SYSTEMATIC TREATMENT OF PHOTO-NUCLEAR CROSS-SECTION CALCULATIONS

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Abstract

The photoneutron cross-sections for twelve radioactive elements are analyzed and are shown to follow an empirical formula. The aim of suggesting this mathematical formula is to be used in γ-ray incineration calculations, where the numerical effort is greatly reduced. The method used in the present research is simple and direct. The results of calculating incineration rate of selected radioactive isotopes were fitted to a simple polynomial form that takes into account the properties of the specified nucleus and the energy and flux intensity of the used γ-rays. Direct application of the present results on the selected radioactive isotopes shows that the suggested formula is efficient and accurate within accepted range of error. The radioactive elements selected represent a group of actinides resulting from nuclear fission reaction. A verity of γ-ray fluxes are selected that span a proper range of practical calculations. This leads to sixty fitted equations, with each equation containing five terms. This will provide enough accuracy and reliability for the suggested method. Fair comparisons with published work revealed that the current method gives satisfactory results.

Keywords: photonuclear reactions, photon absorption and scattering, photo-production reactions, radioactive waste management.

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Introduction

There have been an unlimited interest in nuclear reactions since the early days of nuclear age. Basically, the information that nuclear reactions provide are used to explore and reveal the nuclear structure and properties. The nuclear reactions extend from simple particles to heavy ions in both projectile and ejectile type. This verity of nuclear reactions has already disclosed to us many important features of the nuclear structure. During the path of this exploration, there have been many applications that were based on certain types of nuclear reactions.

One of the most important applications of nuclear reactions is the management of nuclear reactor waste. Incineration and transmutation of these harmful wastes into more stable elements is a subject of great interest to all countries that look forward to use the nuclear energy as a permanent source. There are many practical studies in this course that are based on theoretical [1-5] and experimental [6-10] calculations. The comprehensive study for γ-ray incineration technique on many radioactive isotopes [2, 8, 11] declared the importance of this method in treating nuclear reactor wastes. In all cases, however, the aim was toward finding the most suitable method for treating nuclear reactor waste with less effort and expense. Some scientists also recommend using both incineration method and geological burial [11,12], so that one can achieve the best result of reducing the harm of these wastes with as less expense as possible.

There are many methods used to achieve incineration, depending essentially on the type of the incident particle [11,13,14]. Among these methods, γ-ray incineration method shows its importance. This is because:

(a) It is possible to have very intense γ-ray sources with a relatively simple technology, joined with less expenses compared to proton sources for example. This attains part of the economical profit required from nuclear reactor waste treatment.

(b) The efficiency of using γ-rays incineration method is high because it depends mainly on shielding. In fact, collimation and shielding of γ-rays needs less technological efforts than other nuclear beams.

(c) Because of the nature of γ-rays, the interaction with the nucleus does not depend mainly on the nuclear charge, but it will depend more on the size of the target nuclei.

The most important quantity needed for incinerating nuclear rector wastes using γ-ray is the reaction cross-section. Basically, (γ,n) and (γ, total) cross-sections are the most important [2,6,7]. This physical quantity enters directly in the calculations required for finding the ratio between incinerated number of nuclei, $N_{inc.}$, to the original number, N_{org}. The incineration ratio, labeled by $\zeta = N_{inc}/N_{org.}$, gives the efficiency of the transmutation process. As ζ becomes with the value of a unity for each selected element, then the nuclear reactor waste is no longer harmful and can be disposed safely. The practical estimation of ζ varies between ~ 80% to better than 90%, depending on the flux of γ-rays and the type of the radioactive isotope.

The cross section of the (γ, n) nuclear reaction was calculated in earlier research for many nuclei that compose an important fraction of the nuclear reactor waste. It was shown that there is a direct relation between γ-flux and the ratio ζ . At high γ-ray flux, the value of ζ becomes better than~98% for some radioactive nuclei, while in all the cases this ratio is better than ~90% [1].

In the present paper, the results of ref.[1] are treated systematically. The aim of this study is to write down a unified formula that gives the direct incineration ratio from knowing the type of the radioactive nucleus type (its mass number A and atomic number *Z*), the energy of incident γ-rays, the flux of γ-rays ϕ, and irra-diation period *t*.

The formula suggested in the present paper (eq.5 and 6 below) is aimed for the selected types of nuclei, and may not give satisfactory results for other types of radioactive elements. Other than that, more elements of radioactive isotopes produced from nuclear fission should also be studied and joined with the results of this paper so that more general form can be deduced.

