 2\alpha$, and the right-hand side is $x_{\widetilde{A}R}(\alpha) = a_3 - (a_3 - a_2)\alpha = 5 - 2\alpha$.

Similarly, for $\widetilde{B}$ the left-hand side is $x_{\widetilde{B}L}(\alpha) = b_1 + (b_2 - b_1)\alpha = 2 + 2\alpha$, and the right- hand side is $x_{\widetilde{B}R}(\alpha) = b_3 - (b_3 - b_2)\alpha = 6 - 2\alpha$.

Then the distance between $\widetilde{A}$ and $\widetilde{B}$ is:

$$\begin{aligned}\rho(\widetilde{A}, \widetilde{B}) &= \frac{1}{2}\int_0^1 |x_{\widetilde{A}L}(\alpha) - x_{\widetilde{B}L}(\alpha)| d\alpha + \frac{1}{2}\int_0^1 |x_{\widetilde{A}R}(\alpha) - x_{\widetilde{B}R}(\alpha)| d\alpha \\ &= \frac{1}{2}\int_0^1 |1 + 2\alpha - 2 - 2\alpha| d\alpha + \frac{1}{2}\int_0^1 |5 - 2\alpha - 6 + 2\alpha| d\alpha \\ &= \frac{1}{2}\int_0^1 1 d\alpha + \frac{1}{2}\int_0^1 1 d\alpha \\ &= 1\end{aligned}$$

Theorem 2.18[9]:

Let $\rho: F \times F \to \mathbb{R}$, then the distance $\rho(\widetilde{A}, \widetilde{B})$ on F satisfies the following three axioms of the distance function:

(a) $\rho(\widetilde{A}, \widetilde{B}) = 0$ if and only if $\widetilde{A} = \widetilde{B}$,

(b) $\rho(\widetilde{A}, \widetilde{B}) = \rho(\widetilde{B}, \widetilde{A})$,

(c) $\rho(\widetilde{A}, \widetilde{B}) + \rho(\widetilde{B}, \widetilde{C}) \ge \rho(\widetilde{A}, \widetilde{C})$.

So, (F, ρ) is fuzzy metric space.

Remark 2.19[9]:

It is important to notice that $(F_P(1), \rho)$ is isometric to $(\mathbb{R}, \rho')$, where ρ' is as given in Definition (2.5), which is denoted by $(F_P(1), \rho) \cong (\mathbb{R}, \rho')$, since there is f: $(F_P(1), \rho) \to (\mathbb{R}, \rho')$ such that $f(\tilde{a}) = a \; \forall \; \tilde{a} \in F_P(1)$, is one-

one, onto mapping and $\rho(\tilde{a},\tilde{b}) = |b-a| = \rho'(a,b) = \rho'\left(f(\tilde{a}), f(\tilde{b})\right)$.

3-Fixed Point Theorem on Fuzzy Metric Space (F, ρ)

In order to investigate fixed point theorem in the fuzzy metric space (F,ρ), we must firstly show that the fuzzy metric space (F,ρ) is complete. So some additional basic definitions are introduced.

Definition 3.1:

Let $\{\widetilde{P}_n\}$ be a sequence of fuzzy points in F, then $\{\widetilde{P}_n\}$ is said to be converge to $\widetilde{P}_0$ (written as $\widetilde{P}_n \to \widetilde{P}_0$) if and only if for each $\epsilon > 0$, $\exists k \in \mathbb{N}$ such that $\rho(\widetilde{P}_n, \widetilde{P}_0) < \epsilon, \forall k > n$.

Definition 3.2:

The sequence $\{\widetilde{P}_n\}$ is called Cauchy sequence in (F,ρ) if and only if for each $\epsilon > 0, \exists k \in \mathbb{N}$ such that $\rho(\widetilde{P}_n, \widetilde{P}_m) < \epsilon$, $\forall n, m > k$.

Definition 3.3:

A fuzzy metric space (F,ρ) is said to be complete if and only if every Cauchy sequence in (F,ρ) is converge.

Theorem 3.4:

The fuzzy metric space (F,ρ) is complete.

