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SOME PROPERTIES OF CENTRALLY CLOSED LIE IDEALS OF CENTRALLY PRIME RINGS

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Abstract

In this paper, the definitions of a centrally closed Lie ideal and central identity property are introduced and the behaviors of this type of ideals in 2- torsion free centrally prime rings are studied, also we study the effects of derivation on them, so we will prove some properties of these ideals.

keywords: derivations, localizations, prime rings, centrally prime rings, Lie ideals.

Introduction

Let R be a ring. A non-empty subset S of R is said to be a multiplicative closed set in R if a, b $\in$ S implies ab $\in$ S, and a multiplicative closed set S is called a multiplicative system if $0 \notin S$, [Larsen and McCarthy ,1971].

Let S be a multiplicative system in R such that [S, R] = {0},where $[S,R]=\{[s,r]: s\in S, r\in R\}$ and [s, r] is the commutator defined by $sr-rs$. Define a relation ~ on $R\times S$ as follows :

If (a,s) and (b,t) are in $R\times S$, then $(a,s)\sim(b,t)$ if and only if there exists $x\in S$ such that $x(at-bs)=0$. Since $[S,R]=\{0\}$, it can be shown that ~ is an equivalence relation on $R\times S$.

Now denote the equivalence class of (a,s) in $R\times S$ by a_S, and the set of all equivalence classes determined under this equivalence relation by R_S, that is let $a_S=\{(b,t)\in R\times S:(a,s)\sim(b,t)\}$ and $R_S=\{a_s:(a,s)\in R\times S\}$.

On R_S we define addition (+) and multiplication (.) as follows:

$a_s+b_t=(at+bs)_{st}$ and $a_s.b_t=(ab)_{st}$, for all $a_s,b_t\in R_S$.

It can be shown that these two operations are well-defined and that $(R_S,+,.)$ forms a ring which we call the localization of R at S.

If R is a commutative ring with identity then the equivalence class a_S is also denoted by $\frac{a}{s}$ [Larsen and McCarthy,1971] or by $s^{-1}a$,[Ranicki, 2006], and R_S is also denoted by $S^{-1}R$ [Larsen and McCarthy,1971 and Ranicki,2006].

If R is a ring and S is a multiplicative system in R such that $[S,R]=\{0\}$, then:

i- R_S has the identity element though R does not have, in fact if $s\in S$ then s_S is the identity element of R_S and this identity does not depend on the choice of the elements of S, that is $s_S=t_t$, for all s, t$\in$ S. [Jabbar, 2006]

ii- If $a,b\in R$ and $s\in S$, then $a_S+b_S=(a+b)_S$.In fact this result can be generalized to any n elements of R, that is if $a_1,a_2,\ldots,a_n\in R$ and $s\in S$ then $(a_1)_S+(a_2)_S+\ldots+(a_n)_S=(a_1+a_2+\ldots+a_n)_S$.

iii- For all $s\in S, 0_S$ is the zero of the ring R_S and this zero does not depend on the choice of the elements of S, that means $0_S=0_t$, for all $s,t\in S$.

iv- If $a_S\in R_S$, where $a\in R$ and $s\in S$ then $(-a)_S$ is the additive identity of a_S in R_S, that is $-a_S=(-a)_S$.

v- If $a_S=0$ in R_S, where $a\in R, s\in S$, then there exists $t\in S$ such that $ta=0$.

vi- If $A\subseteq R$, then by A_S we mean the set $A_S=\{a_s: a\in A, s\in S\}$. [Jabbar, 2006].

Now we mention to some basic definitions: Let R be a ring. Then:

i- R is called a prime ring if, whenever $a,b \in R$ such that $aRb=\{0\}$ then $a=0$ or $b=0$, where $aRb=\{arb : r \in R\}$ [Herstein,1969, Tsai, 2004 and Ashraf,2005].

ii- An additive mapping $D:R \to R$ is called a derivation on R if

$D(ab)=D(a)b+aD(b)$, for all $a,b \in R$ [Martindale and Miers,1983 , Vukman, 1999, Jung and Park,2006], in other words a mapping $D:R \to R$ is called a derivation on R if:

1: $D(a+b)=D(a)+D(b)$ and

2: $D(ab)=D(a)b+aD(b)$, for all $a,b \in R$.

iii- If n is a positive integer then R is called an n-torsion free ring if for $x \in R$ nx=0 implies $x=0$[Vukman, 2001].

iv-:By the center of R we mean the set $Z(R)=\{x \in R : xr=rx,$ for all $r \in R\}$.