The Basic Theory

The calculation of the ratio ζ depends mainly on finding the cross-section for the reactions (γ, n) and (γ, total). The total γ-ray nuclear cross-section (γ, total) means the sum of (γ,n), (γ,p), (γ,2np), (γ,2n2p) ..etc. Using the reaction (γ,n) only is still convenient for practical applications in nuclear reactors [15, 16] and accelerator-driven-system (ADS) [17]. This is because (γ,n) represents a typical reaction that γ-ray can go through. A preferred method is to include all the possible γ-ray interactions with the nucleus [1], and this technique will require detailed knowledge about all the γ-ray inter-action cross-sections at various γ-ray energies. Two of the most important interactions are (γ,n) and (γ,np), then the total cross section will approximated by the integrated photo-neutron reaction cross section, found from the following Thomas-Reciche-Kuhn (TRK) summation relation, given as follows [15],

$$\sigma = \int_0^\infty \sigma(E)\, dE = 2\frac{(\pi e \hbar)^2}{mc}\left(\frac{N\,Z}{A}\right) \cong 60\left(\frac{N\,Z}{A}\right) \quad (\mathrm{MeV.mb}) \qquad \text{.................................} (1)$$

where: m is the mass of the nucleon, c is the speed of light, e is the unit charge and Z is the atomic number. .

The first moment of the integrated cross section, σ_{-1}, or the “Bremsstrahlung-Weighted Cross Section”, is also needed during the calc-ulations of γ-ray incineration. This cross section is given by [16],

$$\sigma_{-1} = \int_0^\infty \frac{\sigma(E)}{E} dE = \frac{4\pi^2 e^2}{3\hbar c}\frac{N\,Z}{A-1} < r^2 > \qquad \text{.............} (2)$$

where $<r^2>$ is the mean-square radius of the nuclear charge distribution. The total cross section of γ-rays with nuclei can use the Lorentz curve, [17], given in general form,

$$\sigma(E) = \frac{\sigma_m}{1 + \left[\frac{E^2 - E_m^2}{E^2\Gamma^2}\right]^2} \qquad \text{.................................} (3)$$

where σ_m, E_m and Γ are the parameters of the Lorentz curve for peak cross section, resonance energy and the full-width at half maximum, respectively.

Finally, to calculate the number of nuclei, N_{inc}, that will go through transmutation by γ-ray incineration, the shape of Lorentz line is considered and is obtained from the following relation,

$$N(t + \Delta t) = 1 + (-1)^i \left(\frac{(\Delta t)^i}{i!}\right)(\lambda_j + \sigma_j \phi)^i \qquad \text{.........} (4)$$

and λ_j is the j^{th} transition rate, σ_j is the cross section of the j^{th} reaction, and ϕ is the γ-ray flux (particles per unit area per second). In order to calculate the γ-ray incineration, the values of σ_{-1} were adopted in the present calculations.

This theory is sufficient to perform numerical calculations to find the ratio ζ for any radioactive isotope nucleus if one knows the specifications of the interaction, namely, the reaction cross-section and the specifications of the incident γ-ray.

The Unified (Empirical) Formula

Below we outline the method suggested by us in this paper. We aim to find a fitting equation to the results of incinerating various types of radioactive isotopes using γ-rays. The fit should, of course, depend on the types and specification of the selected nuclei plus the intensity of the incident rays, as well as incineration time.

The selected radioactive isotopes in the present paper are those used in refs. [1,11,14,18], namely, ^{87}Rb, ^{90}Sr, ^{93}Zr, ^{106}Ru, ^{123}Sn, ^{127}Te, ^{129}I, ^{137}Cs, ^{144}Ce, ^{144}Nd, ^{151}Sm, and ^{155}Eu. These nuclei range from intermediate to high mass numbers. This range gives a good confidence when one seeks numerical fitting of the results. These radioactive isotopes were basically selected because they are considered to form an important ratio from the fission reactor wastes. In ref.[1], these radioactive isotopes were all treated with γ-ray fluxes ranging from 10^{16} to 10^{20} γ/cm^2.sec. This is another good range that can be used with ease in the curve fitting. The data of the cross-section are also joined with those from EXFOR library [18] which provided more reliability for the present results. The data of the γ-ray incineration were all fitted to the form,

$$\zeta = \sum_{i=0}^{4} X_i^i t^i \quad \text{.......................................} (5)$$

or more specifically,

$$\zeta = X_0 + X_1 t + X_2^2 t^2 + X_3^3 t^3 + X_4^4 t^4 \quad \text{............} (6)$$

where X_i are numerical coefficients and t is the γ-ray irradiation time.

