# TRANSITION CHARGE DISTRIBUTIONS IN NUCLEAR COLLECTIVE MODES

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**Abstract**

An Expression for the transition charge density is investigated where the deformation in nuclear collective modes is taken into consideration besides the shell model transition density. The inelastic longitudinal form factors C2 are calculated using this transition charge density with excitation of the levels in $^{10}B$ $(J_f^{\pi},T)$: $(1^+\ 0)$ at $E_x$=0.718 MeV, $(1^+\ 0)$ at $E_x$=2.154 MeV, $(2^+\ 0)$ at $E_x$=3.587 MeV, $(3^+\ 0)$ at $E_x$=4.774 MeV, $(2^+\ 0)$ at $E_x$=5.920 MeV and $(4^+\ 0)$ at $E_x$=6.025 MeV. In this work, the core polarization transition density is evaluated by adopting the shape of Tassie model togther with the derived form of the ground state two-body charge density distributions (2BCDD's). It is noticed that the core polarization effects which represent the collective modes are essential in obtaining the remarkable agreement between the calculated inelastic longitudinal *F(q)*'s and those of experimental data.

## Introduction

Comparison between calculated and measured longitudinal electron scattering form factors has long been used as stringent tests of models of transition densities. Various microscopic and macroscopic theories have been used to study excitations in nuclei. Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. In spite of the success of the 1p-shell model on static properties of nuclei in this region , it fails to describe electron scattering data at high momentum transfer [1,2]. Extending the model space to include the $2\hbar\omega$ configurations improves the agreement with the transverse form factors in the beginning of the p-shell, but towards the end of the p-shell the situation deteriorates[2]. Calculations of form factors [3] using the model space wave function alone is inadequate for reproducing the data of electron scattering. Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations. These effects can be considered as a polarization of core protons by the valence protons and neutrons. Core polarization effects can be treated either by connecting the ground state to the *J*-multipole $n\hbar\omega$ giant resonances [3], where the shape of the transition densities for these excitations is given by Tassie model [4], or by using a microscopic theory [5-8] which permits one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these longitudinal excitations. Core polarization effects were incorporated within the p-shell wave function by Sato et al. [9], where the effects greatly improved the agreement with the experimental data. Coulomb form factors of $E4$ transitions in the sd-shell nuclei were discussed taking into account core polarization effects using self-consistent Hartree-Fock plus random phase approximation calculations, which gave a good agreement with experimental form factors [10].

The aim of the present work is to study the inelastic longitudinal form factor C2 for the $^{10}B$ nucleus. Calculations of form factors using the many particle shell model space alone were nown to be inadequaqte in describing electron scattering data. So, effects out of the model space (core-polarization) are necessary to be included in the calculations.The shape of the transtion density for the excitation considered in this work is given by the Tassie model [4], this model is connected with the ground state charge density, where the ground state charge density of the present work is to derive an expression for the ground state two-body charge density distributions (2BCDD), based on the use of the two-body wave functions of the harmonic oscillator and the full two–body correlation functions FC's.

**Theory**

The interaction of the electron with charge distribution of the nucleus gives rise to the longitudinal or Coulomb scattering. The longitudinal form factor is related to the charge density distribution (CDD) through the matrix elements of multipole operators $\hat{T}_J^L(q)$[3].

$$\left| F_J^L(q) \right|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle \right|^2 \left| F_{cm}(q) \right|^2 \left| F_{fs}(q) \right|^2 \quad \ldots\ldots (1)$$

where $Z$ is the atomic number of the nucleus, $F_{cm}(q)$ is the center of mass correction, which removes the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [11]:

$$F_{cm}(q) = e^{q^2b^2/4A} \quad \ldots\ldots (2)$$

where $A$ is the nuclear mass number and $b$ is the harmonic oscillator size parameter. The function $F_{fs}(q)$ is the free nucleon form factor and assumed to be the same for protons and neutrons and takes the form[12]:

$$F_{fs}(q) = e^{-0.43q^2/4} \quad \ldots\ldots (3)$$

The longitudinal operator is defined as [13]:

$$\hat{T}^L_{Jt_z}(q) = \int dr\, j_J(qr) \mathrm{Y}_J(\Omega)\, \rho(r,t_z) \quad \ldots\ldots (4)$$

where $j_J(qr)$ is the spherical Bessel function, $\mathrm{Y}_J(\Omega)$ is the spherical harmonic wave function and $\rho(r,t_z)$ is the charge density operator. The reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system, including the configuration mixing, are given in terms of the One Body Density Matrix (OBDM) elements times the single particle matrix elements of the longitudinal operator[3],i.e.