It can be shown that $Z(R)$ is a subring of R[Jabbar, 2006].

v- An additive subgroup U of R is said to be a Lie ideal of R if $[U,R] \subseteq U$ [Kaya, Golbasi, and Aydin 2001, Haetinger, 2002, Ali and Kumar, 2007].

vi- R is called a centrally prime ring if R_S is a prime ring for each multiplicative system S in R with $[S,R]=\{0\}$ [Jabbar, 2006].

vii- A derivation $D:R \to R$ is called a centrally-zero derivation on R if $D(S)=\{0\}$ for each multiplicative system S in R with $[S,R]=\{0\}$ [Jabbar, 2006].

viii- :By an endomorphism of R we mean a homomorphism of R into itself, and by an automorphism of R we mean an isomomorphism of R onto itself.

ix- If $D:R \to R$ is a mapping then by D^2 we mean $D \circ D$.In general, if n is any positive integer, then D^n will mean $D \circ D \circ ... \circ D$ (n times), and finally if $x \in R$ then by xD we mean the mapping $xD:R \to R$ which is defined by $(xD)(r)=x(D(r))$, for all $r \in R$.

We will try to extend the following results on 2-torsion free prime rings to 2-torsion free centrally prime rings, the proofs of which can be found in [Ashraf, 2005].

Theorem

Let R be a 2-torsion free prime ring and U a Lie ideal of R such that $U \not\subset Z(R)$. If $a,b \in R$ such that $aUb=\{0\}$, then $a=0$ or $b=0$.

Theorem

Let R be a 2-torsion free prime ring and U a Lie ideal of R. If R admits

a derivation D such that $D(u^n)=0$, for all $u \in U$, where $n \geq 1$ is a fixed integer, then $D(u)=0$, for all $u \in U$.

Theorem

Let R be a 2-torsion free prime ring and U be a Lie ideal of R such that $u^2 \in U$, for all $u \in U$. Suppose that θ is an automorphism of R. If $D:R \to R$ is an additive mapping satisfying $D(u^2)=2\theta(u)D(u)$, for all $u \in U$, then:

1. $D(U)=\{0\}$ or $U \subseteq Z(R)$,
2. $D(uv)=\theta(u)D(v)+\theta(v)D(u)$, for all $u,v \in U$.

Definitions

Now it is the time to introduce the following definitions: Let R be a ring, then:

Definition

We call a Lie ideal U of R a centrally closed Lie ideal if $US \subseteq U$ for each multiplicative system S in R with $[S,R]=\{0\}$.(Note that by US we mean the set $US=\{us : u \in U, s \in S\}$).

Definition

We say a mapping $f:R \to R$ satisfies the central identity property (CIP) if f

acts as the identity map on each multiplicative system S in R with $[S,R]=\{0\}$, that is if S is any multiplicative system in R such that $[S,R]=\{0\}$, then $f(s)=s$ for

all $s \in S$.

Examples

If R is any ring then:

(1): Every ideal of R is a centrally closed Lie ideal, since if U is any ideal of R, then clearly $(U,+)$ is a subgroup of $(R,+)$ and $[U,R] \subseteq U$.Thus U is a Lie ideal of R, and if S is any multiplicative system in R with $[S,R]=\{0\}$, then $UR \subseteq U$, so we get

$US \subseteq U$. Hence U is a centrally closed Lie ideal of R.

(2): Clearly the identity map on R satisfies the central identity property.

Throughout this paper all rings under consideration are with non zero center $Z(R)$, and now we begin with the following lemmas which lead to our first result.

Lemma

Let R be a ring and S amultiplicative system with $[S,R]=\{0\}$.