In this work, we suggest to use eq.(6) for the first time as a proper fit to pre-calculated data of γ-ray incineration. This is justified by the results obtained below.

Eq.(6) was fitted for the entire selected radioactive isotopes and for all the selected fluxes. This method, although relatively simple, but it reduces the effort of detailed calculations to determine the ratio ζ for these elements.

Basically, the fitting was to assume that initial values of the series, eq.(6), represents the natural decay of the nuclear reactor waste elements. We must mention that selecting only one term for the dependence of the time t can be efficient for this treatment. This will be true, however, if the problem was concerning one radioactive element where the reduction of the number of excited nuclei can be approximated to have linear relation with the flux of γ-ray. Since in this paper we chose a verity of radioactive isotopes, then we think that higher terms should be considered in order to have more reliable and accurate results. This complicates the present calculations; however, the results are quite satisfactory. In all cases, the coefficients were carefully selected so that the final results represent better than 98% of the calculated ζ ratios.

One can go though the detailed calculations of the ratio z as found in refs. [1, 4, 5, 12] and expand each term using Taylor series for example, and then collect the terms with similar powers to find a theoretical expression for the numerical coefficients of eq.(6). However, this will need extensive mathematical treatment. Instead, the data were plotted, and after that they were deduced and the numerical coefficients were found (see Section IV). This method, although includes detailed calculations, is made for one time then the resultant equations are ready to be applied on any practical need. The only requirement here is to know the time t of the expose to incident γ-rays.

It must be mentioned that this treatment implicitly includes the reaction cross-section evaluation for the target nuclei. As seen from the preceding Section III, the cross-section plays the leading rule in calculating the ratio ζ. This ratio in fact is a direct measure of the cross-section for (γ,n) and (γ,total) reactions. Therefore, eq.(6) actually can be used to treat the cross-section empirically.

Results and Discussions

In the present paper, the aim is to find an empirical formula that can be used for γ-ray incineration method of radioactive isotopes resulting from a nuclear reactor. This formula will highly simplify and reduce the numerical efforts needed to perform thee calculations. Therefore, this goal represents practical and useful application.

The radioactive elements chosen in this work include the following elements: ${}^{84}_{37}Rb_{40}$, ${}^{90}_{38}Sr_{52}$, ${}^{93}_{40}Zr_{53}$, ${}^{106}_{44}Ru_{62}$, ${}^{123}_{50}Sn_{73}$, ${}^{127}_{52}Te_{75}$, ${}^{129}_{53}I_{76}$, ${}^{137}_{55}Cs_{82}$, ${}^{144}_{58}Ce_{86}$, ${}^{144}_{60}Nd_{84}$, ${}^{151}_{62}Sm_{89}$ and ${}^{155}_{63}Eu_{92}$. This group actually represents a typical family of the fission nuclear reactor waste. The calculations assumed incident γ-ray on selected nuclei with five fluxes, namely, 10^{16}, 10^{17}, 10^{18}, 10^{19} and 10^{20} γ/cm^2.sec. Treating this system numerically will result in sixty equations of the form of eq.(6), and each equation contain five terms. This will provide good accuracy for the present method. The results are summarized in Tables (1 to 5).

From these tables, one can see that at low fluxes, i.e., 10^{16} and 10^{17} γ/cm^2.sec., the numerical coefficients fluctuate in sign from one element to another. However, as the γ-rays flux increases, the numerical coefficients tend to have more stable behavior; where as seen for γ-rays with fluxes 10^{18} to 10^{20} γ/cm^2.sec., the fluctuation is disappeared. This strongly indicates that the present treatment is appropriate for estimating the ratio ζ, and that the accuracy of this method is better as the γ-rays flux increases.

In all cases, the goodness (which describes the general accuracy of fitting) is better than ~ 99%. This strongly suggests that the selected form for the present work, eq.(6), is quite satisfactory. Convergence was worse when other forms were chosen, such as polynomial of third order or exponential dependence, ..etc. On the other hand, when choosing polynomials with higher degrees, e.g., of fifth or sixth orders or higher, the results were not better than those given in the Tables (1 to 5) above. So, the choice of fourth degree polynomial was wise for this purpose.

