Journal of Al-Nahrain University Vol.13 (3), September, 2010, pp.161-169 Science

THE SEMI–CLASSICAL SOLUTION FOR A CLASS OF SEMILINEAR INITIAL VALUE CONTROL PROBLEMS IN BANACH SPACE

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Abstract

In this paper, the local existence and uniqueness of S-classical solution of a class of semilinear initial value control problems which defined as:

$$\frac{d}{dt}\left[z_w(t)+g(t,z_w(t))\right]=Az_w(t)+f(t,z_w(t))+Bw(t),\ t>0$$

$$z_w(0)=z_0$$

have been discussed and proved in suitable Banach spaces. All results are obtained by using Banach contraction principle and depending on the theory of analytic semigroup of control problems.

Keywords: S-classical solution, control problem in infinite dimensional spaces, fixed point theorem and analytic semigroup theory.

Introduction

Byszewski in 1991 [1], has study the local existence and uniqueness of the mild solution to the semilinear initial value problem:

$$\frac{dz}{dt}+Az(t)=f(t,z(t)) \quad \text{..............................(1)}$$
$$z(0)=z_0$$

Where A is the infinitesimal generator of a C_0 semigroup (strongly continuous semigroup) defined from $D(A)\subset X$ into X (X is suitable Banach space) and f is a nonlinear continuous map define from $[0,r)\times X$ into X. Eduardo [2] in 2001, has study the local existence and uniqueness of the of S-classical solution to the problem defined in (1).

Definition 1.1 [2]:

A function $z\in C([0,r):X)$ is said to be an S-classical solution to the semilinear initial value problem defined in (1), if $z(t)$ has the following form:

$$z(t)=T(t)z_0+\int_0^t T(t-s)f(s,z(s))ds \quad \text{................(2)}$$

satisfies the following conditions: $z(0)=z_0$, $\frac{d}{dt}z(t)$ is continuous on (0, r), $z(t)\in D(A)$ for all $t\in(0,r)$ and z(.) satisfies equation (1) on (0, r). In the present paper, the following semilinear initial value control problem (abstract Cauchy control problem) has been concerned:

$$\frac{d}{dt}\left[z_w(t)+g(t,z_w(t))\right]=Az_w(t)+f(t,z_w(t))+Bw(t),\ t>0$$
$$z_w(0)=z_0$$
$$\text{..............................(3)}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ on a suitable Banach space X; $f,g:(0,r]\times X_\alpha\to X_\alpha$ are nonlinear continuous maps, where X_α is a Banach space being dense in X and B is a bounded linear operator define from O into X, where O is a Banach space and $w(.)$ be the arbitrary control functions with $\|w(t)\|_O\leq K_1$, for $0\leq t<r$. Throughout this paper X will be a Banach space equipped with the norm $\|.\|$ and the operator $A:D(A)\subset X\to X$ will be the infinitesimal generator of an analytic semigroup of bounded linear operators $\{T(t)\}_{t\geq 0}$ on X. For the theory of analytic semigroup, refer to Pazy [3], Jerome [4] and Balachandran [5]. The books of Pazy [3], Krien [6] and Fitzgibbon [7] contained therein, give a good account of important results. We mention here only some notation and properties essential to our purpose, In particular, we assume that $\{T(t)\}_{t\geq 0}$ is an analytic semigroup generated by infinitesimal generator A and $0\in\rho(A)$, ($\rho(A)$ stands for

resolvent set).In this case it is possible to define the fractional power $(-A)^{\alpha}$, for 0<α<1, as a closed linear operator with domain $D((-A)^{\alpha})$ is dense in X and the expression $\|x\|_{\alpha}=\|(-A)^{\alpha}x\|_X$, defines a norm on $D((-A)^{\alpha})$. Hereafter we represent by X_{α} the space $D((-A)^{\alpha})$ endowed with the norm $\|.\|_{\alpha}$. The S-classical solution of the semilinear initial value control problem defined in (3) will be developed by the following definition.

Definition 1.2:

A function $z_w \in C([0,r):X)$ is said to be S-classical solution to the semilinear initial value problem defined in (3), if $z_w(t)$ has the following form:

$$z_w(t)=T(t)(z_0+g(0,z_0))-g(t,z_w(t))$$
$$-\int_0^t AT(t-s)g(s,z_w(s))\,ds+\int_0^t T(t-s)\left[f(s,z_w(s))+Bw(s)\right]ds$$

satisfies the following conditions:

1. $z_w(0)=z_0$,
2. $\frac{d}{dt}\left[z_w(t)+g(t,z_w(t))\right]$ is continuous on (0, r)
3. $z_w(t)\in D(A)\ \forall t\in(0,r)$
4. $z_w(.)$ satisfies equation (3) on (0, r),where the continuous function $z_w \in C([0,r):X)$ depend on arbitrary control function $w(.)\in L^p([0,r):O)$.

Preliminaries

Definition 2.1 [8]:

A family $\{T(t)\}_{t\geq 0}$ of bounded linear operators on a Banach space X is called a semigroup on X if it satisfies the following conditions:

$T(t+s)=T(t)T(s), \forall t,s\geq 0$

T(0)=I ,(I is the identity operator on X).

Definition 2.2 [8]:

A family $\{T(t)\}_{t\geq 0}$ is said to be an analytic semigroup if the following conditions are satisfy:

(i) $t\rightarrow T(t)$ is analytic in some sector Δ, where Δ is a sector containing the nonnegative real axis.

(ii) T(0)=I and $\lim_{t\downarrow 0}\|T(t)x-x\|_X=0, \forall x\in X$.

(iii) $T(t+s)=T(t)T(s), \forall t,s\geq 0$.

Definition 2.3 [8]:

A semigroup $\{T(t)\}_{t\geq 0}$ on a Banach space X is called strongly continuous semigroup of bounded linear operators or (C_0 semigroup) if The map $R^+ \ni t \longrightarrow T(t)\in L(X)$, satisfies the following conditions:

(i) $T(t+s)=T(t)T(s), \forall t,s\geq 0$.

(ii) T(0) = I.

(iii) $\lim_{t\downarrow 0}\|T(t)x-x\|=0$, for every x∈X.