If U is a centrally closed Lie ideal of R, then U_S is a Lie ideal of R_S.

Proof:

$S \neq \phi$ implies that there exists

$s \in S$. Since $0 \in U$ so $0_S \in U_S$ and thus $\phi \neq U_S \subseteq R_S$. If $u_s, v_t \in U_S$, where $u, v \in U$ and $s,t \in S$. Then we have

$u_s - v_t = u_s + (-v)_t = (ut + (-vs))_{st} =$

$(ut - vs)_{st} \in U_S$

(Since $ut, vs \in U$ so $ut - vs \in U$ and $s,t \in S$ implies $st \in S$). Hence U_S is a subgroup of R_S.

To show $[U_S, R_S] \subseteq U_S$. Let $u_s \in U_S, r_t \in R_S$, where $u \in U, r \in R, s,t \in S$. Then $[u_s, r_t] = u_s r_t - r_t u_s =$

$(ur - ru)_{st} = ([u,r])_{st} \in U_S$ (Since $[U,R] \subseteq U$ and $st \in S$). Hence $[U_S, R_S] \subseteq U_S$, and thus U_S is a Lie ideal of R_S.

Lemma

R is an n-torsion free ring and S is a multiplicative system in R such that $[S,R]=\{0\}$, then R_S is also an n-torsion free ring.

Proof:

Let $a_S \in R_S$, where $a \in R$ and

$s \in S$, be such that $na_s = 0$. Then

$a_S + a_S + ... + a_S = 0$ (n times) or $(a + a + ... + a)_S = 0$. Hence there exists $t \in S$ such that $t(a + a + ... + a) = 0$, that is $ta + ta + ... + ta = 0$ (n times) and so $nta = 0$, and since R is an n-torsion free we get $ta = 0$, and then

$a_s = t_t a_s = (ta)_{ts} = 0_{ts} = 0$. Hence R_S is an n-torsion free.

Lemma

If R is a ring in which $Z(R)$ has no proper zero divisors then

$S = Z(R) - \{0\}$ is a multiplicative system in R with $[S,R]=\{0\}$.

Proof:

Since $Z(R)$ is non zero so there exists an $0 \neq s \in Z(R)$. Hence $S = Z(R) - \{0\} \neq \phi$, and clearly $0 \notin S$. Now if $a,b \in S$, then $0 \neq a \in Z(R)$ and $0 \neq b \in Z(R)$, and since $Z(R)$ has no proper zero divisors so $ab \neq 0$ and since $Z(R)$ is a subring so $0 \neq ab \in Z(R)$, that is $ab \in Z(R) - \{0\} = S$, which establishes that $S = Z(R) - \{0\}$ is a multiplicative system in R, and finally it is easy to cheek that $[S,R]=\{0\}$.

Now we are able to give the first property of centrally closed Lie ideals in 2-torsion free centrally prime rings.

Theorem

Let R be a 2-torsion free centrally prime ring in which Z(R) has no proper zero divisors and U is a centrally closed Lie ideal of R such that $U \not\subset Z(R)$. If $a,b \in R$ are such that $aUb = \{0\}$, then $a = 0$ or $b = 0$.

Proof:

By Lemma 3, we have $S = Z(R) - \{0\}$ is a multiplicative system in R and $[S,R]=\{0\}$. Also by Lemma 2, we get R_S is a 2-torsion free prime ring, and from Lemma 1, we get U_S is a Lie ideal of R_S. Fixing an $s \in S$, we get $a_S, b_S \in R_S$. Then for all $u_t \in U_S$, where $u \in U, t \in S$, we have $a_S u_t b_S = (aub)_{sts} = 0_{sts} = 0$, so that $a_S U_S b_t = \{0\}$. The next step is to show that $U_S \not\subset Z(R_S)$. Since $U \not\subset Z(R)$, so there exists $x \in U$ and $x \notin Z(R)$. Now if $U_S \subseteq Z(R_S)$, then $x_s \in U_S \subseteq Z(R_S)$. Then for all $r \in R$, we have $([x,r])_{SS} = [x_S, r_S] = 0$, which means that there exists $v \in S$ such that $v[x,r] = 0$.