$$\left\langle f \left\| \hat{T}^L_{JT} \right\| i \right\rangle = \sum_{a,b} OBDM^{JT}(i,f,J,a,b) \left\langle b \left\| \hat{T}^L_{JT} \right\| a \right\rangle \quad \ldots\ldots (5)$$

The many particle reduced matrix elements of the longitudinal operator, consists of two parts one is for the model space and the other is for core polarization matrix element [5,7]:

$$\left\langle f \left\| \hat{T}^L_J(\tau_Z,q) \right\| i \right\rangle = \left\langle f \left\| \hat{T}^{ms}_J(\tau_Z,q) \right\| i \right\rangle + \left\langle f \left\| \hat{T}^{cor}_J(\tau_Z,q) \right\| i \right\rangle \quad \ldots (6)$$

where the model space matrix element in eq.(3.6.1) has the form [3]:

$$\left\langle f \left\| \hat{T}^L_J(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty d\mathrm{r}^2 j_J(q\mathrm{r})\, \rho^{ms}_{J,\tau_Z}(i,f,\mathrm{r}) \quad \ldots\ldots (7)$$

where $\rho^{ms}_J(i,f,\mathrm{r})$ is the transition charge density of model space given by [3]:

$$\rho^{ms}_{J,\tau_Z}(i,f,\mathrm{r}) = \sum_{jj'(ms)} OBDM(i,f,J,j,j',\tau_z) \left\langle j \left\| \mathrm{Y}_J \right\| j' \right\rangle R_{nl}(\mathrm{r})\, R_{n'l'}(\mathrm{r}) \quad \ldots\ldots (8)$$

The core- polarization matrix element is given by[3]:

$$\left\langle f \left\| \hat{T}^L_J(\tau_Z,q) \right\| i \right\rangle = e_i \int_0^\infty d\mathrm{r}\, \mathrm{r}^2 j_J(q\mathrm{r})\, \rho^{core}_J(i,f,\mathrm{r}) \quad \ldots (9)$$

where $\rho^{core}_J$ is the core-polarization transition density which depends on the model used for core polarization. To take the core-polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density becomes

$$\rho_{J\tau_Z}(i,f,r) = \rho^{ms}_{J\tau_Z}(i,f,r) + \rho^{core}_{J\tau_Z}(i,f,r) \quad \ldots\ldots (10)$$

where $\rho^{core}_J$ is assumed to have the form of Tassie shape and given by [4].

$$\rho^{core}_{Jt_z}(i,f,r) = N\frac{1}{2}(1+\tau_z)\, r^{J-1} \frac{d\rho(i,f,r)}{dr} \quad \ldots (11)$$

where $N$ is a proportionality constant. Itisdetermined by adjusting the reduced transition probability *B(CJ)* and given by [14]:

$$N=\frac{\int_0^\infty drr^{J+2}\rho^{ms}_{J\tau_z}(i,f,\mathrm{r})-\sqrt{(2J_i+1)B(CJ)}}{(2J+1)\int_0^\infty drr^{2J}\rho(i,f,\mathrm{r})} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Here, $\rho(i,f,r)$ is the ground state charge density distribution. An effective two-body charge density operator (to be used with uncorrelated wave functions) can be produced by folding the two-body charge density operator with the two-body correlation functions $\tilde{f}_{ij}$ as:

$$\hat{\rho}^{(2)}_{eff}(\vec{r})=\frac{\sqrt{2}}{2(A-1)}\sum_{i\neq j}\tilde{f}_{ij}\left\{\delta\left[\sqrt{2}\vec{r}-\vec{R}_{ij}-\vec{r}_{ij}\right]+\delta\left[\sqrt{2}\vec{r}-\vec{R}_{ij}+\vec{r}_{ij}\right]\right\}\tilde{f}_{ij} \quad \ldots(13)$$

where $\vec{\mathrm{r}}_{\mathrm{ij}}$ and $\vec{R}_{\mathrm{ij}}$ are relative and center of mass coordinates and the form of $\tilde{f}_{ij}$ is given by [15]:

$$\tilde{f}_{ij}=f(r_{ij})\Delta_1+f(r_{ij})\left\{1+\alpha(A)S_{ij}\right\}\Delta_2 \ldots. (14)$$

