Signal Denoising Using Double Density Discrete Wavelet Transform

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Abstract

Reality signals do not exist without noise. Wavelet transform based denoising seem to be a powerful tool for suppressing noise in signals. In this paper, we investigate the using of double density discrete wavelet transform “DD-DWT” which based on one scaling function and two wavelet functions, for signal denoising and comparing its performance with the traditional DWT. Three groups of additive White Gaussian Noise levels (5 dB, 3 dB, 2 dB) are added to some standard test signals with both hard and soft threshold function to evaluate the performance of each method in term of Root Mean Square Error (RMSE) and Signal to Noise Ratio (SNR). Experiment results show that DD-DWT performs better than traditional DWT in both RMSE and SNR especially at low SNR. [DOI: 10.22401/JNUS.20.4.19]

Keywords: DWT, DD-DWT, Signal, Denoising.

1. Introduction

In the last two decades, the Discrete Wavelet Transform (DWT) has been founded to be a valuable appliance for analyzing the signal in different fields like engineering, medicine, science, economics and also mathematics, and biology. [11]

Denoising had already been a traditional problem in image processing and signals. Images and signals had usually corrupted by noise throughout transmission, capturing, and storage. A variety of denoising techniques have been proposed so far, among this, wavelet based denoising give a superior performance, due to its properties such as scarcity, energy compaction, and multiresolution structure. Wavelet transform tends to separate the signal into two components: one defining the approximation of the signal” low-frequency subband" and the other defining details “high-frequency subband”. Generally, most of noise components concentrate in the high frequency subband; while the main information components concentrate in the low frequency subband. [11]

Donoho and his coworkers pioneered applying a wavelet-denoising scheme, which is also known as “wavelet thresholding” [1, 2]. Their basic idea is to forcing to zero the coefficients with amplitude lower than the selected threshold value, and preserving or shrinking any coefficient, with amplitude above this threshold, then reconstructed the original based on these thresholding coefficients. [1, 2]

Based on this technique several researchers have explored the use of wavelets for different signals de-noising, such as speech signal [9], electrocardiogram (ECG) [10, 4], encephalogram [5], digital communications signals [6], sonar signals [3] etc.

In 2001 Selesnick et al. proposed a new discrete wavelet transform depend on oversampled filter bank using maximally flat FIR filters. This system, which is analogous to Daubechies orthogonal wavelet, is satisfying certain polynomial property but with oversampled case is called the double density DWT. [7]

This new approach shows a good performance when used for image denoising as in [12] that it is used to denoise standard images and in [3] where it is used to denoise SAR images.

The DD-DWT has specific extra characteristics in comparison with traditional DWT; first, it utilizes single scaling function and two wavelets, which are intended to be offset from one another; second it is over complete by two; third it is almost time-invariant. In additional the DD-DWT overcome the complexity of the Dual-Tree Wavelet Transform (DT-DTW). [12]

In this paper, we will try to use the DD-DWT in denoising 1-D signal and investigate its performance with respect to the traditional discrete wavelet transform (DWT).

The rest of the paper is organized as; Section ٢presents the concept of wavelet thresholding denoising. In section 3 DD-DWT is introduced in details. In section 4 the proposed method and experiment results are present. Finally, conclusion and discussion are given in Section 5.

2. Wavelet Thresholding Denoising:

Consider a noisy signal with the form of

$$y = x + n \quad \text{......................................} (1)$$

Where $y, x,$ and n represent the noisy signal, standard signal and white Gaussian noise with zero mean and variance σ^2 respectively. In this case, the noise is assume to be independent similarly distributed (i, i, d) [1, 2].

The denoising objective is to remove the noise, or "de-noise" y, and to obtain a predicted $\hat{x}$ of x, with minimum mean square error If W and W^{-1} be the two-dimensional orthogonal discrete wavelet transforms [DWT] and its inverse [IDWT] matrices correspondingly. Then, the wavelet transform w was applied to (1) we had

$$Y = X + N \quad \text{................................} (2)$$

Where $Y = Wy$,$X = Wx$, and $N = Wn$

Since " W" is orthogonal transform; "N" is also a (i, i. d) Gaussian random variable. [1,2]

If we consider D (.) as the threshold function. The predicted $(\widehat{x})$ of the main signal (x) after thresholding is achieved as follows

$$\hat{x} = W^{-1}(D(Wy)) \quad \text{............................} (3)$$

Here the predicted $(\widehat{x})$ is obtained from the corrupted signal (y) by using wavelet transform at the beginning; then applied thresholding and at the end, we reversed wavelet transform to earn the clear signal. [1, 2]