Definition 2.4 [3]:

f −A is the infinitesimal generator of bounded analytic semigroup

Then the fractional power $A^{-\alpha}$ exist for α>0.

Definition 2.5 [3]:

Let −A be the infinitesimal generator of an analytic semigroup T(t) if 0 ∈ ρ(A), then:

(a)T(x): X ⟶ $D(A^{\alpha})$, for every t > 0 and α ≥ 0.

(b)For every $x\in D(A^{\alpha})$, we have $T(t)A^{\alpha}x = A^{\alpha}T(t)x$.

(c)For every t ≥ 0, the operator $A^{\alpha}T(t)$ is bounded and $\|A^{\alpha}T(t)\|\leq M_{\alpha}t^{-\alpha}$.

(e)Let 0<α≤1 and $x\in D(A^{\alpha})$ then $\|T(t)x-x\|\leq C_{\alpha}t^{\alpha}\|A^{\alpha}x\|_X$,where C_{α} is the positive constant depend onα.

Theorem 2.6 [9]:

Let M is a closed nonempty set in the Banach space X over• , where • are a scalar field and the operator T: M ⟶ M is strict contraction operator then T has a unique fixed point.

Definition 2.7 [3]:

Let I be an interval, A function f: I ⟶ X, where X is a Banach space is said to be Hölder continuous with exponent ϑ, $0<\vartheta<1$ on I, if there is a constant L such that $\|f(t)-f(s)\|_X\leq L|t-s|^{\vartheta}$, for s, t ∈ I.

Main Result

It should be notice that the local existence and uniqueness of S-classical solution of the semilinear initial value control problem (abstract Cauchy control problem) defined in (3) developed, by assuming the following assumptions:

(A1) $-A$ be the infinitesimal generator of bounded analytic semigroup $\{T(t)\}_{t\geq 0}$ and $0\in\rho(-A)$. Where the operator $-A$ define from $D(-A)\subset X$ into X, where X is a Banach space.

(A2) Let U be an open subset of $[0, r)\times X_\alpha$, for $0<r\leq\infty$, where X_α is a Banach space being dense in X.

(A3) For every $(t,x)\in U$, ther exist a neighborhood $G\subset U$ of (t,x) and there exist L_1, $L_2>0$, $0<\gamma_1<1, 0<\gamma_2<1$ and $\beta\in(0,1)$ such that the nonlinear maps f, g: $[0, r)\times X_\alpha$ into X, satisfy the following conditions:

$$\left\|(-A)^\beta g(t,x_1)-(-A)^\beta g(s,x_2)\right\|\leq L_1\left\{|t-s|^{\gamma_1}+\|x_1-x_2\|_\alpha\right\},$$

$$\|f(t,x_1)-f(s,x_2)\|\leq L_2\left\{|t-s|^{\gamma_2}+\|x_1-x_2\|_\alpha\right\},$$

$\forall(t,x_1)$ and $(s,x_2)\in G$

(A4) For $t''>0$ and $\beta\in(0,1)$, $\|(-A)^\beta g(t,v)\|_X\leq C_1, \|f(t,v)\|_X\leq C_2$, $\forall$ $0\leq t\leq t''$ and $v\in X_\alpha$, where $C_1, C_2>0$.

(A5) For $t'''>0$ and $\delta'>0$, $\left\|(T(t)-I)(-A)^\alpha(x_0+g(0,x_0))\right\|_X\leq\delta'$, $\forall$ $0\leq t\leq t'''$, where $\delta'<\delta$.

(A6) Let $w(.)$ be the arbitrary control function is given in $L^p([0,r):O)$, a Banach space of control functions with O as a Banach space and here B is a bounded linear operator define from O into X and $\|w(t)\|_O\leq K_1$, for $0\leq t<r$.

(A7) Let $t_1\in R^+$, {where R^+ the set of all positive real numbers}, such that $t_1=\min\{t',t'',t'',r\}$ and α, $\beta\in(0, 1)$ with $\alpha\neq\beta$ and η, $\Gamma>0$ satisfy the following conditions:

(A7.i)

$$t_1\leq\left\{\frac{\left(1-\left\|(-A)^{\alpha-\beta}\right\|L_1\right)\delta-\delta'}{\left\|(-A)^{\alpha-\beta}\right\|L_1+\frac{C_{1-\beta+\alpha}C_1}{\beta-\alpha}+\frac{C_\alpha(C_2+k_0K_1)}{1-\alpha}}\right\}^{\frac{1}{\eta}}$$

$$\Rightarrow t_1^\eta\leq\left\{\frac{\left(1-\left\|(-A)^{\alpha-\beta}\right\|L_1\right)\delta-\delta'}{\left\|(-A)^{\alpha-\beta}\right\|L_1+\frac{C_{1-\beta+\alpha}C_1}{\beta-\alpha}+\frac{C_\alpha(C_2+k_0K_1)}{1-\alpha}}\right\}$$

(A7.ii) $$t_1\leq\left\{\frac{1-L_1\left\|(-A)^{\alpha-\beta}\right\|-\frac{\delta'}{\delta}}{\frac{L_1C_{1-\beta+\alpha}}{\beta-\alpha}+\frac{C_\alpha L_2}{1-\alpha}}\right\}^{\frac{1}{\Gamma}}$$

$$\Rightarrow t_1^\Gamma\leq\left\{\frac{1-L_1\left\|(-A)^{\alpha-\beta}\right\|-\frac{\delta'}{\delta}}{\frac{L_1C_{1-\beta+\alpha}}{\beta-\alpha}+\frac{C_\alpha L_2}{1-\alpha}}\right\}$$

(A8) For α, $h\in(0, 1)$

$$\left\|\left(g\left(t,(-A)^{-\alpha}x_w(t)\right)-g\left(t+h,(-A)^{-\alpha}x_w(t+h)\right)\right)(-A)^\alpha\right\|_X\leq Nh^{1-\alpha}$$

where N is a positive constant.

Lemma:

Let $Y=C([0,t_1]:X)$, where Y is a Banach space with the supremum norm: $\|y\|_Y=\sup_{0\leq t\leq t_1}\|y(t)\|_X$ and S_w be the nonempty subset of Y depend on the control function $w(.)\in L^p([0,t_1]:O)$, define as follow:

$$S_w=\left\{y_w\in Y: y_w(0)=(-A)^\alpha y_0,\ \|y_w(t)-(-A)^\alpha y_0\|_X\leq\delta, 0\leq t\leq t_1\right\}.$$

Then S_w is a closed subset of Y for every control function $w(.)\in L^p([0,t_1]:O)$.