Furthermore, the cross-sections of these radioactive elements, namely, σ_{-1} and σ_{in}, taken from refs.[1, 18 and 19] are plotted as a functions of the atomic number Z as in Figs.(1 and 2), respectively. The selected elements were those providing three energies, namely, E=10, 20 and 30 MeV. The radioactive isotopes that do not have reported values at these energies were omitted from these curves. These limits of energy were chosen in order to have well-defined photonuclear cross-section that depend mainly on the excitation mechanism caused by the giant dipole resonance. It is known that at energies below ~30 MeV, the main and dominant excitation is due to the giant dipole resonance, while at energies above~150 MeV the excitation is due to pion creation [19]. Therefore, the selected energy range provide safe dependence on giant dipole resonance.

Table (1)
The numerical coefficients used in eq.(6) for γ-ray flux ϕ=10^{16} γ/cm^2.sec. R(%) is the accuracy of the fitting.

Element	x^4	x^3	x^2	x^1	x^0	R (%)
$^{84}_{37}Rb_{40}$	0.0003	- 0.0006	0.0003	0.0183	2E-06	100
$^{90}_{38}Sr_{52}$	0.3195	- 0.1367	- 1.2584	1.9433	0.0471	99.98
$^{93}_{40}Zr_{53}$	0.0009	- 0.0014	0.0006	0.0169	2E-06	100
$^{106}_{44}Ru_{62}$	0.0049	0.0269	- 0.2297	0.7075	0.0002	100
$^{123}_{50}Sn_{73}$	-0.2130	0.8818	- 1.6978	1.8777	0.0003	100
$^{127}_{52}Te_{75}$	-0.4550	1.6964	2.7436	2.4146	0.0006	100
$^{129}_{53}I_{76}$	0.00190	- 0.0033	0.0014	0.027	5E-06	100
$^{137}_{55}Cs_{82}$	0.00100	- 0.0018	- 0.0006	0.0569	6E-06	100
$^{144}_{58}Ce_{86}$	-0.01710	0.1208	- 0.4351	0.9435	3E-05	100
$^{144}_{60}Nd_{84}$	0.0025	- 0.0055	0.0032	0.0369	2E-05	100
$^{151}_{62}Sm_{89}$	-0.0011	0.0026	- 0.0019	0.0522	- 3E-06	100
$^{155}_{63}Eu_{92}$	0.0068	- 0.0039	- 0.0791	0.4220	0.0001	100

Table (2)
The numerical coefficients used in eq.(6) for γ-ray flux ϕ=10^{17} γ/cm^2.sec.

Element	x^4	x^3	x^2	x^1	x^0	R (%)
$^{84}_{37}Rb_{40}$	0.0088	-0.0187	-0.003	0.1805	0.0001	100
$^{90}_{38}Sr_{52}$	0.0001	0.0015	- 0.0236	0.2200	2.0E-05	100
$^{93}_{40}Zr_{53}$	-0.0164	0.0498	- 0.0593	0.1832	- 0.0004	99.99
$^{106}_{44}Ru_{62}$	-0.0187	0.1280	- 0.4521	0.9609	4.0E-05	100
$^{123}_{50}Sn_{73}$	-0.3130	1.2381	- 2.1797	2.1378	0.0005	100
$^{127}_{52}Te_{75}$	-0.6731	2.3457	- 3.4675	2.7311	0.0013	100
$^{129}_{53}I_{76}$	-0.0150	0.0326	- 0.0551	0.2758	- 0.0001	100
$^{137}_{55}Cs_{82}$	-0.0288	0.0710	- 0.1116	0.3738	- 0.0003	100
$^{144}_{58}Ce_{86}$	-0.0566	0.3205	- 0.8571	1.3309	0.0001	100
$^{144}_{60}Nd_{84}$	0.0067	- 0.0064	- 0.0607	0.3763	0.0001	100
$^{151}_{62}Sm_{89}$	-0.0045	0.0128	- 0.0914	0.4409	0.0001	100
$^{155}_{63}Eu_{92}$	-0.0189	0.0978	- 0.3334	0.8068	- 0.0001	100
$^{155}_{63}Eu_{92}$	-2.2866	6.7863	- 7.7832	4.2632	0.0080	99.99

Table (3)
The numerical coefficients used in eq.(6) for γ-ray flux $\phi=10^{18}$ γ/cm^2.sec.