Since $0 \neq v \in S \subseteq Z(R)$ and $Z(R)$ has no proper zero divisors we get $[x,r]=0$, that means $x \in Z(R)$, which is a contradiction and hence $U_S \not\subset Z(R_S)$.Hence R_S is a 2-torsion free prime ring and U_S is a Lie ideal of R_S with $U_S \not\subset Z(R_S)$. Also $a_s, b_s \in R_S$ such that $a_s U_S b_s = \{0\}$. Hence we get $a_s = 0$ or $b_s = 0$ (see Theorem 1). If $a_s = 0$, then there exists $l \in S$ such that $la = 0$. If $a \neq 0$

then l is a proper zero divisor of $Z(R)$, which is a contradiction, and hence $a = 0$, and if $b_s = 0$, by the same argument we can get $b = 0$.

Before we give the second property of centrally closed Lie ideals we mention to the following lemma.

Lemma

Let R be a ring and S a multiplicative system in R such that $[S,R]=\{0\}$.If $D: R \to R$ is a centrally zero derivation on R, then $D_*: R_S \to R_S$, defined by $D_*(r_s) = (D(r))_s$, for all $r_s \in R_S$, is a derivation on R_S.(is called the induced derivation by D).

Theorem

Let R be a 2-torsion free centrally prime ring in which $Z(R)$ has no proper zero divisors and U is a centrally closed Lie ideal of R. If R admits a centrally zero derivation $D: R \to R$ such that $D(u^n) = 0$, for all $u \in U$, where n ≥ 1 is a fixed integer, then $D(u) = 0$, for all $u \in U$.

Proof:

By Lemma, we get $S = Z(R) - \{0\}$ is a multiplicative system in R with $[S,R] = \{0\}$, and by Lemma 2, R_S is a 2-torsion free prime ring. Also by Lemma 1, we get U_S is a Lie ideal of R_S. If $u_s \in U_S$ is any element, where $u \in U, s \in S$, then we have $D_*((u_s)^n) = D_*(u^n_{s^n}) = (D(u^n)_{s^n} = 0_{s^n} = 0$,where $D_*: R_S \to R_S$ is the derivation of Lemma 5.Hence R_S is a 2-torsion free prime ring, U_S is a Lie ideal of R_S such that D_* is a derivation of R_S with $D_*((u_s)^n) = 0$ for all $u_s \in U_S$. Thus by Theorem 2, we get $D_*(u_s) = 0$, for all $u_s \in U_S$.Now fixing $t \in S$.If $x \in U$ is any element then $x_t \in U_S$, and hence $(D(x))_t = D_*(x_t) = 0$, and thus there exists $k \in S$ such that $kD(x) = 0$. Since $Z(R)$ contains no proper zero divisors and $0 \neq k \in S = Z(R) - \{0\}$, we get $D(x) = 0$, and this result is true for all $x \in U$.Thus $D(u) = 0$, for all $u \in U$.

Before we prove the next theorem we need to prove the following lemma.

Lemma

Let R be a ring and S is a multiplicative system in R with $[S,R] = \{0\}$.If $\theta: R \to R$ is an endomorphism of R which acts as the identity map on S, then $\theta': R_S \to R_S$, defined by $\theta'(r_s) = (\theta(r))_s$, for all $r_s \in R_S$, is an endomorphism of R_S, and furthermore, if θ is an automorphism of R, then θ is an automorphism of R_S.

Proof:

Let $a_s = b_t \in R_S$, where $a, b \in R$ and $s, t \in S$. Then $(a,s) \sim (b,t)$.

Thus there exists $k \in S$ such that $k(at - bs) = 0$. So $kat = kbs$, and then $\theta(kat) = \theta(kbs)$.

Hence we get $\theta(k)\theta(a)\theta(t) = \theta(k)\theta(b)\theta(s)$, which gives $k\theta(a)t = k\theta(b)s$ or $k(\theta(a)t - \theta(b)s) = 0$, and so that $(\theta(a))_s = (\theta(b))_t$, that is $\theta'(a_s) = \theta'(b_t)$.