Proof:

Let $y_w^n\in S_w$ such that $\lim_{n\to\infty}y_w^n=y_w$, where y_w^n is a sequence of continuous function depend on $w(.)$, we must prove that $y_w\in S_w$. Since $y_w^n\in S_w$ $\Rightarrow$ $y_w^n\in Y$, $y_w^n(0)=(-A)^\alpha y_0$ and $\|y_w^n(t)-(-A)^\alpha y_0\|_X\leq\delta, 0\leq t\leq t_1$.

Since $y_w^n\xrightarrow{U.C}y_w$ as $n\to\infty$, hence $y_w\in Y$, where (U.C) stands for the uniform convergence and also since $y_w^n\xrightarrow{U.C}y_w$ $\Rightarrow$ $\|y_w^n-y_w\|_Y\to 0$, as $n\to\infty$

Notice that: $\|y_w^n-y_w\|_Y=\sup_{0\leq t\leq t_1}\|y_w^n(t)-y_w(t)\|_X\to 0$ as $n\to\infty$, which imply that $\|y_w^n(t)-y_w(t)\|_X\to 0$ as $n\to\infty$, $\forall 0\leq t\leq t_1$, i.e. $\lim_{n\to\infty}y_w^n(t)=y_w(t)$, $\forall 0\leq t\leq t_1$,

$\lim_{n\to\infty}y_w^n(0)=y_w(0)$, we get $\lim_{n\to\infty}(-A)^\alpha y_0=y_w(0)$

$(-A)^\alpha y_0=y_w(0)$. Notice that:

$$\left\|y_w(t)-(-A)^\alpha y_0\right\|_X=\left\|\lim_{n\to\infty}y_w^n(t)-(-A)^\alpha y_0\right\|_X$$

$$=\left\|\lim_{n\to\infty}y_w^n(t)-\lim_{n\to\infty}(-A)^\alpha y_0\right\|_X$$

$= \left\| \lim_{n\to\infty} \left(y_w^n(t) - (-A)^\alpha y_0 \right) \right\|_X$

$= \lim_{n\to\infty} \left\| y_w^n(t) - (-A)^\alpha y_0 \right\| \le \lim_{n\to\infty} \delta$, because $x_w^n \in S_w$,

we get: $\left\| y_w(t) - (-A)^\alpha y_0 \right\|_X \le \lim_{n\to\infty} \delta = \delta$

$\left\| y_w(t) - (-A)^\alpha y_0 \right\|_X \le \delta$, $\forall 0 \le t \le t_1$.

We have got S_w is closed subset of Y for every control function $w(.) \in L^p([0,t_1]:O)$.

Theorem:

Assume the hypotheses (A_1) to (A_8) hold. Then for every $v_0 \in X_\alpha$, there exist a fixed number $t_1, 1 < t_1 < r$, such that the semilinear initial value control problem (abstract Cauchy control problem) defined in (3) has unique S-classical solution $z_w \in C([0,t_1]:X)$, for every control function $w(.) \in L^p([0,t_1]:O)$.

Proof:

Without loss of generality, we may suppose $r < \infty$, because we are concerned here with the local existence only.

For a fixed point $(0, z_0)$ in the open subset U of $[0, r) \times X_\alpha$, we choose $t' > 0$ and $\delta > 0$ such that the neighborhood G of the point $(0, z_0)$ define as follow:

$$G = \left\{ (t,x) \in U : 0 \le t \le t', \left\| (-A)^\alpha z - (-A)^\alpha z_0 \right\|_\alpha \le \delta \right\} \subset U$$

because U is an open subset of $[0, r) \times X_\alpha$. It is clear that $\|(-A)^\alpha T(t)\| \le C_\alpha t^{-\alpha}$, for t>0, see {theorem (1.8.7) in [3]}.Where C_α is a positive constant depending on α.

Define a map $\psi_w : S_w \to Y$, where ψ_w depend on the control function $w(.) \in L^p([0,t_1]:O)$, as follow:

$$(\psi_w y_w)(t) = T(t)(-A)^\alpha \left[y_0 + g(0,y_0) \right] - (-A)^\alpha g\left(t, (-A)^{-\alpha} y_w(t)\right) + \int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta g\left(s, (-A)^{-\alpha} y_w(s)\right) ds + \int_0^t (-A)^\alpha T(t-s)\left(f\left(s,(-A)^{-\alpha} y_w(s)\right) + Bw(s)\right) ds \quad (4)$$

Our aim is to prove that there exist a unique fixed point y_w of ψ_w in S_w, i.e. there is a unique $y_w \in S_w$ such that $\psi_w y_w = y_w$ for arbitrary control function $w(.) \in L^p([0,t_1]:O)$. we must applied the Banach contraction principle, i.e. we must prove the following steps:

Step (1) proves that S_w is closed subset of Y for every control function $w(.) \in L^p([0,t_1]:O)$.

Step (2) proves that $\psi_w(S_w) \subseteq S_w$ .Step (3) proves that ψ_w is a strict contraction on S_w, this will ensure the existence of a unique fixed point x_w of the map ψ_w on S_w.

By the above lemma step (1) holds. To prove step (2), let y_w be the arbitrary element in S_w such that $\psi_w y_w \in \psi_w(S_w)$, to prove that $\psi_w y_w \in S_w$ for every control function $w(.) \in L^p([0,t_1]:O)$. From the definition of the map ψ_w, it is clear that $\psi_w y_w \in Y$ and from equation (4), it is clear that $(\psi_w y_w)(0) = (-A)^\alpha y_0$.