There are two familiar thresholding functions, named "soft thresholding function" and "hard thresholding function". Their essential idea is to delete the small-scale wavelet coefficients in the details subband only, while keeping the low-resolution (approximate) coefficients unaltered. These two threshold functions are defined as [1, 2].

i. Hard thresholding function: which zeros every coefficient smaller than the threshold value, while keeping values larger than the threshold.

$$D(Y,T) = \begin{cases} Y & |Y| > T \\ 0 & otherwise \end{cases} \quad \text{.........} (4)$$

ii. Soft thresholding function: which zeros every coefficient smaller than the threshold value, while shrunk coefficient bigger than the threshold value by T.

$$D(Y,T) = sgn(Y) max(0, |Y| - T) \text{ ....} (5)$$

Where $sgn(\ )$ is the sign of the variable, and max( ) is the maximum value between 0 and the second value.

The transfer function of both functions is shown in Fig.(1). The Hard thresholding is "the simplest method but soft thresholding has nice mathematical charachtrestices and provide better denoising performance in many applications".

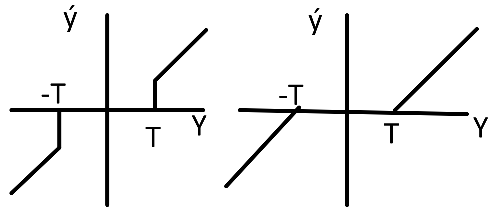

Fig.(1): Hard and Soft Thresholding transfer function.

3. Double Density DWT

The double density DWT is the kind of wavelet transform in which, it had single scaling function and two wavelets; these wavelets are shown to be smoother than that in traditional DWT. Having more wavelets than essential give tight space between neighboring wavelets within the similar scale, which led to better signal representation in wavelet domain. [7].

To assemble a double density (DWT) with filters; the filter bank with oversampled of has been used as in Fig.(2). The filter g_0 (n) will be a low-pass (scaling) filter; while both of g_1 (n) and g_2 (n) will be high-pass (wavelet) filters.

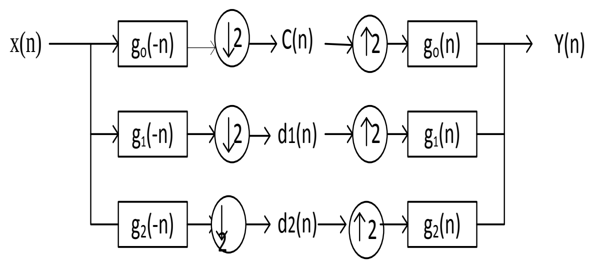

Fig.(2) :The decompassion and reconstruction oversampled FIR filter banks.

The filter coefficients used are shown in Table (1), which has been proven to assure the perfect reconstruction rule [7].

$$\mathbf{G0(z)G0(1/z) + G1(z)G1(1/z) + G2(z)G2(1/z) = 2} ..(6)$$

$$\mathbf{G0(z)G0(-1/z) + G1(z)G1(-1/z) + G2(z)G2(-1/z) = 0} .. (7)$$

Table (1)
The FIR filter coefficients of a double density discrete wavelet transform.

N	af 1	af 2	af 3
0	0.14301535070442	-0.01850334430500	-0.04603639605741
1	0.51743439976158	-0.06694572860103	-0.16656124565526
2	0.63958409200212	-0.07389654873135	0.00312998080994
3	0.24429938448107	0.00042268944277	0.67756935957555
4	-0.07549266151999	0.58114390323763	-0.46810169867282
5	-0.05462700305610	-0.42222097104302	0

4. Proposed Method and Experiment Results

I. Algorithm:

The algorithm of the proposed signal denoising consist the following steps:

a) Applied forward wavelet transform to the noisy signals using both traditional DWT and DD-DWT to get the decomposed signal.
b) Calculate the threshold value from detail coefficients at level 1.
c) Apply both of soft and hard threshold function.
d) Inverse the wavelet transform (IWT) for the thresholded value to obtain the predicted signal.

These steps are summarized in the block diagram is shown in figure below

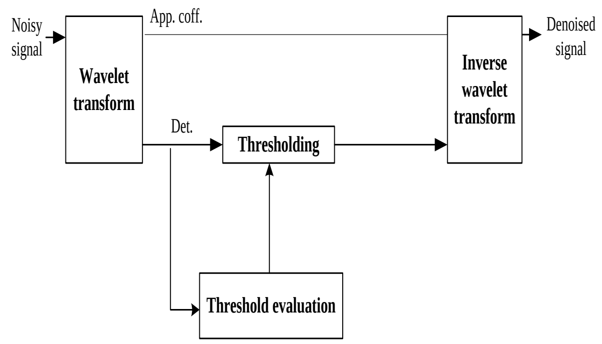

Fig.(3): Proposed block diagram.