Proof:

To prove (F,ρ) is complete, we must prove any Cauchy sequence in F is convergent. Let $\{\tilde{p}_n\}$ be any Cauchy sequence of fuzzy points, such that $\tilde{p}_1 = x_{1_{\lambda_1}}, \tilde{p}_2 = x_{2_{\lambda_2}}, \ldots, \tilde{p}_n = x_{n_{\lambda_n}}, \ldots$.

Consider the following three cases:

1-If $\{\tilde{p}_n\} \in F_P(1)$, for all $n \in \mathbb{N}$.

Then $\tilde{p}_1 = x_1, \tilde{p}_2 = x_2, \ldots, \tilde{p}_n = x_n, \ldots$

Since $\{\tilde{p}_n\}$ is a Cauchy sequence, then we have $\forall \epsilon > 0, \exists k \in \mathbb{N}$ such that $\rho(\tilde{p}_n, \tilde{p}_m) < \epsilon$, $\forall n, m > k$.

And since $\{\tilde{p}_n\} \in F_P(1)$ for all $n \in \mathbb{N}$, implies from (Properties (2.16)part(2))

$\rho(\tilde{p}_n, \tilde{p}_m) = |x_n - x_m| < \epsilon \quad \forall n, m > k$.

Thus $\{x_n\}$ is Cauchy sequence in $\mathbb{R}$, but $\mathbb{R}$ is complete metric space, then $\{x_n\}$ is a convergent sequence.

2-If $\{\tilde{p}_n\} \in F_P(\lambda)$; $0 < \lambda < 1$

Then $\tilde{p}_1 = x_{1_\lambda}, \tilde{p}_2 = x_{2_\lambda}, \ldots, \tilde{p}_n = x_{n_\lambda}, \ldots$

Since $\{\tilde{p}_n\}$ is a Cauchy sequence, and $\lambda > 0$, then from Archimedean property for any $\lambda\epsilon > 0, \exists k \in \mathbb{N}$ such that $\rho(\tilde{p}_n, \tilde{p}_m) < \lambda\epsilon$, $\forall n, m > k$.

For $\tilde{p}_n = x_{n_\lambda}, \tilde{p}_m = x_{m_\lambda} \in F_P(\lambda)$; $0 < \lambda < 1$, their α-cuts, $0 < \alpha \leq 1$, and from the definition of the fuzzy points, are :

$x_{\tilde{p}_n L}(\alpha) = x_{\tilde{p}_n R}(\alpha) = x_n$, if $0 < \alpha \leq \lambda$

$x_{\tilde{p}_n L}(\alpha) = x_{\tilde{p}_n R}(\alpha) = 0$, if $\lambda < \alpha \leq 1$

$x_{\tilde{p}_m L}(\alpha) = x_{\tilde{p}_m R}(\alpha) = x_m$, if $0 < \alpha \leq \lambda$

$x_{\tilde{p}_m L}(\alpha) = x_{\tilde{p}_m R}(\alpha) = 0$, if $\lambda < \alpha \leq 1$

Therefore, by (Properties (2.16) part(1)) the distance between $\tilde{p}_n, \tilde{p}_m$ is

$$\rho(\tilde{p}_n, \tilde{p}_m) = \int_0^\lambda |x_n - x_m| d\alpha + \int_\lambda^1 |0| d\alpha = \lambda |x_n - x_m|$$

Since $\rho(\tilde{p}_n, \tilde{p}_m) < \lambda\epsilon$, $\forall n, m > k$, implies

$\rho(\tilde{p}_n, \tilde{p}_m) = \lambda|x_n - x_m| < \lambda\epsilon, \forall n, m > k$, $0 < \lambda < 1$

Therefore $|x_n - x_m| < \epsilon$, $\forall n, m > k$

And hence $\{x_n\}$ is Cauchy sequence in $\mathbb{R}$, but $\mathbb{R}$ is a complete metric space, then $\{x_n\}$ is a convergent sequence.