Notice that: $\left\| (\psi_w y_w)(t) - (-A)^\alpha y_0 \right\|_X =$

$$\Big\| T(t)(-A)^\alpha \left[y_0 + g(0,y_0) \right] - (-A)^\alpha g\left(t,(-A)^{-\alpha} y_w(t)\right) + \int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta g\left(s,(-A)^{-\alpha} y_w(s)\right) ds + \int_0^t (-A)^\alpha T(t-s)\left(f\left(s,(-A)^{-\alpha} y_w(s)\right) + Bw(s)\right) ds - (-A)^\alpha y_0 \Big\|_X$$

$$\left\| (\psi_w y_w)(t) - (-A)^\alpha y_0 \right\|_X =$$

$$\Big\| T(t)(-A)^\alpha y_0 + T(t)(-A)^\alpha g(0,y_0) - (-A)^\alpha g\left(t,(-A)^{-\alpha} y_w(t)\right) + \int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta g\left(s,(-A)^{-\alpha} y_w(s)\right) ds + \int_0^t (-A)^\alpha T(t-s)\left(f\left(s,(-A)^{-\alpha} y_w(s)\right) + Bw(s)\right) ds - (-A)^\alpha y_0 + (-A)^\alpha g(0,y_0) - (-A)^\alpha g(0,y_0) \Big\|_X$$

$$\left\| (\psi_w y_w)(t) - (-A)^\alpha y_0 \right\|_X =$$

$$\Big\| (T(t) - I)(-A)^\alpha \left(y_0 + g(0,y_0) \right) + (-A)^\alpha g(0,y_0) - (-A)^\alpha g\left(t,(-A)^{-\alpha} y_w(t)\right) + \int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta g\left(s,(-A)^{-\alpha} y_w(s)\right) ds$$

$$+\int_0^t (-A)^{\alpha} T(t-s)\Big(f\big(s,(-A)^{-\alpha} y_w(s)\big)+Bw(s)\Big)ds\Big\|_X$$

$$\big\|(\psi_w y_w)(t)-(-A)^{\alpha} y_0\big\|_X$$
$$\le \big\|(T(t)-I)(-A)^{\alpha}(y_0+g(0,y_0))\big\|_X$$
$$+\big\|(-A)^{\alpha} g(0,y_0)-(-A)^{\alpha} g\big(t,(-A)^{-\alpha} y_w(t)\big)\big\|_X$$
$$+\int_0^t \big\|(-A)^{1-\beta+\alpha} T(t-s)(-A)^{\beta} g\big(s,(-A)^{-\alpha} y_w(s)\big)\big\|_X ds$$
$$+\int_0^t \big\|(-A)^{\alpha} T(t-s)\big(f\big(s,(-A)^{-\alpha} y_w(s)\big)+Bw(s)\big)\big\|_X ds$$

After a series of simplifications and using the conditions A_3, A_4, A_5 and A_6 with the properties $\|(-A)^{\alpha}T(t)\|\le C_{\alpha}t^{-\alpha}$, we get:

$$\big\|(\psi_w y_w)(t)-(-A)^{\alpha} y_0\big\|_X \le \delta'+\big\|(-A)^{\alpha-\beta}\big\| L_1\left(t_1^{\gamma_1}+\delta\right)+ C_{1-\beta+\alpha}C_1\frac{t_1^{\beta-\alpha}}{\beta-\alpha}+ C_{\alpha}(C_2+K_0K_1)\frac{t_1^{1-\alpha}}{1-\alpha}$$

$$\big\|(\psi_w y_w)(t)-(-A)^{\alpha} y_0\big\|_X \le \delta'+\big\|(-A)^{\alpha-\beta}\big\| L_1 t_1^{\gamma_1}+ \big\|(-A)^{\alpha-\beta}\big\| L_1\delta+C_{1-\beta+\alpha}C_1\frac{t_1^{\beta-\alpha}}{\beta-\alpha}+C_{\alpha}(C_2+K_0K_1)\frac{t_1^{1-\alpha}}{1-\alpha}$$

$$\big\|(\psi_w y_w)(t)-(-A)^{\alpha} y_0\big\|_X \le \delta'+\big\|(-A)^{\alpha-\beta}\big\| L_1 t_1^{\eta}+ \big\|(-A)^{\alpha-\beta}\big\| L_1\delta+ C_{1-\beta+\alpha}C_1\frac{t_1^{\eta}}{\beta-\alpha}+C_{\alpha}(C_2+K_0K_1)\frac{t_1^{\eta}}{1-\alpha}$$

where $\eta=\max\{\gamma_1,\beta-\alpha,1-\alpha\}$.

$$\big\|(\psi_w y_w)(t)-(-A)^{\alpha} y_0\big\|_X \le \delta'+\big\|(-A)^{\alpha-\beta}\big\| L_1\delta +\left[\big\|(-A)^{\alpha-\beta}\big\| L_1+\frac{C_{1-\beta+\alpha}C_1}{\beta-\alpha}+\frac{C_{\alpha}(C_2+K_0K_1)}{1-\alpha}\right]t_1^{\eta}$$

Using the condition (A_7.i), we get:

$$\big\|(\psi_w y_w)(t)-(-A)^{\alpha} y_0\big\|_X \le \delta,\ \forall 0\le t\le t_1 .$$

Hence $\psi_w y_w \in S_w$, $\forall w(.)\in L^p([0,t_1]:O)$, which implies that $\psi_w : S_w \to S_w$. To prove step (3); let, $\bar{y}_w,\bar{\bar{y}}_w \in S_w$, where $\bar{y}_w,\bar{\bar{y}}_w$ are the continuous function depend on the control function $w(.)\in L^p([0,t_1]:O)$, then:

$$\big\|(\psi_w\bar{y}_w)(t)-(\psi_w\bar{\bar{y}}_w)(t)\big\|_X =$$
$$\Big\|T(t)(-A)^{\alpha}\left[y_0+g(0,y_0)\right]-(-A)^{\alpha} g\big(t,(-A)^{-\alpha}\bar{y}_w(t)\big)$$
$$+\int_0^t (-A)^{1-\beta+\alpha}T(t-s)(-A)^{\beta} g\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)ds$$
$$+\int_0^t (-A)^{\alpha}T(t-s)\Big(f\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)+Bw(s)\Big)ds -$$
$$T(t)(-A)^{\alpha}\left[y_0+g(0,y_0)\right]+(-A)^{\alpha} g\big(t,(-A)^{-\alpha}\bar{\bar{y}}_w(t)\big)$$
$$-\int_0^t (-A)^{1-\beta+\alpha}T(t-s)(-A)^{\beta} g\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)ds$$
$$-\int_0^t (-A)^{\alpha}T(t-s)\Big(f\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)+Bw(s)\Big)ds\Big\|_X$$