Hence θ' is well-defined.Now let $a_s, b_t \in R_S$, where $a, b \in R$ and $s, t \in S$.Then

$\theta'(a_s + b_t) = \theta'((at + bs)_{st}) =$

$(\theta(at + bs))_{st} = (\theta(at) + \theta(bs))_{st} =$

$(\theta(a)t + \theta(b)s)_{st} = (\theta(a)t)_{st} + (\theta(b)s)_{ts}$

$= (\theta(a))_s t_t + (\theta(b))_t s_s =$

$(\theta(a))_s + (\theta(b))_t = \theta'(a_s) + \theta'(b_t)$.

Also we have

$\theta'(a_s b_t) = \theta'((ab)_{st}) = (\theta(ab))_{st} =$
$(\theta(a)\theta(b))_{st} = (\theta(a))_s(\theta(b))_t =$
$\theta'(a_s)\theta'(b_t)$. Hence θ' is an endomorphism.

It remains to show that if θ is bijective then θ' is also bijective.

Let $a_s \in \mathrm{Ker}\theta'$. Then $\theta'(a_s) = 0$. So that $(\theta(a))_s = 0$. Hence there exists $t \in S$ such that $t\theta(a) = 0$. Then $\theta(t)\theta(a) = 0$. Therefore $\theta(ta) = 0$, so that $ta \in \mathrm{Ker}\theta$, and since θ is one to one, so we get $ta = 0$.Then

$a_s = t_t a_s = (ta)_{ts} = 0_{ts} = 0$.

So $\mathrm{Ker}\theta' = \{0\}$, and hence θ' is one to one.If $y_u \in R_S$, for $y \in R$ and $u \in S$. Then there exists $x \in R$ such that $\theta(x) = y$. Then $x_u \in R_S$ and $\theta'(x_u) = (\theta(x))_u = y_u$. So that θ' is onto.Hence θ' is a bijective mapping and hence it is an automorphism of R_S.

Remark:

We call θ' in Lemma 7, the induced endomorphism (resp. the induced automorphism) of R_S.

Now we are able to prove the next theorem.

Theorem

Let R be a 2-torsion free centrally prime ring in which Z(R) has no proper zero divisors and U is a centrally closed Lie ideal of R such that $u^2 \in U$, for all $u \in U$.If θ is an automorphism of R which satisfies (CIP) and $D: R \to R$ is an endomorphism which satisfies (CIP) and $D(u^2) = 2\theta(u)D(u)$, for all $u \in U$, then either $D(U) = \{0\}$ or $U \subseteq Z(R)$.

Proof:

By Lemma 3, we have $S = Z(R) - \{0\}$ is a multiplicative system in R with $[S, R] = \{0\}$, and by Lemma 2, we get R_S is a 2-torsion free prime ring, and by Lemma 1, we have U_S is a Lie ideal of R_S. Since $u^2 \in U$, for all $u \in U$, so we can show $(u_s)^2 \in U_S$, for all $u_s \in U_S$. Define $D': R_S \to R_S$, by $D'(r_s) = (D(r))_s$ for all $r_s \in R_S$.Let $a_s, b_t \in R_S$, where $a, b \in R$ and $s, t \in S$.If $a_s = b_t$, then there exists $k \in S$ such that $k(at - bs) = 0$ or $kat = kbs$.

Then we have

$D'(a_s) = D'(k_k a_s t_t) = D'((kat)_{kst}) =$
$D'((kbs)_{kts}) = D'(k_k b_t s_s) = D'(b_t)$

Hence D' is well defined.Also we have
$D'(a_s + b_t) = D'((at + bs)_{st}) =$
$(D(at + bs))_{st} = (D(at) + D(bs))_{st} =$
$(D(a)D(t) + D(b)D(s))_{st} = (D(a)t + D(b)s)_{st}$
$= (D(a)t)_{st} + (D(b)s)_{ts} =$
$(D(a))_s t_t + (D(b))_t s_s = (D(a))_s + (D(b))_t =$
$D'(a_s) + D'(b_t)$.So D' is an additive mapping.If $u_s \in U_S$ is any element, where $u \in U$ and $s \in S$, then we have