3-If $\{\tilde{p}_n\} \in F_P$, then $\tilde{p}_1 = x_{1_{\lambda_1}}, \tilde{p}_2 = x_{2_{\lambda_2}}, \ldots, \tilde{p}_n = x_{n_{\lambda_n}}, \tilde{p}_m = x_{m_{\lambda_m}}, \ldots$ such that $0 < \lambda_i \leq 1$, $\forall i = 1,2,3,\ldots$, where from Properties (2.16) part(1)it is supposed that $\{\lambda_i\}$ is a monotonic increasing sequence of levels, then for $m > n$, we have $\lambda_m > \lambda_n$ or $\lambda_m - \lambda_n > 0$. Since $\{\tilde{p}_n\}$ is a Cauchy sequence in F, and suppose that:

$\lambda_n \epsilon + (\lambda_m - \lambda_n)|x_m| > 0$, then $, \exists k \in \mathbb{N}$ such that

$\rho(\tilde{p}_n, \tilde{p}_m) < \lambda_n \epsilon + (\lambda_m - \lambda_n)|x_m| \quad \forall n, m > k$,

by (Proposition (2.16) part(1)), we have:

$$\begin{aligned}\rho(\tilde{p}_n, \tilde{p}_m) &= \lambda_n |x_n - x_m| + (\lambda_m - \lambda_n)|x_m| \\ &< \lambda_n \epsilon + (\lambda_m - \lambda_n)|x_m| \quad \forall n, m > k \\ &= \lambda_n |x_n - x_m| < \lambda_n \epsilon \quad \forall n, m > k\end{aligned}$$

$= |x_n - x_m| < \epsilon \qquad \forall n, m > k$

$= \rho'(x_n, x_m) < \epsilon \qquad \forall n, m > k$

Implies $\{x_n\}$ is Cauchy sequence in $\mathbb{R}$, but $\mathbb{R}$ is a complete metric, therefore $\{x_n\}$ is a convergent sequence.

Thus in all cases $\{\tilde{p}_n\}$ is a convergent sequence, and therefore (F, ρ) is a complete fuzzy metric space.

Now, we are ready to investigate the fixed point theorem in fuzzy metric space (F, ρ) .

Theorem 3.5:

Let (F, ρ) be a complete fuzzy metric space, and f: $(F, \rho) \to (F, \rho)$ satisfy $\rho(f(\tilde{p}), f(\tilde{q})) \leq r\rho(\tilde{p}, \tilde{q})$, where $0 \leq r < 1$. Then f has a unique fixed point $\tilde{p}$.

Proof:

For any fuzzy point $\tilde{p}_0 \in F$. Let $\tilde{p}_1 = f(\tilde{p}_0),\ \tilde{p}_2 = f(\tilde{p}_1) = f(f(\tilde{p}_0)) = f^2(\tilde{p}_0), \ldots, \tilde{p}_n = f^n(\tilde{p}_0)$

So, we have a sequence of fuzzy points $\{\tilde{p}_0, \tilde{p}_1, \tilde{p}_2, \ldots\}$.

Now, we show that for each $\tilde{p}_0$, $\{\tilde{p}_n\}$ is Cauchy sequence.

If $m > n$, $\rho(\tilde{p}_m, \tilde{p}_n) = \rho(f^m(\tilde{p}_0), f^n(\tilde{p}_0))$

$\leq r^n \rho(f^{m-n}(\tilde{p}_0), \tilde{p}_0)$

$= r^n \rho(\tilde{p}_{m-n}, \tilde{p}_0)$

$\leq r^n\{\rho(\tilde{p}_0, \tilde{p}_1) + \rho(\tilde{p}_1, \tilde{p}_2) + \cdots + \rho(\tilde{p}_{m-n-1}, \tilde{p}_{m-n})\}$

$\leq r^n \rho(\tilde{p}_0, \tilde{p}_1)\ \{1 + r + r^2 + \cdots + r^{m-n-1}\}$

$\leq \frac{r^n \rho(\tilde{p}_0, \tilde{p}_1)}{1-r}$

Since $r < 1$, $\forall \epsilon > 0, \exists n \in \mathbb{N}$ and as $n \to \infty$, then $\frac{r^n \rho(\tilde{p}_0, \tilde{p}_1)}{1-r} < \epsilon$

Then $\rho(\tilde{p}_m, \tilde{p}_n) < \frac{r^n \rho(\tilde{p}_0, \tilde{p}_1)}{1-r} < \epsilon \quad \forall n, m > k$

Thus $\{\tilde{p}_n\}$ is Cauchy sequence. Since (F, ρ) is complete fuzzy metric space, there exists $\tilde{p}$ such that $\{\tilde{p}_n\} \to \tilde{p}$.