$$\big\|(\psi_w\bar{y}_w)(t)-(\psi_w\bar{\bar{y}}_w)(t)\big\|_X =$$
$$\Big\|(-A)^{\alpha}\left[g\big(t,(-A)^{-\alpha}\bar{\bar{y}}_w(t)\big)-g\big(t,(-A)^{-\alpha}\bar{y}_w(t)\big)\right]$$
$$+\int_0^t (-A)^{1-\beta+\alpha}T(t-s)(-A)^{\beta}\left[g\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)-g\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)\right]ds +$$
$$\int_0^t (-A)^{\alpha}T(t-s)\left[f\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)-f\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)\right]ds\Big\|_X$$

$$\big\|(\psi_w\bar{y}_w)(t)-(\psi_w\bar{\bar{y}}_w)(t)\big\|_X$$
$$\le\Big\|\left[(-A)^{\alpha} g\big(t,(-A)^{-\alpha}\bar{\bar{y}}_w(t)\big)-(-A)^{\alpha} g\big(t,(-A)^{-\alpha}\bar{y}_w(t)\big)\right]\Big\|_X$$
$$+\int_0^t \big\|(-A)^{1-\beta+\alpha}T(t-s)\big\|_X \big\|(-A)^{\beta} g\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)-(-A)^{\beta} g\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)\big\|_X ds+$$
$$\int_0^t \big\|(-A)^{\alpha}T(t-s)\big\|_X \big\|f\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)-f\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)\big\|_X ds$$

$$\big\|(\psi_w\bar{y}_w)(t)-(\psi_w\bar{\bar{y}}_w)(t)\big\|_X \le$$
$$\Big\|(-A)^{\alpha-\beta}(-A)^{\beta} g\big(t,(-A)^{-\alpha}\bar{\bar{y}}_w(t)\big)-(-A)^{\alpha-\beta}(-A)^{\beta} g\big(t,(-A)^{-\alpha}\bar{y}_w(t)\big)\Big\|_X$$
$$+\int_0^t \big\|(-A)^{1-\beta+\alpha}T(t-s)\big\|_X \Big\|(-A)^{\beta} g\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)-(-A)^{\beta} g\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)\Big\|_X ds+\int_0^t \big\|(-A)^{\alpha}T(t-s)\big\|_X$$
$$\big\|f\big(s,(-A)^{-\alpha}\bar{y}_w(s)\big)-f\big(s,(-A)^{-\alpha}\bar{\bar{y}}_w(s)\big)\big\|_X ds$$

After a series of simplifications and using the condition A_3 with the properties $\|(-A)^{\alpha}T(t)\|\le C_{\alpha}t^{-\alpha}$, we get:

$$\left\|(\psi_w \bar{y}_w)(t) - (\psi_w \bar{\bar{y}}_w)(t)\right\|_X \le$$

$$\left\{ L_1 \left\|(-A)^{\alpha-\beta}\right\| + C_{1-\beta+\alpha} L_1 \frac{t_1^{\beta-\alpha}}{\beta-\alpha} + C_\alpha L_2 \frac{t_1^{1-\alpha}}{1-\alpha} \right\} \left\|\bar{y}_w - \bar{\bar{y}}_w\right\|_Y$$

$$\left\|(\psi_w \bar{y}_w)(t) - (\psi_w \bar{\bar{y}}_w)(t)\right\|_X \le$$

$$\left\{ L_1 \left\|(-A)^{\alpha-\beta}\right\| + C_{1-\beta+\alpha} L_1 \frac{t_1^{\Gamma}}{\beta-\alpha} + C_\alpha L_2 \frac{t_1^{\Gamma}}{1-\alpha} \right\} \left\|\bar{y}_w - \bar{\bar{y}}_w\right\|_Y$$

where $\Gamma = \max\{\beta-\alpha, 1-\alpha\}$.

$$\left\|(\psi_w \bar{y}_w)(t) - (\psi_w \bar{\bar{y}}_w)(t)\right\|_X \le$$

$$\left\{ L_1 \left\|(-A)^{\alpha-\beta}\right\| + \left(\frac{C_{1-\beta+\alpha} L_1}{\beta-\alpha} + \frac{C_\alpha L_2}{1-\alpha} \right) t_1^{\Gamma} \right\} \left\|\bar{y}_w - \bar{\bar{y}}_w\right\|_Y$$

By using the condition (A7.ii), we get:

$$\left\|(\psi_w \bar{y}_w)(t) - (\psi_w \bar{\bar{y}}_w)(t)\right\|_X \le \left(1 - \frac{\delta'}{\delta}\right) \left\|\bar{y}_w - \bar{\bar{y}}_w\right\|_Y \quad \text{..............................(5)}$$

Taking the supremum over $[0, t_1]$ of equation (5), we get:

$$\sup_{0 \le t \le t_1} \left\|(\psi_w \bar{y}_w)(t) - (\psi_w \bar{\bar{y}}_w)(t)\right\|_X \le \left(1 - \frac{\delta'}{\delta}\right) \left\|\bar{y}_w - \bar{\bar{y}}_w\right\|_Y$$

$\Rightarrow \left\|\psi_w \bar{y}_w - \psi_w \bar{\bar{y}}_w\right\|_X \le \left(1 - \frac{\delta'}{\delta}\right) \left\|\bar{y}_w - \bar{\bar{y}}_w\right\|_Y$, {by $\|y\|_Y = \sup_{0 \le t \le t_1} \|y(t)\|_X$ }.Thus ψ_w is a strict contraction map from S_w into S_w and therefore by the Banach contraction principle there exist a unique fixed point y_w of ψ_w in S_w , i.e. there is a unique $y_w \in S_w$ such that $\psi_w y_w = y_w$ for arbitrary control function $w(.) \in L^p([0, t_1] : O)$. This fixed point $y_w(.)$ satisfies the integral equation:

$$y_w(t) = T(t)(-A)^\alpha \left[y_0 + g(0, y_0)\right] - (-A)^\alpha g\left(t, (-A)^{-\alpha} y_w(t)\right)$$

$$+ \int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta g\left(s, (-A)^{-\alpha} y_w(s)\right) ds$$

$$+ \int_0^t (-A)^\alpha T(t-s)\left(f\left(s, (-A)^{-\alpha} y_w(s)\right) + Bw(s)\right) ds \quad \text{..............................(6)}$$