There are many different methods that can be used to calculate the threshold value. Universal threshold estimation method being the most widely used due to its capability and simplicity. It can be calculated as:

$$\lambda = \sigma\sqrt{2\ ln\ (N)} \quad .... (8)$$

Where N is the signal length and σ is the noise variance, σ can be calculated as : [1,2]

$$\sigma_n = \frac{median(H_1)}{0.6745} \quad .... (9)$$

Where H_1 represents the details (high frequency) sub-band for first level wavelet decomposition of the signal.

After calculating the threshold value both hard and soft threshold had been applied on details coefficients at each level as mentioned in the beginning of the section.

To validate the proposed de-noising approach four well-known standard signals, namely, Blocks, Bumps, Heavy Sine, and Doppler, have been used as test signals.

The length of these signals is (1024) samples. These signals are corrupted by additive white Gaussian noise with various levels of noise each signal is decomposed using both DD-DWT and DWT with different wavelets families to three level which is shown to be the optimum in many researches.

In order to evaluate the performance of the proposed denoising algorithm, the following parameters have been used:

a) Signal to Noise Ratio (SNR) which can be calculated as:

$$SNR = 10log\left[\frac{\sum_{i=1}^{n} s^2(k)}{\sum_{i=1}^{n}[s(k) - s\hat{}(k)]2}\right] \quad .... (10)$$

Where s (k) is the clear signal and s˘ (k) is the signal to be denoised.

b)Root Mean Square Error (RMSE) represented the likeness between the original signal and recovered signal which is described as:

$$RMSE = \frac{1}{n}\sqrt{\sum_{i=1}^{n}[x(k) \; - x\hat{}(k)]\,2} \;\; .... (11)$$

II.Experiment Results

Table (2) summarize the performance of both the denoising algorithm using DD-DWT and traditional DWT with three family type (db1, db3, and db4). These results show that the DD-DWT based outperform the traditional wavelet transform for the three family type, also one can find that although the results are a little close at high SNR, but with low SNR the different between the result is clear. That is because DWT has one high detail coefficient but DD-DWT has two high detail coefficient, so the thresholding is applied to both high frequency filter output ،which gives an advantage over the traditional DWT.

Subjectively this can also be shown in Figs. (4-7), where Fig.(4, 5) present the original test signal and noisy signals at level 3; while Fig.(6-7) contain the denoising version for these signals by using DWT, and DD-DWT. Analyzing Fig.(6) with Fig.(7); denoised signal resulted by the proposed algorithm is closer to the clear signal than traditional.

DWT, despite whether hard or soft threshold function is used.

Table (2)

MSE and SNR of the noisy and denoised signal at different noise level using DD-DWT and DWT with different wavelet families.

			Noisy signals with SNR of 5 dB				Noisy signals with SNR of 3 dB				Noisy signals with SNR of 1 dB			
			Blocks	Bumps	Heavsin	Doppler	Blocks	Bumps	Heavsin	Doppler	Blocks	Bumps	Heavsin	Doppler
		SNR	5.143	5.143	5.143	5.143	3.143	3.143	3.143	3.143	2.143	2.143	2.143	2.143
		RMSE	2.699	0.991	2.913	0.026	4.278	1.570	4.617	0.041	5.385	1.977	5.813	0.052
DWT denoising using db1	H. T*	SNR	10.819	10.955	10.718	10.981	6.278	9.406	4.617	9.828	4.430	8.392	4.451	9.170
		RMSE	0.730	0.259	0.807	0.006	2.109	0.371	2.362	0.008	3.180	0.468	3.417	0,010
	S. T**	SNR	11.978	11.642	13.476	10.981	10.284	9.886	11.154	9.828	9.205	9.208	9.803	9.170
		RMSE	0.559	0.254	0.427	0.006	0.826	0.332	0.730	0.008	1.059	0.388	0.996	0.010
DWT denoising using db3	H. T	SNR	9.661	12.641	10.115	12.130	9.661	12.641	10.115	12.130	4.619	8.444	4.569	10.023
		RMSE	0.953	0.176	0.927	0.005	0.953	0.176	0.927	0.005	3.044	0.463	3.325	0.008
	S. T	SNR	12.572	12.934	13.815	12.130	10.658	11.334	11.292	10.772	9.487	10.448	9.880	10.023
		RMSE	0.487	0.164	0.395	0.005	0.758	0.238	0.707	0.007	0.992	0.292	0.978	0.008
DWT denoising using db4	H. T	SNR	10.218	12.669	10.844	12.268	10.218	12.669	10.844	12.268	4.822	8.838	4.831	9.910
		RMSE	0.838	0.175	0.784	0.005	0.838	0.175	0.784	0.005	2.906	0.423	3.130	0.008
	S. T	SNR	12.185	12.669	13.439	12.268	10.394	11.004	11.081	10.734	9.300	10.102	9.758	9.910
		RMSE	0.533	0.175	0.431	0.005	0.805	0.257	0.742	0.007	1.036	0.316	1.006	0.008
DDDWT denoising	H. T	SNR	12.899	12.959	14.348	13.083	10.241	11.688	10.907	11.825	8.462	10.979	8.435	11.118
		RMSE	0.452	0.163	0.349	0.004	0.834	0.219	0.772	0.005	1.257	0.258	1.365	0.006
	S. T	SNR	13.152	12.959	15.260	13.083	11.824	11.688	13.223	11.825	11.023	10.979	12.104	11.118
		RMSE	0.426	0.163	0.283	0.004	0.579	0.219	0.453	0.005	0.696	0.258	0.586	0.006