Similarly if $n > m$.

Now, we show that $\tilde{p}$ is fixed point in F, then we must prove $f(\tilde{p}) = \tilde{p}$.

$\rho(f(\tilde{p}), \tilde{p}) \leq \rho(f(\tilde{p}), \tilde{p}_n) + \rho(\tilde{p}_n, \tilde{p})$

$= \rho(f(\tilde{p}), f(\tilde{p}_{n-1})) + \rho(\tilde{p}_n, \tilde{p})$

$\leq r\, \rho(\tilde{p}, \tilde{p}_{n-1}) + \rho(\tilde{p}_n, \tilde{p})$

Since $\{\tilde{p}_n\} \to \tilde{p}$, thus $\rho(f(\tilde{p}), \tilde{p}) \leq r\epsilon + \epsilon = r(1+\epsilon)$. Therefore $\rho(f(\tilde{p}), \tilde{p}) = 0$, implies $f(\tilde{p}) = \tilde{p}$.

Next, we must show that $\tilde{p}$ is the unique fixed point of f, suppose there exists another fixed point $\tilde{q} \in F$, such that $f(\tilde{q}) = \tilde{q}$.

$\rho(\tilde{p}, \tilde{q}) = \rho(f(\tilde{p}), f(\tilde{q})) \leq r\rho(\tilde{p}, \tilde{q})$

Thus $\rho(\tilde{p}, \tilde{q}) \leq r\rho(\tilde{p}, \tilde{q})$

Therefore $\rho(\tilde{p}, \tilde{q}) = 0$ implies $\tilde{p} = \tilde{q}$. Thus, there is a unique fixed point $\tilde{p}$ of f in F.

References:

[1] L. A. Zadeh, "Fuzzy Set", Information and Control, vol.8, no.3, pp.338-353, 1965.

[2] G. J. Klir and B. Yuan, "Fuzzy Set Theory: Foundations and Applications", Prentice Hall PTR, 1997.

[3] M. M. Gupta, R. K. Ragade and R.R. Yager, "Advances in Fuzzy Set Theory and Applications", North-Holland Publishing Company, 1979 .

[4] B. Huttan, "Normality in Fuzzy Topological Spaces J. Math. Anal. Appl., vol.50, pp.74-79, 1975 .

[5] I.Kramosil and J.Michalek, "Fuzzy Metric and Statistical Metric Space", Kybernetika, vol.11, pp.336-344, 1975 .

[6] A.George and P.Veeramani, "On Some Results in Fuzzy Metric Space ", Fuzzy set And systems, vol.64, pp. 395-399, 1994.

[7] M. Grabiec, "Fixed Point in Fuzzy Metric Space", Fuzzy set and systems, vol.27, pp.385-389, 1988.

[8] A. Kaufmann and M.M. Gupta, " Introduction to Fuzzy Arithmetic Theory and Applications", Van Nortrand Rein bold, New York, 1991.

[9] M.A.Aziz, "On Separation Axioms of Fuzzy Topological Spaces", Ph.D. Thesis, College of Education, AL-Mustansiriyah University, 2006.

[10] A.K. Katsaras and D. B. Liu, "Fuzzy Space", J. of Math. Anal. And Appl., vol.58, pp.135-146, 1977 .

[11] S. Gao, Z. Zhang and C. Cao, "Multiplication Operation on Fuzzy Numbers", Journal of software, vol.4, June, pp.331-338, 2009 .

الخلاصة

في هذا البحث قمنا بإثبات إن الفضاء المتري الضبابي (F,ρ) هو فضاء متري كامل (حيث ان F هي عائلة كل المجموعات الضبابية في مجموعة الاعداد الضبابية والنقاط الضبابية, وتقاطع او اتحاد اي عناصر في هذه المجموعة يكون فيها ايضا) ، ومن ثم إعطاء نص وبرهان مبرهنة النقطة الصامدة في الفضاءات المترية الضبابية كنتيجة رئيسية.