For simplification, we set $\tilde{f}(t) = f\left(t, (-A)^{-\alpha} y_w(t)\right)$, $\tilde{g}(t) = g\left(t, (-A)^{-\alpha} y_w(t)\right)$ then equation(6) can be rewritten as follow:

$$y_w(t) = T(t)(-A)^\alpha \left[y_0 + g(0, y_0)\right] - (-A)^\alpha \tilde{g}(t) +$$

$$\int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta \tilde{g}(s) ds$$

$$+ \int_0^t (-A)^\alpha T(t-s)\left(\tilde{f}(s) + Bw(s)\right) ds \quad \text{............(7)}$$

Now To show that $s \to (-A)^\beta \tilde{g}(s)$ and $s \to \tilde{f}(s)$ are locally Hölder continuous maps on $(0, t_1]$. For this, we first show that $y_w(t)$ given by equation (7) is locally Hölder continuous on $(0, t_1]$.Notice that from the theorem IV.7 in [3], it follows that for all $0 \le s \le t \le t_1$, $0 < \beta < 1-\alpha$ and for every $0 < h < 1, \tilde{\sigma} > 0, 0 < \tilde{\sigma} < \min\{1-\alpha, \beta-\alpha\}$, $\tilde{\sigma} + \beta > 1$ and $\tilde{\sigma} < \gamma_1$, we have got: $\forall\ 0 \le s < t$

$$\left\|(T(h) - I)(-A)^\alpha T(t-s)\right\|_X \le C h^{\tilde{\sigma}} (t-s)^{-(\tilde{\sigma}+\alpha)} \quad \text{........(8)}$$

$$\left\|(T(h) - I)(-A)^{1-\beta+\alpha} T(t-s)\right\|_X \le C h^{\tilde{\sigma}} (t-s)^{\beta-\alpha-\tilde{\sigma}-1} \quad \text{....(9)}$$

For $t \in (0, t_1]$ and h>0 sufficiently small,

$$\left\|y_w(t+h) - y_w(t)\right\|_X = \Big\| T(t+h)(-A)^\alpha \left(y_0 + g(0, y_0)\right)$$

$$- (-A)^\alpha \tilde{g}(t+h) +$$

$$\int_0^{t+h} (-A)^{1-\beta+\alpha} T(t+h-s)(-A)^{-\beta} \tilde{g}(s) ds +$$

$$\int_0^{t+h} (-A)^\alpha T(t+h-s)\left[\tilde{f}(s) + Bw(s)\right] ds$$

$$- T(t)(-A)^\alpha \left(y_0 + g(0, y_0)\right) + (-A)^\alpha \tilde{g}(t) -$$

$$\int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^{-\beta} \tilde{g}(s) ds -$$

$$\int_0^t (-A)^\alpha T(t-s)\left[\tilde{f}(s) + Bw(s)\right] ds \Big\|_X$$

$$\left\|y_w(t+h) - y_w(t)\right\|_X = \Big\|(T(h) - I)(-A)^\alpha T(t)$$

$$\left(y_0 + g(0, y_0)\right) - (-A)^\alpha \tilde{g}(t+h) +$$

$$\int_0^t (-A)^{1-\beta+\alpha} T(t+h-s)(-A)^\beta \tilde{g}(s) ds +$$

$$\int_t^{t+h} (-A)^{1-\beta+\alpha} T(t+h-s)(-A)^\beta \tilde{g}(s) ds + (-A)^\alpha \tilde{g}(t)$$

$$- \int_0^t (-A)^{1-\beta+\alpha} T(t-s)(-A)^\beta \tilde{g}(s) ds -$$

$$\int_0^t (-A)^\alpha T(t-s)\left[\tilde{f}(s) + Bw(s)\right] ds +$$

$$\int_0^t (-A)^{\alpha} T(t+h-s)\left[\tilde{f}(s)+Bw(s)\right]ds + \int_t^{t+h} (-A)^{\alpha} T(t+h-s)\left[\tilde{f}(s)+Bw(s)\right]ds\Big\|_X$$

$$\|y_w(t+h)-y_w(t)\|_X \le$$

$$\left\|(T(h)-I)(-A)^{\alpha} T(t)(y_0+g(0,y_0))\right\|_X +$$

$$\int_0^t \left\|(-A)^{1-\beta+\alpha}(T(h)-I)T(t-s)(-A)^{\beta}\tilde{g}(s)\right\|_X ds +$$

$$\int_0^t \left\|(-A)^{\alpha}(T(h)-I)T(t-s)\left[\tilde{f}(s)+Bw(s)\right]\right\|ds +$$

$$\int_t^{t+h} \left\|(-A)^{1-\beta+\alpha} T(t+h-s)(-A)^{\beta}\tilde{g}(s)\right\|_X ds$$

$$+\int_t^{t+h} \left\|(-A)^{\alpha} T(t+h-s)\left[\tilde{f}(s)+Bw(s)\right]\right\|_X ds$$

$$+\left\|(\tilde{g}(t)-\tilde{g}(t+h))(-A)^{\alpha}\right\|_X \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

We estimate each of the terms of (10) separately.