It is clear that eq. (14) contains two types of correlations:

1. The two body short range correlations presented in the first term of eq. (14) and denoted by $f(r_{ij})$. Here $\Delta_1$ is a projection operator onto the space of all two-body functions with the exception of ${}^3S_1$ and ${}^1D_3$ states. It should be noted that the short range correlations are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances,where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of $f(r_{ij})$ is given as [15]:

$$f(r_{ij})=\begin{cases}0 & for\ r_{ij}\le r_c\\ 1-\exp\left\{-\mu(r_{ij}-r_c)^2\right\} & for\ r_{ij}>r_c\end{cases} \ldots (15)$$

where $r_c$ (in $fm$) is the radius of a suitable hard core and $\mu=25\ fm^{-2}$ [15] is a correlation parameter.

2. The two-body tensor correlations presented in the second term of eq.(14) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here $\Delta_2$ is a projection operator onto ${}^1S_3$ and ${}^1D_3$ states only. $S_{ij}$ is the usual tensor operator, formed by the scalar product of a second-rank operator in intrinsic spin space and coordinate space and is defined by

$$S_{ij}=\frac{3}{r_{ij}^2}(\vec{\sigma}_i.\vec{r}_{ij})(\vec{\sigma}_j.\vec{r}_{ij})-\vec{\sigma}_i.\vec{\sigma}_j \ldots\ldots\ldots\ldots (16)$$

The parameter $\alpha(A)$ is the strength of tensor correlations and it is non zero only in the ${}^1S_3-{}^1D_3$ channels.

**Results and Discussion**

The calculations for the C2 isoscalar transition from the ground state $(J_i^\pi,T):(3^+\ 0)$ to $(J_f{}^\pi T):(1^+\ 0)$ at $E_x$=0.718 MeV, $(1^+\ 0)$ at $E_x$=2.154 MeV, $(2^+\ 0)$ at $E_x$=3.587 MeV, $(3^+\ 0)$ at $E_x$=4.774 MeV, $(2^+\ 0)$ at $E_x$=5.920 MeV and $(4^+\ 0)$ at $E_x$=6.025 MeV of ${}^{10}B$ are shown in Fig. (1). The inelastic longitudinal electron scattering form factors *F(q)'s* are calculated using an expression for the transition charge density of eq.(10). The model space transition density is obtained using eq.(8), where the OBDM elements required by the calculations of the form factors of open shell nuclei are taken from [7] using the interaction matrix elements of Cohen-Kurath (CK) [16] for 1p shell nuclei. For considering the collective modes of the nuclei, the core polarization transition density of eq.(11) is evaluated by adopting the Tassie model [4] together with the calculated ground state 2BCDD of eq.(13). All parameters required in the following calculations of 2BCDD's, $\left\langle \mathrm{r}^2\right\rangle^{1/2}$ and longitudinal *F(q)*'s, such as the values of the harmonic oscillator spacing parameter $\hbar\omega$, the occupation

probabilities $\eta$'s of the states, the values of $\alpha(A)$ for $^{10}B$ are presented in Table (1).

In Fig.(1) the dashed curves represent the contribution of the model space where the configuration mixing is taken into account, the dotted curves represent the core polarization contribution where the collective modes are considered and the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects. The experimental data of Ref. [17] are represented by solid circles.

**The 0.718 MeV ($1^+0$) state:**

The calculations for the C2 isoscalar transition from the ground state $3^+0$ to the $1^+0$ state at $E_x$= 0.718 MeV are shown in Fig. (1). The data are well described by the 1p-shell for $2.4\ fm^{-1} > q > 1\ fm^{-1}$. The inclusion of core-polarization effect enhances the form factor. This enhancement brings the total theoretical results of the longitudinal C2 form factor very close to the experimental data which are plotted versus $q_{eff}$. The experimental value of $B(C2)$ used in the present calculations is equal to $1.7 \pm 0.3\ e^2.fm^4$ [17].

**The 2.154 MeV ($1^+0$) state:**

In this isoscalar transition the result of core-polarization effect increases the C2 longitudinal form factor component by about a factor of 2.0 over the 1p-shell calculation (dashed curve), making the total theoretical form factor results closer to the experimental data [18]. The measured reduced transition strength used in the present calculations $B(C2)$ is equal to $0.4 \pm 0.1\ e^2.fm^4$ [17].