$D'((u_s)^2) = D'((u^2)_{s^2}) = (D(u^2))_{s^2} =$
$(2\theta(u)D(u))_{ss} =$
$(\theta(u)D(u) + \theta(u)D(u))_{ss} = (\theta(u)D(u))_{ss}$
$+ (\theta(u)D(u))_{ss} =$
$(\theta(u))_s(D(u))_s + (\theta(u))_s(D(u))_s =$
$\theta'(u_s)D'(u_s) + \theta'(u_s)D'(u_s) =$
$2\theta'(u_s)D'(u_s)$, where θ' is the induced automorphism of Lemma 7.

Thus R_S is a 2-torsion free prime ring, U_S is a Lie ideal of R_S with $(u_s)^2 \in U_S$, for all $u_s \in U_S$.Also θ' is an automorphism of R_S and D' is an additive mapping of R_S such that $D'((u_s)^2) = 2\theta'(u_s)D'(u_s)$, for all $u_s \in U_S$.Hence $D'(U_S) = \{0\}$ or $U_S \subseteq Z(R_S)$ (see Theorem 3).

If $D'(U_S) = \{0\}$, then since Z(R) has no proper zero divisors we can show $D(U) = \{0\}$, and if $U_S \subseteq Z(R_S)$, we can show $U \subseteq Z(R)$.

Finally we give the last property of the centrally closed Lie ideals.

Theorem

Let R be a 2-torsion free centrally prime ring in which Z(R) has no proper zero divisors and U is a centrally closed Lie ideal of R such that $u^2 \in U$, for all $u \in U$.If θ is an automorphism and D is an endomorphism of R both satisfy (CIP) and $D(u^2)=2\theta(u)D(u)$, for all $u\in U$, then $D(uv)=\theta(u)D(v)+\theta(v)D(u)$, for all $u,v\in U$.

Proof:

By Lemma 3, we have $S=Z(R)-\{0\}$ is a multiplicative

system in R with $[S,R]=\{0\}$. Let θ' and D' be the same as in Theorem 8.

Then exactly as the same technique as in Theorem 8, we can show that R_S is

a 2-torsion free prime ring, U_S is a Lie ideal of R_S such that $(a_s)^2\in U_S$, for all $a_s\in U_S$ and θ' is an automorphism of R_S, and D' is an additive mapping such that $D'(a_s)^2=2\theta'(a_s)D'(a_s)$, for all $u_s\in U_S$. Hence by Theorem 3, we get

$$D'(a_s b_t)=\theta'(a_s)D'(b_t)+\theta'(b_t)D'(a_s),$$

for all $a_s,b_t\in U_S$.Now let $u,v\in U$ be any elements. Fix an s∈S, so that $u_s,v_t\in U_S$.Then

$$(D(uv))_{st}=D'((uv)_{st})=D'(u_s v_t)=$$
$$\theta'(u_s)D'(v_t)+\theta'(v_t)D'(u_s)=$$
$$(\theta(u))_s(D(v))_t+(\theta(v))_t(D(u))_s=$$
$$(\theta(u)D(v)+\theta(v)D(u))_{st}.$$

Hence there exists $k\in S$ such that $k(D(uv)-(\theta(u)D(v)+\theta(v)D(u)))=0$, and since $Z(R)$ contains no proper zero divisors we can show that

$D(uv)-(\theta(u)D(v)+\theta(v)D(u))=0$, that is $D(uv)=\theta(u)D(v)+\theta(v)D(u)$.

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الخلاصة

فى هذا البحث قدمت تعريفى مثاليات لي المغلقة مركزيا و خاصية التحايد المركزي و تمت دراسة سلوك هذا النوع من المثاليات في الحلقات الاولية مركزيا ذات الالتواء الثنائي الحر وكذلك تمت دراسة تأثيرات الاشتقاق عليها و بذلك تمكننا من الحصول على خواص عديدة لهذه المثاليات.