Element	x^4	x^3	x^2	x^1	x^0	R (%)
$^{84}_{37}Rb_{40}$	-0.1994	0.8267	- 1.6127	1.8249	0.0004	100
$^{90}_{38}Sr_{52}$	-0.2188	0.9444	- 1.8225	1.9583	0.0005	100
$^{93}_{40}Zr_{53}$	-0.1347	0.6398	- 1.3795	1.6920	0.0003	100
$^{106}_{44}Ru_{62}$	-1.1744	3.8067	- 5.002	3.3341	0.0028	100
$^{123}_{50}Sn_{73}$	-2.4963	7.3170	- 8.2444	4.4028	0.0092	99.99
$^{127}_{52}Te_{75}$	-4.0057	11.074	- 11.387	5.2912	0.0196	99.95
$^{129}_{53}I_{76}$	-0.6058	2.1703	- 3.2987	2.6664	0.0015	100
$^{137}_{55}Cs_{82}$	-1.1157	3.6617	- 4.8775	3.2948	0.0029	100
$^{144}_{58}Ce_{86}$	-3.0119	8.6261	- 9.3700	4.7351	0.0124	99.98
$^{144}_{60}Nd_{84}$	-1.4911	4.6794	- 5.8527	3.6382	0.0022	100
$^{151}_{62}Sm_{89}$	-2.0510	6.1614	- 7.2166	4.0848	0.0065	99.99

Table (4)
The numerical coefficients used in eq.(6) for γ-ray flux $\phi=10^{19}$ γ/cm^2.sec.

Element	x^4	x^3	x^2	x^1	x^0	R (%)
$^{84}_{37}Rb_{40}$	-2.0E-07	2.0E-05	- 0.0013	0.0511	- 0.0001	100
$^{90}_{38}Sr_{52}$	-0.0008	0.0157	- 0.1271	0.5291	0.0021	100
$^{93}_{40}Zr_{53}$	6.0E-08	1.0E-05	- 0.0010	0.0469	0.0004	100
$^{106}_{44}Ru_{62}$	-5.0E-07	6.0E-05	- 0.0029	0.0779	0.0009	100
$^{123}_{50}Sn_{73}$	-8.0E-07	8.0E-05	- 0.0034	0.0852	0.0013	100
$^{127}_{52}Te_{75}$	-1.0E-06	0.0001	- 0.0050	0.1045	0.0070	99.97
$^{129}_{53}I_{76}$	-5.0E-07	6.0E-05	- 0.0027	0.0752	0.0004	100
$^{137}_{55}Cs_{82}$	-1.0E-06	1.0E-04	- 0.0041	0.0931	0.0013	100
$^{144}_{58}Ce_{86}$	-2.0E-06	0.0002	- 0.0066	0.1198	0.0035	100
$^{144}_{60}Nd_{84}$	-2.0E-06	0.0002	- 0.0057	0.1113	- 0.0272	99.82
$^{151}_{62}Sm_{89}$	-2.0E-06	0.0002	- 0.0065	0.1188	0.0018	99.99
$^{155}_{63}Eu_{92}$	-2.0E-06	0.0002	- 0.0060	0.1148	0.0027	100

Table (5)
The numerical coefficients used in eq.(6) for γ-ray flux ϕ=10^{20} γ/cm^2.sec.

Element	x^4	x^3	x^2	x^1	x^0	R (%)
$^{84}_{37}Rb_{40}$	-0.0009	0.0158	- 0.1207	0.5046	0.0008	100
$^{90}_{38}Sr_{52}$	-0.0008	0.0157	- 0.1271	0.5291	0.0021	100
$^{93}_{40}Zr_{53}$	-0.0007	0.0125	- 0.1031	0.4674	0.0009	100
$^{106}_{44}Ru_{62}$	-0.0025	0.0384	- 0.2383	0.7341	0.0049	100
$^{123}_{50}Sn_{73}$	-0.0028	0.0433	- 0.2614	0.7729	0.0056	100
$^{127}_{52}Te_{75}$	-0.0048	0.0682	- 0.3709	0.9406	0.0129	99.98
$^{129}_{53}I_{76}$	-0.0024	0.0371	- 0.2322	0.7237	0.0043	100
$^{137}_{55}Cs_{82}$	-0.0039	0.0568	- 0.3226	0.8698	0.0094	99.99
$^{144}_{58}Ce_{86}$	-0.0062	0.0869	- 0.4499	1.0539	0.0138	99.98
$^{144}_{60}Nd_{84}$	-0.0036	0.0580	- 0.3459	0.9317	0.0240	100
$^{151}_{62}Sm_{89}$	-0.0035	0.0590	- 0.3461	0.9320	0.0239	100
$^{155}_{63}Eu_{92}$	-0.0059	0.0830	- 0.4332	1.0303	0.0129	99.99