**The 3.587 MeV ($2^+0$) state:**

The 1p-shell calculations describe the experimental data very well at $1.5\ fm^{-1} > q > 0.5\ fm^{-1}$. The core-polarization effect increases the C2 longitudinal form factor component by a very small amount, making the total theoretical form factor agreed with the experimental values which are taken from Ref.[17].

**The 4.774 MeV ($3^+0$) state:**

The total form factors are shifted from the experimental data which are shown as circles and taken from Ref.[18]. In this isoscalar transition, the core-polarization effect results are very close to the experimental data while the 1p-shell model calculations are shifted from the experimental data at $q \geq 0.6\ fm^{-1}$. In the present work, we used the value of $B(C2)$ as $0.03\ e^2.fm^4$ [17].

**The 5.925 MeV ($4^+0$) state:**

The total theoretical results for the longitudinal C2 form factors of this transition are given in Fig.(1) as solid curve. The 1p-shell calculations are very close to the experimental values, so the core-polarization decreases the 1p-shell model space calculations by a small amount to make the total form factor in good agreement with the experimental values for $1.8\ fm^{-1} > q > 0.6\ fm^{-1}$. The experimental values of $B(C2)$ value for this transition is $0.17 \pm 0.05\ e^2.fm^4$ [17].

**The 6.025 MeV ($4^+0$) state:**

The improvement in the description of the form factors for the states considered so far is also reflected in the longitudinal form factor for this state, as shown in Fig.(1). While the core-polarization effect calculations raise the 1p-shell model space calculation making the total theoretical form factor agreed with the experimental values which are taken from Refs.[18,19], but it fails to describe the experimental data in many regions of momentum transfer. The experimental values of $B(C2)$ used in the present calculations is equal to $17.4 \pm 0.7\ e^2.fm^4$ [17].

**Conclusions**

The 1p-shell models, which can describe the static properties and energy levels, are less successful for describing dynamics properties such as C2 transition rates and electron scattering form factors. The core-polarization effect enhances the form factors and makes the theoretical results of the longitudinal form factors closer to the experimental data in the C2 transition which is studied in the present work.

Considering the effect of higher occupation probabilities and full correlation are, generally, essential in getting good agreement between the calculated result of $\langle r^2 \rangle^{1/2}$ and those of experimental data.