Let

$$I_1 = \left\|(T(h)-I)(-A)^{\alpha} T(t)(y_0+g(0,y_0))\right\|_X \le$$

$$\left\|(T(h)-I)(-A)^{\alpha} T(t)\right\|_X \left\|(y_0+g(0,y_0))\right\|_X \le$$

$Ch^{\tilde{\sigma}} t^{-(\tilde{\sigma}+\alpha)} \|y_0+g(0,y_0)\|_X$ {by using equation (8)}

$\le M_1 h^{\tilde{\sigma}}$, where M_1 depends on t, for $0 \le t < t_1$. Where $M_1 = C t^{-(\tilde{\sigma}+\alpha)} \|y_0+g(0,y_0)\|_X$

$$I_2 = \int_0^t \left\|(-A)^{1-\beta+\alpha}(T(h)-I)T(t-s)(-A)^{-\beta}\tilde{g}(s)\right\|_X ds$$

$$\le \int_0^t \left\|(-A)^{1-\beta+\alpha}(T(h)-I)T(t-s)\right\|_X \left\|(-A)^{-\beta}\tilde{g}(s)\right\|_X ds$$

$\le \int_0^t Ch^{\tilde{\sigma}}(t-s)^{\beta-\alpha-\tilde{\sigma}-1}\left\|(-A)^{-\beta}\tilde{g}(s)\right\|_X ds$, {by using equation (9)}

$\le CC_1 h^{\tilde{\sigma}} \int_0^t (t-s)^{\beta-\alpha-\tilde{\sigma}-1} ds$, {by using Assumption A_4}

$$\le CC_1 h^{\tilde{\sigma}} \frac{t^{\beta-\alpha-\tilde{\sigma}}}{\beta-\alpha-\tilde{\sigma}} \le CC_1 h^{\tilde{\sigma}} \frac{t_1^{\beta-\alpha-\tilde{\sigma}}}{\beta-\alpha-\tilde{\sigma}} \le M_2 h^{\tilde{\sigma}},$$

Where $M_2 = \frac{CC_1 t_1^{\beta-\alpha-\tilde{\sigma}}}{\beta-\alpha-\tilde{\sigma}}$ is independent of t, for $0 \le t \le t_1$.

$$I_3 = \int_0^t \left\|(-A)^{\alpha}(T(h)-I)T(t-s)\left[\tilde{f}(s)+Bw(s)\right]\right\|_X ds$$

$$I_3 \le \int_0^t \left\|(-A)^{\alpha}(T(h)-I)T(t-s)\right\|_X \left\|\left[\tilde{f}(s)+Bw(s)\right]\right\|_X ds$$

$I_3 \le \int_0^t Ch^{\tilde{\sigma}}(t-s)^{-(\tilde{\sigma}+\alpha)}\left\|\left[\tilde{f}(s)+Bw(s)\right]\right\|_X ds$, {by using equation (8)}

$I_3 \le Ch^{\tilde{\sigma}}(C_2+K_0K_1)\int_0^t (t-s)^{-(\tilde{\sigma}+\alpha)} ds$, {by using Assumption A_4 and A_6}

$$I_3 \le Ch^{\tilde{\sigma}}(C_2+K_0K_1)\frac{t^{-(\tilde{\sigma}+\alpha)+1}}{-(\tilde{\sigma}+\alpha)+1} \le$$

$$Ch^{\tilde{\sigma}}(C_2+K_0K_1)\frac{t_1^{-(\tilde{\sigma}+\alpha)+1}}{-(\tilde{\sigma}+\alpha)+1} \le M_3 h^{\tilde{\sigma}},$$

Where $M_3 = \frac{C(C_2+K_0K_1)t_1^{-(\tilde{\sigma}+\alpha)+1}}{-(\tilde{\sigma}+\alpha)+1}$ is independent of t, for $0 \le t \le t_1$.

$$I_4 = \int_t^{t+h} \left\|(-A)^{1-\beta+\alpha} T(t+h-s)(-A)^{\beta}\tilde{g}(s)\right\|_X ds$$

$$I_4 \le \int_t^{t+h} \left\|(-A)^{1-\beta+\alpha} T(t+h-s)\right\|_X \left\|(-A)^{\beta}\tilde{g}(s)\right\|_X ds$$

$$I_4 \le C_{1-\beta+\alpha} C_1 \int_t^{t+h} (t+h-s)^{-(1-\beta+\alpha)} ds$$

$\le \frac{C_{1-\beta+\alpha}C_1}{\beta-\alpha} h^{\beta-\alpha} \le M_4 h^{\beta-\alpha}$, {by using Assumption A_4 and $\|(-A)^{\alpha} T(t)\|_X \le C_{\alpha} t^{-\alpha}, \forall t > 0, \alpha > 0$ }.

Where $M_4 = \frac{C_{1-\beta+\alpha}C_1}{\beta-\alpha}$ is independent of t, for $0 \le t \le t_1$. So $I_4 \le M_4 h^{\beta-\alpha} \le M_4 h^{\tilde{\sigma}}$ {since $0<h<1$ and $\tilde{\sigma} < \beta-\alpha$ }.

$$I_5 = \int_t^{t+h} \left\|(-A)^{\alpha} T(t+h-s)\left[\tilde{f}(s)+Bw(s)\right]\right\|_X ds$$

$$I_5 \le \int_t^{t+h} \left\|(-A)^{\alpha} T(t+h-s)\right\|_X \left\|\left[\tilde{f}(s)+Bw(s)\right]\right\|_X ds$$

$I_5 \le C_{\alpha}(C_2+K_0K_1)\int_t^{t+h} (t+h-s)^{-\alpha} ds$, {by using Assumption A_4, A_6 and $\|(-A)^{\alpha} T(t)\|_X \le C_{\alpha} t^{-\alpha}, \forall t > 0, \alpha > 0$ }.

$$I_5 \le C_{\alpha}(C_2+K_0K_1)\frac{h^{-\alpha+1}}{-\alpha+1} \le M_5 h^{1-\alpha},$$

where $M_5 = \frac{C_{\alpha}(C_2+K_0K_1)}{-\alpha+1}$.

So $I_5 \le M_5 h^{1-\alpha} \le M_5 h^{\tilde{\sigma}}$ {since $0<h<1$ and $\tilde{\sigma} < 1-\alpha$ }.

$I_6 = \left\|\left(\tilde{g}(t)-\tilde{g}(t+h)\right)(-A)^{\alpha}\right\|_X \le N h^{1-\alpha}$, {by using Assumption A_8}

$I_6 \le N h^{\tilde{\sigma}}$ {Since $0<h<1$ and $\tilde{\sigma} < 1-\alpha$ }.So $I_6 \le M_6 h^{\tilde{\sigma}}$, where $M_6=N$. Combining equation (10) with these estimates, it follows that there is a constant M such that:

$\left\|y_w(t+h)-y_w(t)\right\|_X \le M h^{\tilde{\sigma}} \le M\left|h^{\tilde{\sigma}}\right|$, where $M=M_1+M_2+M_3+M_4+M_5+M_6$.