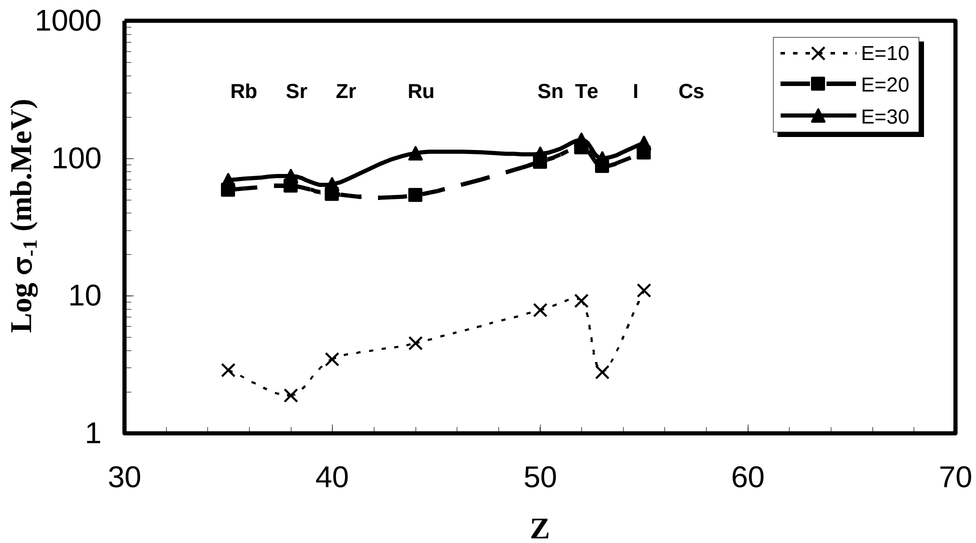

Fig.(1) : The relation between Log(σ_{-1}) and Z for the selected isotopes. Data are taken from Refs.[1, 18 and 19].

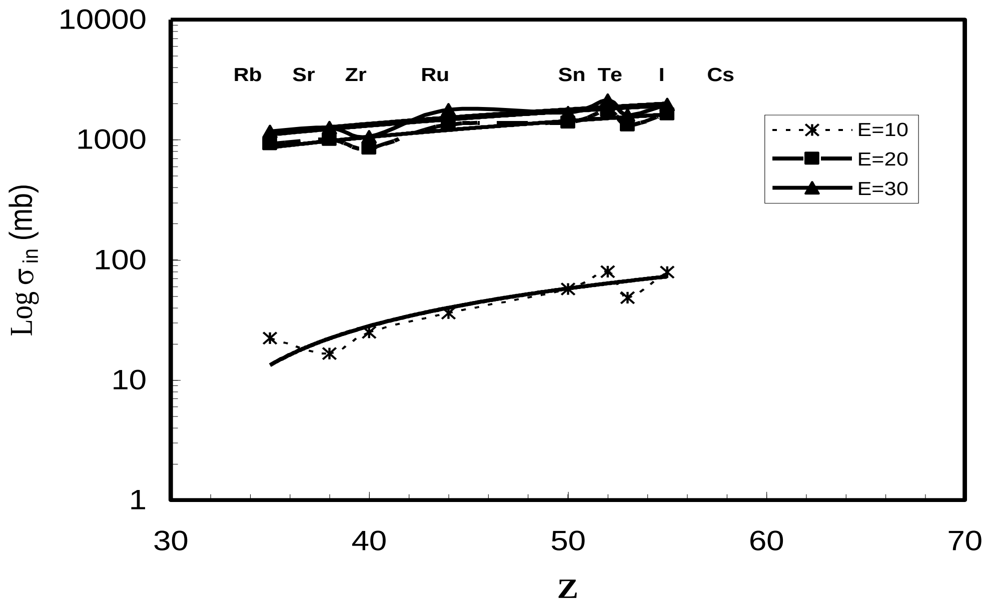

Fig.(2) : The relation between Log(σ_{in}) and Z for the selected isotopes. The values of fitting are: for E=10 MeV: Log(σ_{in}) = 2.9671Z - 90.597 with R^2 = 0.8227; for E=20 MeV: Log(σ_{in}) = 38.097Z-471.1 with R^2 = 0.7774; and for E=30 MeV: Log(σ_{in}) = 42.108Z-350.07 with R^2 = 0.6906. Data are taken from Refs.[1, 18 and 19].