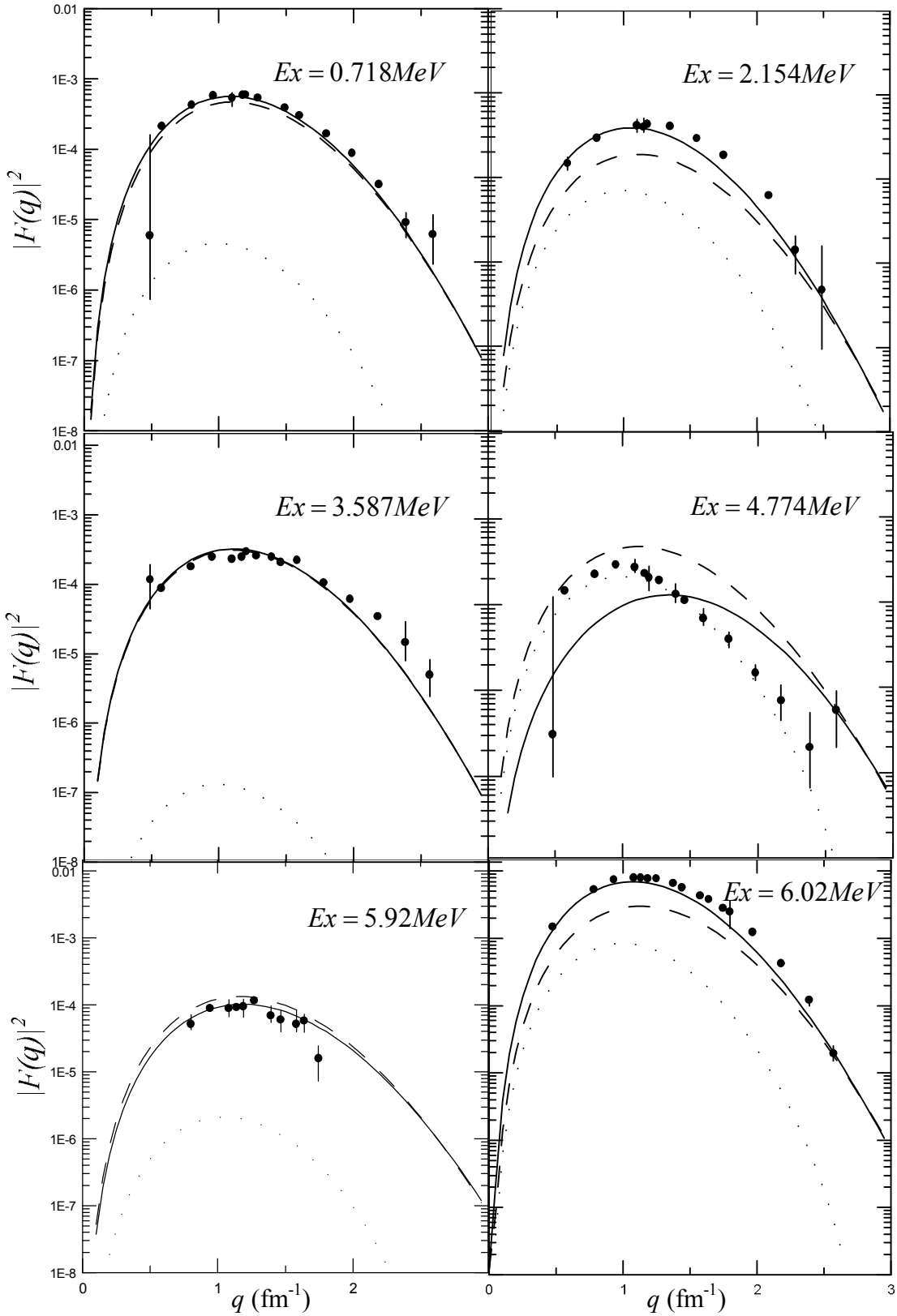


***Fig. (1) : Inelastic longitudinal $C2$ form factors for the transitions in $^{10}B$ nucleus. The dashed curves represent the contribution of the model space, the dotted curves represent the core polarization contribution and the solid curves represent the total form factors obtained by the sum of model space and core polarization contributions. The experimental data of Refs. [17,18] are represented by solid circles.***

***Table(1)***
***Arameters which have been used in the calculations of the present work for the 2BCDD's and longitudinal F(q)'s for*** $^{10}B$.

| **Nucleus** | $^{10}B$ *e* |
|---|---|
| $\hbar\omega$ | 13 MeV |
| $\eta_{1S_{1/2}}$ | 1 |
| $\eta_{1P_{3/2}}$ | 0.5 |
| $\eta_{1P_{1/2}}$ | 0.0 |
| $\eta_{1d_{5/2}}$ | 0.1667 |
| $\alpha(A)$ | 0.1 |
| $\langle r^2 \rangle^{1/2}_{r_c=0.5,\alpha\neq 0.0}$ | 2.44 fm |
| $\langle r^2 \rangle^{1/2}_{exp.}$ [18] | 2.45 fm |

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**الخلاصة**

تم اختبار علاقة كثافة الشحنة للحالات المثارة باخذ التشوه_الحاصل في الأنماط النووية التجميعية بالاضافة الى كثافة الشحنة للحالات الناتجة من انموذج القشرة. استخدمت_هذه العلاقة في حساب عوامل التشكل للاستطارة الالكترونية الطولية C2 للمستويات المثارة : $(1^+\ 0)$ 0.718 MeV, $(1^+\ 0)$ 2.15MeV , $(2^+\ 0)$ 3.587MeV، $(3^+\ 0)$ 4.774MeV , $(2^+\ 0)$ 5.920 MeV و $(4^+\ 0)$ 6.025 MeV لنواة $^{10}B$.

أن تأثيرات استقطاب القلب لكثافة الانتقال حسبت بالاعتماد على شكل أنموذج Tassie الى جانب الصيغة الرياضية المشتقة لتوزيعات كثافة الشحنة النووية ذو صيغة الجسيمين في الحالة الارضية (2BCDD's) . لقد وجد بان تأثير استقطاب القلب الذي يمثل نمط تجميعي يكون جوهريا للحصول على توافق جيد بين حسابات الاستطارة الطولية غير المرنة ($F(q)$'s) و القيم العملية.