Let $r = t + h \Rightarrow h = r - t$, which implies that $\left\|y_w(r)-y_w(t)\right\|_X \le M|r-t|^{\beta}, \forall 0<t<r\le t_1$.

So we have got $y_w(t)$ is locally Hölder continuous on $(0,t_1]$.

Now it easy to show that $s \to (-A)^{\beta}\tilde{g}(s)$ and $s \to \tilde{f}(s)$ are locally Hölder continuous maps on $(0,t_1]$.For t>s,

$\left\|(-A)^{\beta}\tilde{g}(t)-(-A)^{\beta}\tilde{g}(s)\right\|_X =$

$\left\|(-A)^{\beta} g\left(t,(-A)^{-\alpha} y_w(t)\right)-(-A)^{\beta} g\left(s,(-A)^{-\alpha} y_w(s)\right)\right\|_X$

$\le L_1\left\{|t-s|^{\gamma_1}+\left\|(-A)^{-\alpha} y_w(t)-(-A)^{-\alpha} y_w(s)\right\|_{\alpha}\right\}$, {by using Assumption A_3}

$\le L_1\left\{|t-s|^{\gamma_1}+\left\|y_w(t)-y_w(s)\right\|_X\right\}$, {by using the property $\|y\|_{\alpha}=\|A^{\alpha}y\|_X$ }

$\le L_1\left\{|t-s|^{\gamma_1}+M|t-s|^{\beta}\right\}$ {Since $y_w(t)$ is locally Hölder continuous on $(0,t_1]$}

$\le L_1\left\{|t-s|^{\zeta}+M|t-s|^{\zeta}\right\}$, where $\zeta = \min\{\gamma_1,\beta\}$.

$\le L_1(1+M)|t-s|^{\zeta} \le R_0|t-s|^{\zeta}$, where $R_0 = L_1(1+M)$. Also

$\left\|\tilde{f}(t)-\tilde{f}(s)\right\| = \left\|f\left(t,(-A)^{-\alpha} y_w(t)\right)-f\left(s,(-A)^{-\alpha} y_w(s)\right)\right\|_X$

$\le L_2\left\{|t-s|^{\gamma_2}+\left\|(-A)^{-\alpha} y_w(t)-(-A)^{-\alpha} y_w(s)\right\|_{\alpha}\right\}$, {by using Assumption A_3}

$\le L_2\left\{|t-s|^{\gamma_2}+\left\|y_w(t)-y_w(s)\right\|_X\right\}$, {by using the property $\|y\|_{\alpha}=\|A^{\alpha}y\|_X$ }

$\le L_2\left\{|t-s|^{\gamma_2}+M|t-s|^{\beta}\right\}$, {since $y_w(t)$ is locally Hölder continuous on $(0,t_1]$}.

$\le L_2\left\{|t-s|^{\psi}+M|t-s|^{\psi}\right\}$, where $\psi = \min\{\gamma_2,\beta\}$.

$\le L_2(1+M)|t-s|^{\psi} \le R_1|t-s|^{\psi}$, where $R_1 = L_2(1+M)$. from the theorem (2.4.1) in [4], we infer that the function:

$$z_w(t) = T(t)\left[y_0+g(0,y_0)\right]-g\left(t,(-A)^{-\alpha} y_w(t)\right)+ \int_0^t (-A)^{1-\beta}T(t-s)(-A)^{\beta} g\left(s,(-A)^{-\alpha} y_w(s)\right)ds + \int_0^t T(t-s)\left(f\left(s,(-A)^{-\alpha} y_w(s)\right)+Bw(s)\right)ds \quad ....(11)$$

is X_{α} –valued, that the integral terms in (11) are functions in $C^1\left((0,t_1]:X\right)$ and that $z_w(t) \in D(A), \forall t \in (0,t_1]$.Operating on $z_w(.)$ with $(-A)^{\alpha}$, we get:

$$(-A)^{\alpha} z_w(t) = T(t)(-A)^{\alpha}\left[y_0+g(0,y_0)\right]-(-A)^{\alpha} g\left(t,(-A)^{-\alpha} y_w(t)\right) + \int_0^t (-A)^{1-\beta+\alpha}T(t-s)(-A)^{\beta} g\left(s,(-A)^{-\alpha} y_w(s)\right)ds + \int_0^t (-A)^{\alpha}T(t-s)\left(f\left(s,(-A)^{-\alpha} y_w(s)\right)+Bw(s)\right)ds,$$

which implies that $(-A)^{\alpha} z_w(t) = y_w(t)$, i.e., $z_w(t) = (-A)^{-\alpha} y_w(t)$, and hence that $z_w(t)+g\left(t,z_w(t)\right)$ is a C^1 function on $(0,t_1]$.

So we have got $z_w(t)$ be S-classical solution to the semilinear initial value problem defined in (3).

Conclusions

1. The subject of this paper is based mainly on the analytic semigroup theory and the provided as a good approach or tools to solve the problem not only to ensure the existence and uniqueness.
2. The existence and uniqueness have represented the main objects of real life dynamical control system, so studying these titles providing as a good work studying of this subject.
3. Analytic semigroup has become an important tool to define the fractional power $A^{-\alpha}$ to ensure the existence of the S-classical solution to the semilinear initial value control problem (abstract Cauchy control problem) defined in (3).

Future Works

1. Establish the controllability of the semilinear initial value control problem (abstract Cauchy control problem) defined in (3).

2. Developing the present approach to some optimum control in infinite dimensional spaces.
3. Developing numerical procedures to find the solution numerically or even exact in some infinite dimensional spaces.
   Establish and study the classical solution, mild solution and strong solution to the
4. semilinear initial value control problem defined in (3).

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الخلاصة

الوجود المحليّ ووحدانيــة الحَــلّ نــصف كلاســيكي (حَلّ نصف كلاسيكي) لصنف مسألة سيطرة شبه خطية في فضاء باناخ مناسب نوقشتْ وأثبتتْ. إنّ النَتــائِجَ النظريــةَ تَعتمدُ على نظريةِ نصفِ المجموعةَ التحليليةَ ومبدأَ إنكماشِ باناخ.