**** mean thresholding with hard threshold function***
***** mean thresholding with soft threshold function***

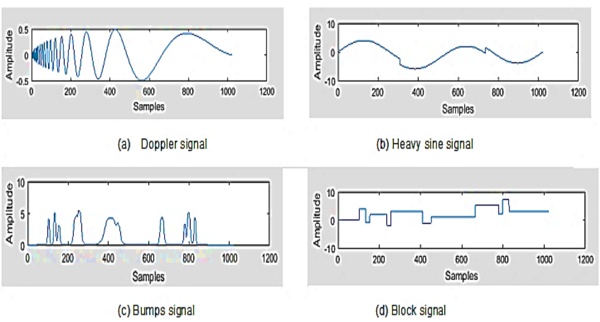

Fig.(4) :Test benchmark signals.

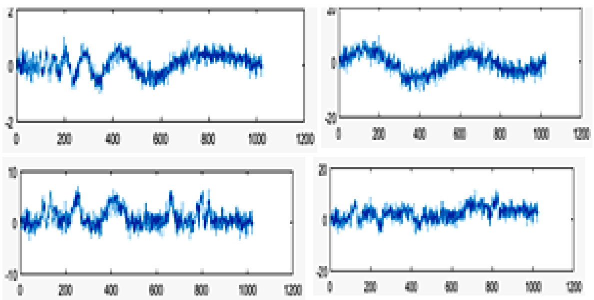

Fig.(5) :The Noisy test signals.

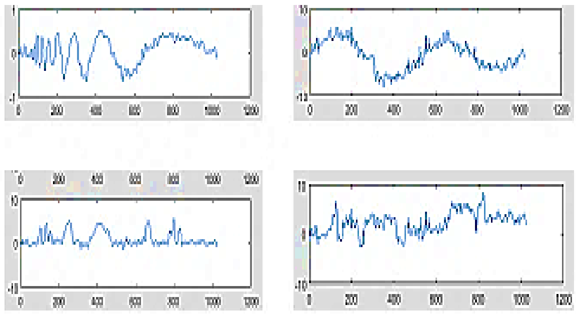

Fig.(6) :Denoised signals by DWT.

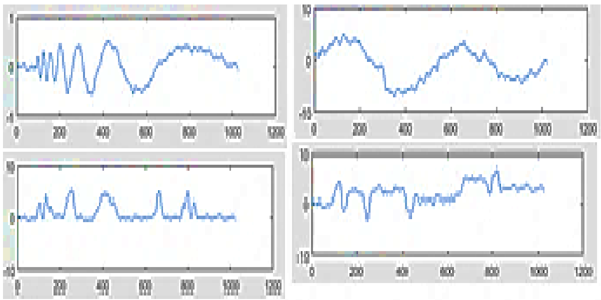

Fig.(7) :Denoised signals by DDDWT.

5. Conclusions

In this paper, we improve the performance of wavelet based denoising by using Double density DWT instead of the tradition DWT this improvement is a result of DD-DWT properties that has two wavelet function which gives a closer spacing between neighbor wavelets with the same scale in addition to its approximate shift invariant.

The experiment results show that our method is more effective than traditional wavelet denoising in term of both SNR and RMSE particularly with low SNR, so this method is recommended in many real applications especially its complexity is not far more than the traditional DWT.

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