The values of σ_{-1} as seen from Fig.(1) are fluctuating with Z so that it was inconvenient for the selected range to have accurate fitting. However, the values of σ_{in}, shown in Fig.(2), were much smoothly varying with Z so that it was possible to have reasonable fit to these data. The fit was chosen to be linear with the form: Log(σ_{in})=$aZ+b$ where a and b are fitting parameters. The parameters of this linear fitting are indicated in the caption of Fig.(2).

Conclusions

In the present work, the photonuclear cross-section was treated systematically in order to obtain an empirical mathematical formula. Eq.(5) was suggested to treat γ-ray incineration by us and the goal was to have this formula applied, so that numerical calculations are greatly simplified. Testing this method with a pre-calculated incineration ratio, ζ, for twelve radioactive isotopes and five different fluxes of γ-rays, showed that the present method is practical and simple providing useful application. The fitting procedure assumed a formula given in eq.(6). A resultant of sixty equations were treated, and the results were given in tabulated form. The important conclusions of suggesting this formula in this work are summarized as:

1- Only fourth order polynomial provides enough accuracy for finding the incineration ratio, ζ, for a reasonable range of radioactive isotopes.
2- As the flux intensity increases, the fitted equations based on eq.(5 or 6) have better agreement with pre-calculated data and thus one concludes that the present treatment will be more accurate for high γ-ray fluxes.
3- The equation suggested here, eq.(6), can be also obtained from analytical derivation of the expansion of individual components of the mathematical components described in eqs.(1-6). However, fitting the results directly will highly simplify the task with reliable accuracy.
4- Furthermore, the reaction cross-sections, σ_{-1} and σ_{in}, were plotted against the atomic number Z and it was shown that as the energy of γ-rays increase, the cross-sections become less dependent on Z.
5- One also concludes that investigation of other radioactive isotopes is quite important in order to have better confidence to the suggested form, eq.(6), and the results of Tables (1 to 5).

6- Finally, it is shown that data evaluation can also be applied successfully to γ-ray incineration, which saves time and effort. The present calculations are assumed as a powerful example of the importance of this technique in treating and management of the radioactive wastes produced from fission reactors.

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الخلاصة

تم تحليل مساحات المقطع العرضي للتفاعل الفوتو–نووي (photo-nuclear cross-section) لأثنا عشر عنصر مشع وقد تم التوصل إلى ربط هذه المساحات معا بمعادلة وضعية. إن الغرض من اقتراح مثل هذه المعالجة الرياضية هو لتوفير علاقة بسيطة تستخدم للأغراض العملية في حسابات معدلات الإحراق النووي، حيث أن المعادلة الحالية توفر الجهد اللازم لأجراء الحسابات بصورة كبيرة. فإن الطريقة المتبعة في هذا البحث تتسم بالمباشرة والبساطة. بعد حساب معدلات الإحراق للنوى المختارة، تمت معاملة النتائج رياضيا لتوضع في إطار معادلة رياضية متعددة الحدود تأخذ بنظر الاعتبار خصائص أشعة كاما الساقطة من طاقة وفيض، وكذلك خصائص النوى الهدف. عند تطبيق هذه المعادلة وجد بأنها معادلة كفوءة وذات دقة مقبولة. إن النوى التي أختيرت للبحث الحالي قد اختيرت كمجموعة من نواتج التفاعل النووي الانشطاري. أما الفيض المفترض فقد أختير لعدد من القيم مما يمثل تطبيقا مناسبا للأغراض العملية والحسابات. لهذا فهناك ستون معادلة تمت معالجتها في هذا البحث كل معادلة تتألف من خمسة حدود. إن هذه المعالجة ستوفر دقة كافية لاعتماد الطريقة الحالية. تمت مقارنة النتائج الحالية بصورة وافية مع نتائج سابقة وأتضح بأن نتائج البحث الحالي جيدة ومرضية.